Fiscal Stimulus with Imperfect Expectations: Spending vs. Tax Policy*

Riccardo Bianchi-Vimercati† Martin Eichenbaum‡ Joao Guerreiro§

January 2024

Abstract

This paper addresses the question: how sensitive is the power of fiscal policy at the Zero Lower Bound (ZLB) to the assumption of rational expectations? We do so through the lens of a standard New Keynesian model in which people are dynamic level-k thinkers. Our analysis weakens the case for using government spending to stabilize the economy when the ZLB binds. The less sophisticated people are, the smaller the government-spending multiplier is. Our analysis strengthens the case for using tax policy to stabilize output when the ZLB is binding. The power of tax policy to stabilize the economy during the ZLB period is essentially undiminished when agents do not have rational expectations. Our results are robust to whether or not Ricardian equivalence holds. Finally, we show that the way in which tax policy is communicated is critical to its effectiveness.

*We thank Bence Bardóczy, Lawrence Christiano, Benjamin Johannsen, Guido Lorenzoni, Pooya Molavi, Matthew Rognlie, Miguel Santana, Thomas Sargent, Pedro Teles, and Michael Woodford for helpful comments. We also thank George-Marios Angeletos, John Leahy, Elisa Rubbo, and Dejanir Silva for excellent discussions that helped improve the paper. This paper is a thoroughly revised version of a previous paper titled "Fiscal Policy at the Zero Lower Bound without Rational Expectations".

†PIMCO; riccardo.bianchivimercati@gmail.com.

‡Northwestern University and NBER; eich@northwestern.edu.

§UCLA; jguerreiro@econ.ucla.edu.

1 Introduction

The *Zero Lower Bound* (ZLB) on interest rate significantly constrains conventional monetary policy.[1] A large literature emphasizes that fiscal policy is particularly useful for stabilizing the aggregate economy when the ZLB binds. According to this literature, the government-spending multiplier is significantly higher than under normal circumstances, see, e.g., Christiano et al. (2011), Eggertsson (2011), and Woodford (2011).[2] In addition, appropriately designed tax policy can mimic the effect of conventional monetary policy on aggregate demand, see Feldstein (2003) and Correia et al. (2013).

In this paper, we address the question: how sensitive is the power of fiscal policy at the ZLB to the assumption of rational expectations? According to our analysis, the efficacy of government spending is quite sensitive to that assumption. Under plausible assumptions, the less sophisticated people are the smaller the multiplier. In contrast, tax policy at the ZLB is less sensitive to deviations from rational expectations. Indeed, in our analysis, tax policy continues to be able to support the flexible-price allocation even when agents are boundedly rational and the ZLB is binding.

We reach these conclusions using a simple sticky-wage, representative-agent New Keynesian (NK) model. As in Correia et al. (2013), we assume that there is an unanticipated shock to people's discount factor at time zero that lasts for T periods. As a result, the subjective discount rate falls below zero, driving the nominal interest rate to the ZLB.

In our benchmark model, wages are fully rigid, and the price level is constant. Since there is no inflation, this model is useful to highlight the effect of bounded rationality on the income effects of government spending and the direct relative price effects of tax policy. We also develop an extended model that allows for time-varying prices and wages to incorporate inflation effects into our analysis. Those effects strongly reinforce our conclusions about the relative sensitivity of government spending and tax policy as effective policy tools when agents are boundedly rational.

We depart from rational expectations by assuming people form beliefs about future endogenous variables via *dynamic-level-k thinking*. As in standard level-k thinking models, individuals understand the economy's structure. However, they are limited in their ability to predict the behavior of other people and, as a result, the time path for the endogenous variables in the economy (e.g., aggregate output). Starting from an initial belief for the least sophisticated agents, individuals update their expectations about changes in

[1] We understand that interest rates can be negative. But there is some effective lower bound on interest rates. To facilitate comparisons with the literature we work with the ZLB, with the understanding that our key results would obtain when the effective lower interest rate is binding.

[2] See also the analyses in Werning (2011) and Farhi and Werning (2016).

the future based on a finite reasoning process about other people's behavior, involving k iterations. In standard level-k models, people do not improve their ability to predict other peoples' behavior over time, so their 'prediction' errors do not get smaller over time. This feature raises potential issues about the robustness of the standard level-k models' predictions for the dynamic effects of a shock. To assess the robustness issue, we proceed as in Iovino and Sergeyev (2018), and modify the standard level-k thinking model to allow cognitive sophistication to grow over time. This dynamic extension enables us to investigate how the power of fiscal policy depends on both the level of people's cognitive sophistication and how quickly they learn over time.

In Section 3 we use the benchmark model to evaluate the effects of increased government spending and time-varying consumption taxes when the ZLB is binding. Consistent with earlier work by Woodford and Xie (2019) and Farhi, Petri, and Werning (2020), we establish that the size of the government-spending multiplier depends on agents' level of cognitive sophistication (Proposition 1).[3] The intuition is as follows. Other things equal, higher government spending leads to increased labor demand and higher labor income. The latter effect implies an increase in consumer demand. Under reasonable conditions, the less sophisticated people are, the less they take into account the positive general-equilibrium effects of higher spending. So, aside from special cases, lower levels of cognitive sophistication imply lower values for the government-spending multiplier.

We then turn to an analysis of tax policy at the ZLB. Correia, Farhi, Nicolini, and Teles (2013) show that tax policy is a powerful tool for stimulating demand at the ZLB when people have rational expectations. Following these authors, we consider a policy of lowering an *ad-valorem* tax on consumption as soon as the ZLB binds and then slowly raising that tax to its pre-shock level. This policy has the effect of putting consumption "on-sale" while the ZLB binds. We show that there always exists a time path for consumption taxes that completely stabilizes the economy at its pre-shock level, i.e., it supports the flexible-price allocation.

Suppose the least sophisticated people think that aggregate output will remain at its pre-shock level. Then, the path for consumption taxes that supports the flexible-price allocation is the same regardless of how cognitively sophisticated people are. Critically, the flexible-price allocation is the same as the pre-shock steady state of the economy. So, under the tax policy that supports this allocation, people's initial beliefs are self-confirming, i.e., they do not make any expectational errors. In this sense, the efficacy of this policy

[3]As discussed in the related literature section below, Angeletos and Lian (2018) obtain a similar result stemming from the assumption that people do not share common information about future government actions.

does not exploit people's lack of sophistication.

These results show that tax policy is a powerful and robust way to stabilize the economy when the ZLB binds. The basic intuition is as follows. Suppose the government announces a time path for current and future tax rates. Then, people incorporate these rates into their personal consumption-savings decision and substitute consumption to dates when the tax rate is lower. This basic force is operative regardless of any general-equilibrium (GE) considerations, i.e., people do not need to calculate the GE effects of the announced tax rate to adjust their personal consumption decision to the tax rates. So, the policy boosts consumption demand and supports the flexible-price allocation when the ZLB binds, even if people are very unsophisticated.

Recall that our benchmark model assumes that the price level is constant. This assumption does not hold in more general versions of the NK model. In those models, the impact of government spending on inflation and the real interest rate plays an important role in magnifying the size of the government-spending multiplier. When the ZLB is binding, increases in government spending lead to upward pressure on prices, which lowers the real interest rate and boosts the demand for consumption. To the extent that people do not understand these equilibrium effects, the size of the government-spending multiplier should be smaller (see Angeletos and Lian, 2018 and Farhi et al., 2020). It is not obvious how a variable price level affects the efficacy of tax policy under bounded rationality.

To study these issues, we redo our analysis in a framework where prices and wages are not constant. In section 4, we assume that nominal wages are set subject to Calvo-style frictions as in Erceg, Henderson, and Levin (2000).[4] Since wages are not constant, neither is the price level. We show numerically that the key results of Proposition 1 are stronger because they hold for a wider set of model parameter values. The reason is as follows. The simple model focuses on the income effects of a shock to government spending and abstracts from the impact of government spending on inflation. The extended model allows for both effects. As it turns out, expected inflation effects powerfully reinforce our results. There are some special cases in the fixed-price model where the efficacy of government spending is unaffected by bounded rationality. These cases are eliminated once we allow for inflation effects.

Turning to tax policy, we suppose the government can impose time-varying tax rates on consumption and labor income. With this proviso, we show that it is still true that an appropriately designed tax policy can support the flexible-price allocation. As in the

[4] Appendix D redoes the analysis of section 4 under the assumption that nominal prices, rather than nominal wages, are subject to Calvo-style frictions.

benchmark economy, the policy that supports the flexible-price allocation does not depend on the level of cognitive sophistication, provided the least sophisticated agents continue to expect the economy to remain at steady state. We also use the model to study the implications of two alternative forms of tax policy: (1) a policy that does not condition on the "right beliefs", and (2) a policy in which the government changes consumption tax rates, but leaves labor taxes unchanged. We show that even under these limitations, tax policy still has a powerful stabilizing force.

With bounded rationality, the way that policy is communicated matters. In the results discussed above, we assumed that the government announced a sequence of consumption tax rates that will apply during the ZLB. Suppose instead that the government announces a rule according to which tax rates are set as a function of the output gap. We show that this form of communication substantially degrades the efficacy of tax rate policy. The intuition for this result is as follows. When the policy is communicated as a rule, individuals must forecast the future level of output to predict what tax rates will be. When individuals are limited in their ability to compute GE effects, they will also be limited in their ability to forecast future tax rates. This limitation translates into a lower efficacy of tax policy in stimulating demand.

In our benchmark and extended models, Ricardian equivalence holds. A natural question is whether our results depend on that feature. To address this question, we break that equivalence by assuming that the least sophisticated people in the economy don't understand the government budget constraint. More sophisticated people understand that constraint, but their views about the variables in that constraint may not coincide with their realized values. The net result is that government debt affects aggregate demand and output in this version of the model. We show quantitatively that our results about the relative efficacy of spending versus taxes are robust to this failure of Ricardian equivalence.

Taken together, our results *weaken* the case for using government spending to stabilize the economy when the ZLB binds. At the same time, our results *strengthen* the case for using tax policy to stabilize output when the ZLB is binding. The power of tax policy to stabilize the economy during the ZLB period is essentially undiminished when agents do not have rational expectations.

The paper is organized as follows. Section 2 discusses the literature related to our paper. Section 3 describes our benchmark NK model with level-k thinking. Section 3.1 analyzes the effects of government spending and the implications of bounded rationality for the government-spending multiplier in the benchmark model. Section 3.2 presents our results on consumption-tax policy in the benchmark model. Section 4 considers the extended model with time-varying wages and prices. Section 5 presents our results on policy communication, and Section 6 presents our results in the model in which Ricardian equivalence fails. Finally, section 7 contains concluding remarks. The proofs for all propositions are in the appendix.

2 Related Literature

In this section we discuss the related theoretical and empirical literature.

2.1 Related theoretical literature

This paper belongs to a growing literature that studies the implications of deviations from rational expectations for the effectiveness of macroeconomic policy. The form of bounded rationality that we consider is based on level-k thinking models originally studied by Nagel (1995) and Stahl and Wilson (1995). Farhi and Werning (2019) use this approach to study how deviations from rational expectations impact the efficacy of forward guidance. García-Schmidt and Woodford (2019) develop a closely related form of deviation from rational expectations, which they refer to as reflective expectations. They apply this form of expectations to study the impact of forward guidance and interest rate pegs on economic activity. Under both level-k thinking and reflective expectations, individuals have a limited ability to understand the general-equilibrium consequences of monetary policy.[5] García-Schmidt and Woodford (2019) and Farhi and Werning (2019) show that this effect limits the power of forward guidance and mitigates some anomalous implications of this policy under rational expectations.[6] Iovino and Sergeyev (2018) apply level-k thinking and reflective expectations to analyze the effects of quantitative easing.

Angeletos and Lian (2017) initially developed the idea that the lack of common knowledge attenuates general-equilibrium effects. Angeletos and Lian (2018) study a rational-expectations environment in which people do not have common knowledge about the relevant news. They show that the absence of common knowledge dampens the general-equilibrium effects of news and the size of the government-spending multiplier. We obtain a similar result about government spending when people have complete information

[5]Similar ideas are captured by the *calculation equilibrium* and *internal rationality* approach to bounded rationality discussed in Evans and Ramey (1992) and Adam and Marcet (2011), respectively.

[6]Similar results are derived in Woodford (2018) in a model in which individuals can only make contingent plans up to a finite number of future periods, i.e., they have *limited foresight*, Gabaix (2020) in a model in which individuals are inattentive to the interest rate, Angeletos and Lian (2018) in a model with informational frictions and imperfect common knowledge, and in Wiederholt (2015) in a model with sticky expectations.

about the shocks but are limited in their ability to forecast the GE consequences of policies. While the mechanism is different, this limitation attenuates the general-equilibrium effects of those shocks as in Angeletos and Lian (2018). In our paper, we adopt a dynamic extension of standard level-k thinking which allows to assess the sensitivity of our results to learning. In that sense, our framework is related to the literature on incomplete information when people accumulate more signals over time, e.g., Woodford (2001) and Angeletos and Huo (2018). For more recent work in incomplete-market economies see Auclert, Rognlie, and Straub (2020), Guerreiro (2022), and Gallegos (2023).

Woodford and Xie (2019) and Farhi et al. (2020) analyzed the consequences of bounded rationality for the size of fiscal multipliers. Following the approach developed by Woodford (2018), Woodford and Xie (2019) assumes that individuals can only plan for a finite number of periods but are fully rational within the planning horizon. They show that this behavioral bias may limit the size of the government-spending multiplier at the ZLB because the stimulus effect of future government spending on current output is zero if it occurs after the relevant planning horizon. Instead, we work with a model in which individuals have an infinite planning horizon but a limited capacity to understand the GE effects of different policies.

Our analysis is closest to Farhi et al. (2020), who also assume that individuals are level-k thinkers. Their primary focus is on the *fiscal-multiplier puzzle* discussed in Farhi and Werning (2016), who note that, in standard representative-agent NK economies, the government-spending multiplier grows explosively as government spending is back-loaded. The key force underlying this result is that back-loaded spending generates more inflation, which lowers the real interest rate when the ZLB is binding. Farhi et al. (2020) examine the fiscal multiplier puzzle in both representative-agent and heterogeneous agents NK models with level-k thinking. They show that the government-spending multiplier is generally lower, the lower the level of cognitive sophistication in the economy and that models with level-k thinking do not exhibit the fiscal-multiplier puzzle.

An important distinction between our paper and the literature just cited is that we study how deviations from rational expectations affect the efficacy of tax policy versus government spending when the ZLB is binding. In addition, we analyze how communication affects the power of tax policy at the ZLB.

Angeletos and Sastry (2020) analyze the implications of policy communication when agents have a particular form of bounded rationality. They analyze whether policy communication should focus on instruments (interest rates) or targets (unemployment). They show that the answer to this question depends on the relative importance of partial versus general-equilibrium effects of a given policy. Their substantive application is forward guidance, while we focus on tax policy. In addition, we look at rules versus instrument settings rather than their focus on instruments versus targets.

We develop a version of our model where Ricardian equivalence fails because individuals do not understand the government budget constraints. In that version of the model, government debt and transfers affect aggregate demand and equilibrium output even in the absence of liquidity constraints. Woodford and Xie (2022) shows that uniform lump-sum transfers can be a powerful stabilization tool in a model in which Ricardian equivalence fails due to bounded rationality.[7] Eusepi and Preston (2018) develop a theory of inflation based on fiscal policy that assumes that people do not understand the government's budget constraint.

2.2 Related empirical literature

A large empirical literature documents deviations from standard notions of rationality. Of direct relevance is experiment-based evidence on the level of people's sophistication. Crawford et al. (2013) review this literature and argue that the experimental evidence is consistent with the distribution of cognitive levels being very concentrated at low levels of k. For example, Camerer et al. (2004) concludes that a substantial fraction of people are well characterized as having levels of k between 0 and 2 and that the median level of k is between 1 and 2.[8] In our model, these levels of k generate very different behavior than rational expectations. In a non-experimental setting, Iovino and Sergeyev (2018) estimate the sophistication level of professional forecasters by looking at survey data about mortgage rates and their response to quantitative easing. They find that 86 percent of forecasters in their data are level-1 thinkers.

There is an extensive literature that characterizes people's expectations of macro variables based on survey evidence, see, e.g., Coibion and Gorodnichenko (2012, 2015), Bordalo, Gennaioli, and Shleifer (2012), and Angeletos, Huo, and Sastry (2021). A key conclusion from this literature is that on average, people's beliefs about macroeconomic aggregates like inflation and real GDP growth tend to under-react to changes in macro fundamentals relative to the rational-expectations benchmark. Our model is consistent with this finding.

Our conclusions about the efficacy of tax policy receive strong support from recent empirical work. D'Acunto, Hoang, and Weber (2020) estimate the impact of forward

[7] Wolf (2021) also considers a general model in which Ricardian Equivalence fails and shows that aggregate allocations that are implementable with interest rate policy can be equivalently implemented with uniform cash transfers.

[8] See also Stahl and Wilson (1995), Ho et al. (1998), Bosch-Domenech et al. (2002), among others.

guidance and consumption tax policies on household inflation expectations and spending. They show that forward guidance policies had little effect on household expectations and behavior. However, consumption tax policies like those that we describe are effective at raising household spending. These empirical results are consistent with our conclusion that tax policy can be a powerful stabilization tool, even if people are not as sophisticated as in the rational-expectations paradigm. Bachmann et al. (2021) provides strong evidence of the efficacy of a temporary VAT cut in Germany when the ZLB was binding. They find that (1) most households were aware of the policy change and (2) that people with different degrees of financial literacy responded roughly the same way to the tax cut. On this basis, they conclude that the tax cut successfully stimulated aggregate consumption spending because of its simplicity and salience.

3 A simple model

In this section, we describe a simple model to highlight the key features of dynamic level k thinking in the context of an NK model. Sections 3.1 and 3.2 analyze the effect of government spending and tax policy, respectively.

Consider a simple NK economy with fully rigid wages. Without loss of generality, we normalize nominal wages to one, $W_t = 1$. There is a continuum of identical households, each of which has preferences over sequences of consumption, C_t, and labor, N_t, are given by:

$$\sum_{t=0}^{\infty} \beta^t \xi_t \left[u\left(C_t\right) - v\left(N_t\right) \right], \tag{3.1}$$

where $u(C) = C^{1-\sigma^{-1}} / \left(1-\sigma^{-1}\right)$ and $v(N) = N^{1+\varphi^{-1}} / \left(1+\varphi^{-1}\right)$. As in Correia et al. (2013), we assume that the steady-state subjective discount factor $\beta \in (0,1)$ is perturbed by a *discount-factor shock:*

$$\xi_t = e^{-\chi(T-t)}, \tag{3.2}$$

for $t = 0, 1, ..., T$ and $\xi_t = 1$ for $t \geq T$. This assumption implies that the household's subjective discount rate between periods t and $t+1$ is

$$\log \frac{\xi_t}{\beta \xi_{t+1}} = \rho - \chi, \quad t \leq T-1,$$

where $\rho \equiv \log \beta^{-1}$. We assume that the shock satisfies $\chi > \rho$, so that the subjective discount rate is negative for $t \leq T-1$.

For simplicity, we assume that the production function is linear in labor, $Y_t = N_t$. The goods market clearing condition is

$$C_t + G_t = Y_t, \tag{3.3}$$

where G_t denotes government spending. We also assume that steady-state government spending is zero.

In this simple economy, the first-best (flexible-price) allocation is

$$Y_t = C_t = N_t = 1.$$

Note that the discount-rate shock does not affect aggregate consumption or production in this allocation. However, implementing this allocation requires a negative real interest rate. So that allocation cannot be achieved using only conventional monetary policy.

Firms Firms are perfectly competitive and maximize profits. An interior solution for the firms' problem requires that $W_t = P_t$. Because wages are fully rigid, there is no inflation:

$$\frac{P_{t+1}}{P_t} = \frac{W_{t+1}}{W_t} = 1. \tag{3.4}$$

Monetary and fiscal policies The monetary authority controls the nominal interest rate, R_t. During $t \leq T-1$ the nominal interest rate is at the *ZLB*,

$$R_t = 1, \tag{3.5}$$

and then goes back to its pre-shock level: $R_t = \beta^{-1}$ for $t = T, T+1, ...$

The fiscal authority sets government spending G_t, consumption taxes τ_t^c, and lump-sum taxes T_t. The government's intertemporal budget constraint is given by:

$$\sum_{h=0}^{\infty} Q_{t,t+h} G_{t+h} + R_{t-1} B_t = \sum_{h=0}^{\infty} Q_{t,t+h} \left[\tau_{t+s}^c C_{t+h} + T_{t+h}\right], \quad \forall t \geq 0. \tag{3.6}$$

Here $Q_{t,t+h}$ is the discount factor between t and $t+h$,

$$Q_{t,t+h} \equiv \prod_{m=t}^{t+h-1} R_m^{-1}$$

for $s \geq 1$, $Q_{t,t} \equiv 1$.

Households and expectations The household has perfect foresight regarding exogenous variables and correctly anticipates the path for the discount rate shock, ξ_t. We assume the government announces sequences of nominal interest rates, R_t, government spending, G_t, and consumption taxes, τ_t^c. The fact that the household correctly anticipates the path for these policy variables is consistent with the idea that they see and understand policy announcements.[9] However, the household is limited in its ability to fully predict the equilibrium changes due to these policies. We denote by $F_t [Y_{t+h}]$ and $F_t [T_{t+h}]$ the household's time-t beliefs about the time $t+h$ values of output and lump-sum taxes, respectively. For simplicity, we assume that these beliefs are what people think those variables will be with probability one.

Our goal in this section is to transparently highlight the consequences of failures in predicting the general-equilibrium implications of fiscal policies for their effectiveness. For clarity of exposition, we assume that given their beliefs for output, the household's expectations for lump-sum taxes are consistent with the government's inter-temporal budget. Formally, we assume that household beliefs for $F_t [T_{t+h}]$ satisfy:

$$\sum_{h\geq 0} Q_{t,t+h} F_t [T_{t+h}] = \sum_{h\geq 0} Q_{t,t+h} \left[G_{t+h} - \tau_{t+h}^c (F_t [Y_{t+h}] - G_{t+h})\right] + R_{t-1} B_t. \tag{3.7}$$

This expression implies that households don't care about the timing of lump-sum taxes. However, they do know that the present value of their taxes will change when there is a change in government spending. However, because they may be incorrect about how changes in G affect changes in Y, they may be wrong about how much the LHS of (3.7) changes when G changes. In section 6, we relax the assumptions that people understand the government's intertemporal budget constraint.

The household enters period t with financial assets B_t earning the interest rate R_{t-1}. As in Farhi and Werning (2019), we assume the household knows its contemporaneous income Y_t and taxes T_t.[10] When solving its dynamic consumption-savings problem, the household maximizes its perceived utility which is evaluated based on today's consumption, C_t, and on its plans for future consumption $\widetilde{C}_{t+h}$ for $h = 1, 2, ...$ To the extent that the household makes mistakes in predicting its future disposable income, actual consumption will deviate from planned consumption.

[9] Bachmann et al. (2021) study an unexpected and temporary VAT cut in Germany that occurred in the second half of 2020. They find that most households were aware of the tax cut, which supports our assumptions.

[10] Our results go through if we assume that the household does not see contemporaneous Y_t and C_t.

The household solves the problem:

$$\max_{\widetilde{C}_{t+s}} \sum_{h\geq 0} \beta^h \xi_{t+h} \frac{\widetilde{C}_{t+h}^{1-\sigma^{-1}}}{1-\sigma^{-1}}, \quad \text{subject to}$$

$$\sum_{h\geq 0} Q_{t,t+h} \left(1+\tau^c_{t+h}\right) \widetilde{C}_{t+h} = \sum_{s\geq 0} Q_{t,t+h} \left\{F_t\left[Y_{t+h}\right] - F_t\left[T_{t+h}\right]\right\} + R_{t-1}B_t.$$

Since wages are rigid, equilibrium output and labor are demand determined. The solution to the household's problem implies C_t satisfies

$$C_t = \frac{Y_t - T_t + \sum_{h\geq 1} Q_{t,t+h} \left\{F_t\left[Y_{t+h}\right] - F_t\left[T_{t+h}\right]\right\} + R_{t-1}B_t}{\left(1+\tau^c_t\right)\left[1+\sum_{h\geq 1}\left(\beta^h \frac{\xi_{t+h}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+h}\frac{1+\tau^c_{t+h}}{1+\tau^c_t}\right]^{1-\sigma}\right]}.$$

Replacing the present value of lump-sum taxes using equation (3.7), we obtain:

$$C_t = \frac{\left(Y_t - G_t\right) + \sum_{h\geq 1} Q_{t,t+h}\frac{1+\tau^c_{t+h}}{1+\tau^c_t}\left[F_t\left[Y_{t+h}\right] - G_{t+h}\right]}{1+\sum_{h\geq 1}\left(\beta^h \frac{\xi_{t+h}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+h}\frac{1+\tau^c_{t+h}}{1+\tau^c_t}\right]^{1-\sigma}}. \tag{3.8}$$

Temporary and rational-expectations equilibria We start by defining a *temporary equilibrium*. Because this general-equilibrium concept does not impose any restrictions on agents' expectations, it is a good starting point for our analysis. Formally, for given beliefs $\{F_t\left[Y_{t+h}\right]\}_{t,h\geq 0}$, a temporary equilibrium is a sequence of allocations that satisfy private optimality for households and firms and the budget constraint of the government. In addition, markets clear. Equation (3.8) and the market clearing condition, $Y_t = C_t + G_t$, imply that the temporary equilibrium output is given by

$$Y_t = \mathcal{Y}_t\left(\{F_t\left[Y_{t+h}\right]\}_{h\geq 0}\right) = G_t + \frac{\sum_{h\geq 1} Q_{t,t+h}\frac{1+\tau^c_{t+h}}{1+\tau^c_t}\left[F_t\left[Y_{t+h}\right] - G_{t+h}\right]}{\sum_{h\geq 1}\left(\beta^h \frac{\xi_{t+h}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+h}\frac{1+\tau^c_{t+h}}{1+\tau^c_t}\right]^{1-\sigma}}, \tag{3.9}$$

for all t.

A *full-information and rational-expectations equilibrium* (FIRE) is a temporary equilibrium in which expectations are consistent with the equilibrium path for these variables: $F_t\left[Y_{t+h}\right] = Y_{t+h}$. The RE equilibrium, Y^*_t, solves the fixed-point problem

$$Y^*_t = \mathcal{Y}_t\left(\left\{Y^*_{t+h}\right\}_{h\geq 1}\right),$$

for all t. Without uncertainty, the FIRE equilibrium is the perfect-foresight equilibrium.

Dynamic level-k equilibria To evaluate the full dynamic response of the economy to rare and unprecedented events, we modify the standard level-k thinking model to allow cognitive sophistication to grow over time, similar to Iovino and Sergeyev (2018).[11]

We begin by describing the equilibrium under standard level-k. Let Y_t^k denote the time-t output level in an economy where all agents are level k, and let $F_t^k[Y_{t+h}]$ denote the beliefs of level-k individuals. To compute the level-k outcome, we must ascribe to people views about the equilibrium populated by level-$(k-1)$ people. The recursion takes as given what people believe in a level-1 economy (see Farhi and Werning 2019). We denote these beliefs by $F_t^1[Y_{t+h}]$ for $t \geq 0$. For convenience, we refer to these beliefs as belonging to level-1 people, understanding that such people don't exist in a level $k \geq 2$ economy. Analogous to priors in Bayesian analyses, these beliefs are essentially free parameters.

Given these beliefs, a level-1 outcome is given by

$$Y_t^1 = \mathcal{Y}_t\left(\left\{F_t^1[Y_{t+h}]\right\}_{h\geq 1}\right).$$

In the standard level-k thinking model, individuals believe that all other agents are exactly one level below them regarding cognitive ability. So level-2 people believe the economy is entirely populated by level-1 people. So, level-2 people think that output is given by $F_t^2[Y_{t+h}] = Y_{t+h}^1$. Output in a level-2 economy is given by

$$Y_t^2 = \mathcal{Y}_t\left(\left\{F_t^2[Y_{t+h}]\right\}_{h\geq 1}\right) = \mathcal{Y}_t\left(\left\{Y_{t+h}^1\right\}_{h\geq 1}\right).$$

Level-3 people think that output is given by $F_t^3[Y_{t+h}] = Y_t^2$. So output in a level-3 economy is given by

$$Y_t^3 = \mathcal{Y}_t\left(\left\{Y_{t+h}^2\right\}_{h\geq 1}\right).$$

More generally, level-k people think that output is given by $F_t^k[Y_{t+h}] = Y_{t+h}^{k-1}$. So output in a level-$k$ economy is

$$Y_t^k = \mathcal{Y}_t\left(\left\{Y_{t+h}^{k-1}\right\}_{h\geq 1}\right). \tag{3.10}$$

We now use the definition of a standard level-k equilibrium to define a dynamic level-k equilibrium.

[11] See also Bianchi-Vimercati (2022) for an analysis of the effects of forward guidance in a model of integrated reasoning where individuals combine level-k thinking with adaptive learning.

Definition 1. A *dynamic level-k equilibrium* with initial level $k_0 \in \mathbb{N}$ and step $\mu \in \mathbb{N}_0$ is a temporary equilibrium, such that

$$F_t\left[Y_{t+h}\right] = F_t^{k_t}\left[Y_{t+h}\right],$$

where $k_t \equiv k_0 + \mu t$.

Here $F_t[Y_{t+h}]$ denotes the time t belief of the representative agent about output at time $t+h$. There are two ways of controlling the degree of sophistication in the economy. First, we can set k_0 to be higher so that people are initially more sophisticated. Second, we can set μ higher so that the level of sophistication grows more quickly over time. If $\mu > 0$, then peoples' beliefs converge to rational expectations. The dynamic level-k equilibrium nests a standard level-k equilibrium if $\mu = 0$.

3.1 Government-spending multipliers

In this section we consider an increase in government spending, ΔG_t, during the ZLB periods, i.e., for $t \leq T-1$. In addition we assume that consumption taxes are equal to their steady-state level $\tau_t^c = \tau^c$ for all periods.

Rational expectations In the simple model, the monetary authority pegs the real interest rate. It is widely understood that, under such a policy, there are multiple equilibria in the standard rational-expectations NK model. As in Farhi and Werning (2019), we focus on rational-expectations equilibria for which $Y_t \to 1$ as $t \to \infty$, i.e., the equilibrium converges to the pre-shock steady state. The household's Euler equation then implies that

$$C_t = C_{t+1} = C_{t+2} = \lim_{h \to \infty} C_{t+h} = 1$$

for all $t \geq T$.

During the ZLB period, the real interest rate is higher than the subjective discount rate. So consumption is lower than in the pre-shock steady-state:

$$C_t = \left(\beta e^{\chi}\right)^{-\sigma} C_{t+1} = \ldots = e^{-\sigma(T-t)(\chi-\rho)}. \tag{3.11}$$

Here, $\rho \equiv -\log(\beta)$. The rational expectation equilibrium level of output is given by

$$Y_t^* = G_t + e^{-\sigma(T-t)(\chi-\rho)}.$$

Consistent with Bilbiie (2011) and Woodford (2011), equation (3.11) implies that government spending does not affect consumption in the rational-expectations equilibrium. So the government-spending multiplier is exactly equal to one

$$\frac{\Delta Y_t^*}{\Delta G_t} = 1. \tag{3.12}$$

Here ΔY_t denotes the difference in output relative to the output level in the equilibrium without government spending.

Bounded rationality Relation (3.9) implies that the temporary equilibrium is given by

$$\mathcal{Y}_t\left(\{F_t\left[Y_{t+s}\right]\}\right) = G_t + \frac{\sum_{h\geq 1} Q_{t,t+h}\left[F_t\left[Y_{t+h}\right] - G_{t+h}\right]}{\sum_{h\geq 1}\left(\beta^s \frac{\xi_{t+h}}{\xi_t}\right)^{\sigma} Q_{t,t+h}^{1-\sigma}}.$$

It seems natural to assume that level-1 people believe the economy goes back to its steady state after the shock reverts to its pre-shock value, i.e., $F_t^1\left[Y_{t+h}\right] = 1$ for $t+h \geq T$. This assumption implies that Y_t is equal to its steady-state level for $t \geq T$. It follows that $F_t\left[Y_{t+h}\right] = F_t^{k_t}\left[Y_{t+h}\right] = 1$ for all $t+h \geq T$. So we can write the equilibrium level of output for $t \leq T-1$ as follows

$$Y_t = G_t + \Omega_t\left\{\sum_{h=1}^{T-t-1}\left[F_t\left[Y_{t+h}\right] - G_{t+h}\right] + \frac{1}{1-\beta}\right\},$$

where $\Omega_t \equiv \left[e^{\sigma(\chi-\rho)}\left[\frac{1-e^{\sigma(\chi-\rho)(T-t-1)}}{1-e^{\sigma(\chi-\rho)}} + \frac{e^{\sigma(\chi-\rho)(T-t-1)}}{1-\beta}\right]\right]^{-1} \in (0,1]$.

Lemma 1. *In a temporary equilibrium, the government-spending multiplier is given by*

$$\frac{\Delta Y_t}{\Delta G_t} = 1 + \Omega_t \sum_{h=1}^{T-t-1}\left[\frac{\Delta F_t\left[Y_{t+h}\right]}{\Delta G_{t+h}} - 1\right]\frac{\Delta G_{t+h}}{\Delta G_t}. \tag{3.13}$$

Note that the time t government-spending multiplier in a temporary equilibrium depends on people's beliefs regarding future income. This dependency is not a feature of the rational-expectations equilibrium for our simple model.

The intuition about how beliefs about future government spending affect the time t multiplier is as follows. First, if expectations for future incomes do not change with the policy ($\Delta F_t\left[Y_{t+h}\right]=0$), then the effect of future spending on current output is negative,

$$-\Omega_t\sum_{h=1}^{T-t-1}\frac{\Delta G_{t+h}}{\Delta G_t}.$$

We refer to this effect as the *partial-equilibrium effect* of government spending*:* higher taxes associated with higher current and future expenditures lead to a negative wealth effect that causes people to reduce consumption. This effect arises because people understand the government budget constraint, an assumption that we relax in subsection 6.

The *general-equilibrium effect* of government spending is given by

$$\Omega_t\sum_{h=1}^{T-t-1}\frac{\Delta F_t\left[Y_{t+h}\right]}{\Delta G_{t+h}}\frac{\Delta G_{t+h}}{\Delta G_t}. \tag{3.14}$$

Higher future spending leads people to believe their future incomes will be higher. The associated positive wealth effect leads to an increase in current consumption. Other things equal, this increase leads to a rise in actual current output. The fact that the government-spending multiplier is one under rational expectations reflects that the partial and general-equilibrium effects exactly offset each other in this model.

We now consider the dynamic level-k economy and show that, under plausible conditions, the less sophisticated people are, the less they consider GE effects when making decisions. This effect leads to a lower government-spending multiplier. Suppose that level-1 people think that the multiplier is η, i.e., $\Delta F_t^1\left[Y_{t+h}\right]/\Delta G_{t+h}=\eta$ for all t and h.[12] The parameter η allows us to flexibly parameterize the beliefs of level-1 people. When $\eta=0$, level-1 people think that their pre-tax income is unaffected by government spending. When $\eta\in(0,1)$, level-1 people expect pre-tax income to rise in response to higher government spending. When $\eta=1$, level-1 people think that their post-tax income is unaffected by government spending.

Proposition 1. *Suppose that level-*1 *people believe* $\Delta F_t^1\left[Y_{t+h}\right]/\Delta G_{t+h}=\eta$ *for all* $t+h\leq T-1$*. Then, for any* $k_0\in\mathbb{N}$ *and* $\mu\in\mathbb{N}_0$*:*

1. *If* $0\leq\eta<1$*, then the time-*$(T-1)$ *government-spending multiplier is lower than the rational-expectations multiplier,*

$$\frac{\Delta Y_t}{\Delta G_t}\leq\frac{\Delta Y_t^*}{\Delta G_t}.$$

[12] For simplicity, we assume that η does not vary with t and h. Our results would extend if we allowed η to depend on those timing parameters.

Furthermore, if $1 - \Omega_t \sum_{s=1}^{T-t-1} \frac{\Delta G_{t+s}}{\Delta G_t} \geq 0$ *for all* t*, then the government-spending multiplier is increasing in* k_0 *for all* $t \geq 0$ *and in* μ *for* $t \geq 1$*.*

2. *If* $\eta = 1$*, then the time-*$(T-1)$ *government-spending multiplier is exactly equal to its value under rational expectations,*

$$\frac{\Delta Y_t}{\Delta G_t} = \frac{\Delta Y_t^*}{\Delta G_t}.$$

According to the previous Proposition, for finite k_0 and μ, the government-spending multiplier is lower than under rational expectations for all $\eta < 1$. When $\eta = 0$, level-1 people believe that pre-tax labor income is unaffected by government spending. In this case, the multiplier is at its lowest. When $\eta = 1$, level-1 people believe that their *after-tax* income is unaffected by government spending, i.e., changes in government spending map one-to-one to changes in *pre-tax* income. In this case, the government-spending multiplier is unaffected by the level of cognitive reasoning k_0 or μ. This result follows from the fact that level-1 individuals expect the multiplier to be the same as in the rational-expectations equilibrium.

With $\Delta F_t^1 [Y_{t+h}] / \Delta G_{t+h} = \eta$, the GE effect in the government-spending multiplier, (3.14) is given by

$$\eta \Omega_t \sum_{h=1}^{T-t-1} \frac{\Delta G_{t+h}}{\Delta G_t}.$$

It follows from the Proposition that the multiplier is increasing in η because the GE effect of increasing government spending is larger. Note that η could be larger than 1, i.e., people believe their after-tax income will rise due to increased government spending. In this case, the multiplier is larger than one.

Recall that, based on survey evidence, Coibion and Gorodnichenko (2012, 2015), Bordalo et al. (2020), and Angeletos et al. (2021) find that average beliefs about macroeconomic aggregates like inflation and real GDP growth tend to *underreact* to changes in macro fundamentals relative to the rational-expectations benchmark.[13] These findings support the view that η is a relatively small number, strictly less than one.

According to Proposition 1, the size of the multiplier is higher the more sophisticated people are. Sophistication comes in two forms: a level effect (higher k_0) and a dynamic effect (higher μ). In either event, the more sophisticated people are, the more they understand the GE effect which, implies that their consumption is higher.

In the extended model of Section 4 we show that the efficacy of government spending is reduced even in the limiting case of $\eta = 1$. In that model, wages and prices are time-

[13]Interestingly, Bordalo et al. (2020) find evidence of overreaction of individual expectations to news. However, in our model, average expectations are the key determinants of aggregate outcomes.

varying. So, expectations regarding future inflation and its impact on real interest rates are important determinants of aggregate demand. This extra general-equilibrium force eliminates the sensitivity of our multiplier results to the case of $\eta = 1$.

3.2 Tax policy

This section discusses the efficacy of consumption-tax policy when the ZLB is binding. Following Correia et al. (2013), we show that consumption-tax policy can implement the flexible-price allocation under rational expectations. We then evaluate the efficacy of consumption-tax policy under dynamic level-k thinking and show that a policy always exists that supports that allocation. Moreover, under plausible assumptions, that policy does not depend on peoples' level of sophistication (k_0 and μ), and its success does not depend on people making systematic errors in their beliefs about economy-wide variables.

Assume that government spending does not respond to the discount rate shock so that G_t remains at its steady-state value of zero. Consumption taxes change during the ZLB period and converge back to their pre-shock level, τ^c, once the economy exits the ZLB ($t = T$).

Rational expectations With time-varying consumption taxes, the household's Euler equation for $t \leq T-1$ can be written as

$$Y_t = Y_{t+1}\left(\beta \frac{\xi_{t+1}}{\xi_t} R_t \frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right)^{-\sigma}$$

where we have set $C_t = Y_t$. This expression makes clear that the relevant relative price of consumption at time t versus time $t+1$ is the real interest rate times the ratio of consumption taxes, $R_t\left(1+\tau_t^c\right) / \left(1+\tau_{t+1}^c\right)$.

We write this Euler equation in log terms,

$$y_t = y_{t+1} - \sigma\left(r_t + \log\left(\frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right) - (\rho - \chi)\right), \tag{3.15}$$

where $r_t = \log R_t = 0$. Note that, for $t \geq T$, the real interest rate returns to its pre-shock level, $r_t = \rho$, and $y_t = 0$ (or $Y_t = 1$).

Suppose that at time 0, the government announces that taxes will follow the path

$\tau_t^c = \tau_t^{c,*}$, where

$$\tau_t^{c,*} = (1 + \tau^c)\, e^{-(T-t)(\chi - \rho)} - 1 \tag{3.16}$$

for $t \leq T$. With this specification, the consumption tax falls at time 0 and then slowly converges back to its pre-shock value. Also, note that:

$$\log\left(\frac{1 + \tau_t^{c,*}}{1 + \tau_{t+1}^{c,*}}\right) = \rho - \chi.$$

Under this assumption, the relative price of consumption is equal to the subjective discount rate even if the nominal interest rate is at the ZLB.

Equation (3.15) implies that, under this policy, $y_t = y_{t+1}$ for all t. Since $y_t \to 0$ in the limit, this tax policy implements the flexible-price allocation, i.e., $y_t^* = 0$ for all t. The conclusion that tax policy can effectively circumvent the ZLB and achieve the flexible-price allocation is the key result in Correia et al. (2013).

Bounded rationality Suppose that the government announces a path for consumption taxes, τ_t^c, such that taxes go back to their pre-shock level as soon as the economy exits the ZLB, i.e., $\tau_t^c = \tau^c$ for $t \geq T$. In addition, suppose that everyone expects the economy to return to its pre-shock steady state once the ZLB is no longer binding. Then the temporary equilibrium level of output is given by:

$$Y_t = \frac{\sum_{s\geq 1} Q_{t,t+s} \frac{1+\tau_{t+s}^c}{1+\tau_t^c} F_t\left[Y_{t+s}\right]}{\sum_{s\geq 1}\left(\beta^s \frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+s}\frac{1+\tau_{t+s}^c}{1+\tau_t^c}\right]^{1-\sigma}} \equiv \mathcal{Y}_t\left(\left\{F_t\left[Y_{t+s}\right]\right\}\right). \tag{3.17}$$

Equation (3.17) highlights the effect of time-varying consumption taxes on consumption and equilibrium output. For $t = T - 1$, we can write this equation as

$$Y_{T-1} = \left(\frac{1+\tau^c}{1+\tau_{T-1}^c}\right)^{\sigma} e^{-\sigma(\chi-\rho)}.$$

This expression makes clear that setting $\tau_{T-1}^c = (1+\tau^c)\, e^{-(\chi-\rho)} - 1$ implements $Y_{T-1} = 1$.

It follows directly from (3.17) that, for exogenous beliefs, there always exists an appropriate choice of τ_t^c for which $Y_t = 1$ for all t. Of course, beliefs are endogenous to the policy that is implemented. Proposition 2 shows that for every parameterization of dynamic level-k thinking (k_0 and μ), there is a path for consumption taxes that implements the flexible-price allocation. As agents become more sophisticated, this policy approaches the rational-expectations optimal policy, $\tau_t^{c,*}$. In general, the path of consumption taxes that implements the flexible-price allocation depends on k_0 and μ. However, if level-1 people think their income will remain equal to its steady-state level, then the policy that achieves full stabilization is the same regardless of k_0 and μ. Moreover, that policy coincides with the optimal policy under rational expectations.

Proposition 2. *Suppose level-*1 *people believe the economy goes back to steady state after the ZLB period, i.e.,* $F_t^1 [Y_{t+h}] = 1$ for $t \geq T$.

1. *For each* (k_0, μ)*, there exists a policy announcement* $\{\tau_t^c\}$ *which implements the flexible-price allocation.*

2. *Suppose that* $F_t^1 [Y_{t+h}] = 1$ *for all* t *and* $h \geq 0$*, then the policy* $\{\tau_t^{c,*}\}$ *implements the flexible-price allocation for all* (k_0, μ)*.*

In the appendix, we prove the first result. Specifically, we show how to construct the path for consumption taxes that implements the flexible-price allocation for a given level of cognitive sophistication.

A simple proof of the second result is as follows. Recall that under the tax policy $\{\tau_t^{c,*}\}$, the rational-expectations equilibrium is $Y_t^* = 1$. By definition, this equilibrium is a fixed point of the temporary equilibrium relation (3.17). Suppose level- 1 individuals expect their pre-tax income to remain at its steady-state level. In that case, they will adjust their behavior so that $Y_t^1 = 1$. Since $F_t^2 [Y_{t+h}] = Y_{t+h}^1 = 1$, the level-2 equilibrium is the same as the level-1 equilibrium. The same logic applies for any k_t. We conclude that the belief $F_t^1 [Y_{t+h}] = 1$ is self-confirming under the proposed tax policy. So, the proposed tax policy does not rely on people making mistakes. On the contrary: the tax policy leads to an equilibrium in which people's beliefs coincide with actual outcomes.

Discussion We can summarize the key mechanisms behind the way spending and tax policies affect the economy in the following way. A rise in government spending increases aggregate demand, output, and labor income. So, government spending has a positive effect on household demand via a general-equilibrium response of individual income. Under rational expectations, people understand this general-equilibrium effect and increase their personal consumption, magnifying the rise in aggregate demand, output, and labor in income. Under level-k thinking, people do not internalize these general-equilibrium effects. So, the government spending multiplier is smaller. In contrast, a decline in the consumption tax rate directly impacts the "price of consumption." People do not need to understand anything about the general-equilibrium effects of the policy to see that consumption is effectively on sale. It follows that household demand directly responds to the change in policy, even absent any general-equilibrium considerations. Our proposition shows that the government can exploit this fact to implement the first-best allocation via an appropriate tax policy.

A natural question is whether our results are robust to alternative ways of modeling bounded rationality. In Appendix D, we redo our analysis of the benchmark model using two alternatives to the level-k thinking approach. The first alternative is that people have *reflective expectations* as in García-Schmidt and Woodford (2019). The second alternative is that people display *shallow reasoning* as developed in Angeletos and Sastry (2020). We show that Propositions 1 and 2 continue to hold for both cases.

We conclude this subsection by contrasting the efficacy of tax rate and interest-rate policy. When Ricardian equivalence holds, changing the announced path of tax rates and interest rates affects the equilibrium in the same way. However, there is one crucial difference. The ZLB constrains the class of feasible announced paths for interest rates. So, monetary policy can only boost consumption demand via forward guidance, i.e., a promise to lower interest rates in the future after the ZLB is no longer binding. Farhi and Werning (2019) and García-Schmidt and Woodford (2019) show that the strong stimulative power of forward guidance relies heavily on general-equilibrium effects. Those effects become muted when people are boundedly rational. Instead, consumption taxes can be changed as soon as the ZLB becomes binding. So, tax policy can effectively counteract the effects of the discount-factor shock and support the flexible-price allocation. This flexibility implies that, even when Ricardian equivalence holds, consumption-tax rates have a significant advantage relative to interest-rate policy in circumstances where the ZLB is binding. In section 6, we return to the case in which Ricardian equivalence does not hold.

4 A model with Calvo-style wage rigidities

This section extends the baseline model to allow for time-varying prices and wages. We introduce Calvo-style wage rigidities as in Erceg et al. (2000) and Schmitt-Grohé and Uribe (2005). In Appendix E, we show that our results are robust to assuming Calvo-style price rigidities.

The model economy is populated by a continuum of households, unions, goods producers, and the government. Each household has a continuum of workers who have different labor skills. Output can be used for private or government consumption so that the aggregate resource constraint is still given by (3.3).

Goods producer The final good is produced by a representative firm using the Cobb-Douglas technology using a fixed stock of capital, $\overline{K}$, and a composite labor input, N_t:

$$Y_t = A\overline{K}^{\alpha} N_t^{1-\alpha}, \tag{4.1}$$

Here N_t denotes a composite labor input and $A > 0$ represents total-factor productivity. For simplicity we assume that the capital stock is fixed at the level $\overline{K}$.[14] The capital share, α, is between 0 and 1.

The composite labor input N_t is generated using a continuum of labor varieties according to the technology:

$$N_t = \left[\int_0^1 n_{u,t}^{\frac{\theta-1}{\theta}} du\right]^{\frac{\theta}{\theta-1}}, \tag{4.2}$$

where $\theta > 1$ captures the elasticity of substitution across the labor varieties. The firm, which is perfectly competitive in both the goods and the labor market, maximizes

$$P_t Y_t - \int_0^1 w_{u,t} n_{u,t} du$$

subject to (4.1) and (4.2). Here P_t denotes the price of the consumption good and $w_{u,t}$ denotes the wage of $n_{u,t}$. The solution to this problem is given by:

$$n_{u,t} = \left(\frac{w_{u,t}}{W_t}\right)^{-\theta} N_t, \tag{4.3}$$

where

$$W_t = \left[\int_0^1 w_{u,t}^{1-\theta} du\right]^{\frac{1}{1-\theta}}, \tag{4.4}$$

and

$$\frac{W_t}{P_t} = (1-\alpha) A \left(\frac{\overline{K}}{N_t}\right)^{\alpha}. \tag{4.5}$$

Households The household enters period t with financial assets B_t, which earn the interest rate R_{t-1}. As in section 3, we assume the household knows its time-t income Y_t and taxes T_t. When solving its dynamic consumption-savings problem, the household maximizes its utility which is evaluated based on today's consumption, C_t, and on its plans

[14] This assumption can be rationalized for business cycle dynamic analysis if there are large capital adjustment costs. See for example Rotemberg and Woodford, 1997 and Farhi and Werning, 2019.

for future consumption, $\widetilde{C}_{t+s}$ for $s = 1, 2, ...$ Labor supply is determined by labor unions, as described below. We denote by L_t the total hours worked by the household,

$$L_t = \int_0^1 n_{u,t}.$$

With wage dispersion induced by nominal rigidities, L_t is not to equal N_t.

The representative household maximizes (3.1) subject to

$$\left(1 + \tau^c_{t+h}\right) \widetilde{C}_{t+h} + \widetilde{B}_{t+h+1} = \left(1 - \tau^n_{t+h}\right) F_t \left[\frac{W_{t+h}}{P_{t+h}}\right] F_t \left[N_{t+h}\right] + F_t \left[\Omega_{t+h}\right] - F_t \left[T_{t+h}\right] + \frac{R_{t+h-1}}{F_t \left[P_{t+h}/P_{t+h-1}\right]} \widetilde{B}_{t+h},$$

where Ω_{t+h} denotes lump-sum profits from firms and τ^n_t denotes the time t tax rate on labor income.

The household has perfect foresight with respect to exogenous variables, including the discount rate shock, ξ_t. For now, we assume that the government announces sequences of nominal interest rates, $\{R_t\}$, government spending, $\{G_t\}$, and taxes $\{\tau^c_t, \tau^n_t\}$. Household beliefs for $F_t \left[T_{t+h}\right]$ satisfy:

$$\sum_{h \geq 0} Q_{t,t+h} F_t \left[\frac{P_{t+h}}{P_t}\right] F_t \left[T_{t+h}\right] = \sum_{h \geq 0} Q_{t,t+h} F_t \left[\frac{P_{t+h}}{P_t}\right] \left[G_{t+h} - \tau^c_{t+h} F_t \left[C_{t+h}\right] - \tau^n_{t+h} F_t \left[\frac{W_{t+h}}{P_{t+h}}\right] F_t \left[N_{t+h}\right]\right] + \frac{R_{t-1}}{P_t/P_{t-1}} B_t. \tag{4.6}$$

Along with our other assumptions, (4.6) implies that Ricardian equivalence holds.

In appendix B.1 we show the solution to the household's problem implies

$$C_t = \frac{\sum_{h \geq 0} Q_{t,t+h} F_t \left[\frac{P_{t+h}}{P_t}\right] \frac{1+\tau^c_{t+h}}{1+\tau_t} \left[F_t \left[Y_{t+h}\right] - G_{t+h}\right]}{1 + \sum_{h \geq 1} \left(\beta^h \frac{\xi_{t+h}}{\xi_t}\right)^\sigma \left[Q_{t,t+h} F_t \left[\frac{P_{t+h}}{P_t}\right] \frac{1+\tau^c_{t+h}}{1+\tau^c_t}\right]^{1-\sigma}}. \tag{4.7}$$

Labor market and unions Each household supplies $n_{u,t}$ units of type u labor to a union indexed by $u \in [0, 1]$. Union u sets the wage for type u labor subject to Calvo-style frictions and labor demand given by (4.3).

At each date, $1 - \lambda$ unions are randomly selected to adjust their wage, $w_{u,t}$. For the other λ unions, $w_{u,t} = w_{u,t-1}$. Unions act on behalf of households and choose wages and labor hours to maximize the expected household's valuation of labor income. In the presence of sticky wages, actual employment is demand determined.

In a symmetric equilibrium, unions that can reset their wages choose the same value. We denote the new reset wage by W_t^*. In appendix B.2, we show that W_t^* satisfies

$$\frac{W_t^*}{P_t} = \frac{\theta}{\theta - 1} \frac{\sum_{h=0}^{\infty} (\beta\lambda)^h \xi_{t+h} \left(F_t\left[\frac{P_{t+h}}{P_t}\right]\right)^{\theta} \left(F_t\left[\frac{W_{t+h}}{P_{t+h}}\right]\right)^{\theta} F_t\left[N_{t+h}\right] v'\left(F_t\left[L_{t+h}\right]\right)}{\sum_{h=0}^{\infty} (\beta\lambda)^h \xi_{t+h} \left(F_t\left[\frac{P_{t+h}}{P_t}\right]\right)^{\theta-1} \left(F_t\left[\frac{W_{t+h}}{P_{t+h}}\right]\right)^{\theta} F_t\left[N_{t+h}\right] u'\left(F_t\left[C_{t+h}\right]\right) \frac{1-\tau_{t+h}^n}{1+\tau_{t+h}^c}}. \tag{4.8}$$

The union has perfect foresight with respect to exogenous variables.The union forms beliefs about future aggregate prices, $F_t\left[P_{t+h}/P_t\right]$, wages, $F_t\left[W_{t+h}/P_{t+h}\right]$, consumption, $F_t\left[C_{t+h}\right]$, the labor composite, $F_t\left[N_{t+h}\right]$, and labor input, $F_t\left[L_{t+h}\right]$ using dynamic level-k thinking.

Monetary and fiscal policies Nominal interest rates during and after the ZLB period are as described in the benchmark model. The fiscal authority sets government spending G_t, consumption taxes τ_t^c, labor income taxes τ_t^n, and lump-sum taxes T_t subject to the intertemporal budget constraint:

$$\sum_{h\geq 0} Q_{t,t+h} P_{t+h} G_{t+h} + R_{t-1} B_t = \sum_{h\geq 0} Q_{t,t+h}\left[\tau_{t+h}^c P_{t+h} C_{t+h} + \tau_{t+h}^n W_{t+h} N_{t+h} + T_{t+h}\right]. \tag{4.9}$$

Temporary Equilibrium As in Farhi and Werning (2019), we assume beliefs regarding future nominal prices and wages are scaled by $P_{t+h}/E_t\left[P_{t+h}\right]$. This assumption allows people to incorporate current and past surprise inflation into their beliefs, leaving beliefs about future inflation and real wages unchanged.

For each date t, given beliefs, a temporary equilibrium is a sequence of allocations and prices,

$$\mathcal{A}_t \equiv \left\{Y_t, C_t, N_t, L_t, P_t/P_{t-1}, W_t/P_t\right\},$$

in which households, firms, and unions solve their optimization problem, and goods markets clear for each t. In appendix B, we summarize the equations whose solution defines an equilibrium for this economy. This appendix also shows that the log-linearized temporary equilibrium can be computed using the following equations. First, the household's optimality conditions imply that aggregate consumption is given by:

$$c_t = (1-\beta)\sum_{h=1}^{\infty}\beta^{h-1}\frac{Y}{C}\left\{F_t\left[y_{t+h}\right]-\frac{G}{Y}g_{t+h}\right\} \\ -\sigma\sum_{h=0}^{\infty}\beta^{s}\left\{r_{t+h}-F_t\left[\pi_{t+h+1}\right]-\left(\hat{\tau}^{c}_{t+h+1}-\hat{\tau}^{c}_{t+h}\right)+\chi_{t+h}\right\}, \tag{4.10}$$

where small case letters represent log-deviations of variables from their steady-state values, and $\hat{\tau}^c_t \equiv d\log\left(1+\tau^c_t\right)$. Second, union wage-setting implies that wage inflation $\pi^w_t \equiv w_t - w_{t-1}$ is given by

$$\pi^w_t = \kappa\sum_{h=0}^{\infty}\left(\beta\lambda\right)^h\left\{\left(\varphi+\alpha\right)F_t\left[n_{t+h}\right]+\sigma^{-1}F_t\left[c_{t+h}\right]+\hat{\tau}^n_{t+h}+\hat{\tau}^c_{t+h}\right\}+\frac{1-\lambda}{\lambda}\sum_{h=1}^{\infty}\left(\beta\lambda\right)^h F_t\left[\pi_{t+h}\right], \tag{4.11}$$

where $\kappa \equiv (1-\lambda)(1-\beta\lambda)/\lambda$. Price inflation $\pi_t \equiv p_t - p_{t-1}$ is given by

$$\pi_t = \pi^w_t + \alpha\left(n_t - n_{t-1}\right). \tag{4.12}$$

Finally, using the resource constraint we obtain:

$$\frac{C}{Y}c_t + \frac{G}{Y}g_t = y_t = (1-\alpha)\,n_t, \tag{4.13}$$

Around a zero inflation steady state, $n_t = l_t$.

Generalizing Level-k thinking – Cognitive Hierarchies In our quantitative results, we adopt a simple generalization of the standard level-k thinking model based on the *cognitive hierarchy model* developed in Camerer et al. (2004). Whereas the standard level-k thinking model assumes that people believe all others to be exactly level $k-1$, the cognitive hierarchy model allows people to consider that others may be distributed among the levels of cognitive sophistication below theirs. We adopt this generalization for two reasons. First, Camerer et al. (2004) argue that this model provides a better description of the data. Second, under some parameterizations, this generalization of level-k thinking also fixes well-known peculiar behavior generated by the standard model. In particular, García-Schmidt and Woodford (2019) and Angeletos and Sastry (2020) show that when applied to games featuring strategic substitution, standard models of level-k thinking tend to produce a type of oscillatory behavior which takes the following form. The equilibrium level of output lies below the rational-expectations equilibrium level when for odd levels of k but is above it when k is even.

To develop the generalization, we introduce the concept of a level-0 person. This type of person continues to act as they did before the discount rate shock, i.e., their consumption and pricing decisions are such that $\mathcal{A}_t^0 = \mathcal{A}$, where $\mathcal{A}_t^0$ denotes the equilibrium allocations and prices of an economy populated by level-0 individuals. Level-1 individuals believe that the economy is populated by level-0 people so, $F_t^1 [\mathcal{A}_{t+h}] = \mathcal{A}_{t+h}^0$. Level-2 individuals believe that a fraction $f_2(j)$ of the population is level $j = 0, 1$ and work through the problem of level-0 and level-1 people and then solve the equilibrium under these assumed shares. More generally, level-k people believe that output is the equilibrium when the percentage of people of level $j < k$ is given by $f_k(j)$.

Camerer et al. (2004) assume that the distributions $f_k(\cdot)$ are consistent with the physical distribution of cognitive levels in the economy. In contrast, we maintain the representative agent assumption so everyone shares the same level k. We assume that agents of different cognitive levels agree on the relative proportions of lower cognitive levels. The distributions $f_k(\cdot)$ are such that for any $k_1 < k_2$ and $s, s' < k_1$

$$\frac{f_{k_1}(s)}{f_{k_1}(s')} = \frac{f_{k_2}(s)}{f_{k_2}(s')}. \tag{4.14}$$

Let $\gamma_k \equiv f_k(k-1)$ for all k. In appendix B.4, we show that in the log-linearized model the beliefs of a level-k thinker are given by a weighted average of the beliefs of level-$(k-1)$ agents and the equilibrium that would arise if everyone in the economy was a level-$(k-1)$ thinker:

$$F_t^k [\mathcal{A}_{t+h}] = (1-\gamma_k) F_t^{k-1} [\mathcal{A}_{t+h}] + \gamma_k \mathcal{A}_t^{k-1}. \tag{4.15}$$

Here $\mathcal{A}_t^k$ denotes the equilibrium in an economy where everyone is exactly level $k-1$. The standard level-k thinking model corresponds to the case of $\gamma_k = 1$.

Calibration For our quantitative results, we assume that the elasticity of intertemporal substitution is $\sigma = 0.5$, the steady-state discount factor is $\beta = 0.99$, the discount factor shock is $\chi = 0.02$, and the steady-state spending to GDP ration is given by $G/Y = 0.2$. Consistent with the evidence in Chetty et al. (2011) we set the Frisch elasticity is $\varphi^{-1} = 0.75$. We normalize $\overline{K} = 1$ and set the capital share, α, to 0.33. In addition, we set total factor productivity, A, so that steady-state output equals one. Following Correia et al. (2013), we assume that the elasticity of substitution across labor types θ is equal to 3, the Calvo parameter λ is 0.85, and the steady-state tax rates τ^c and τ^n are equal to 0.05 and

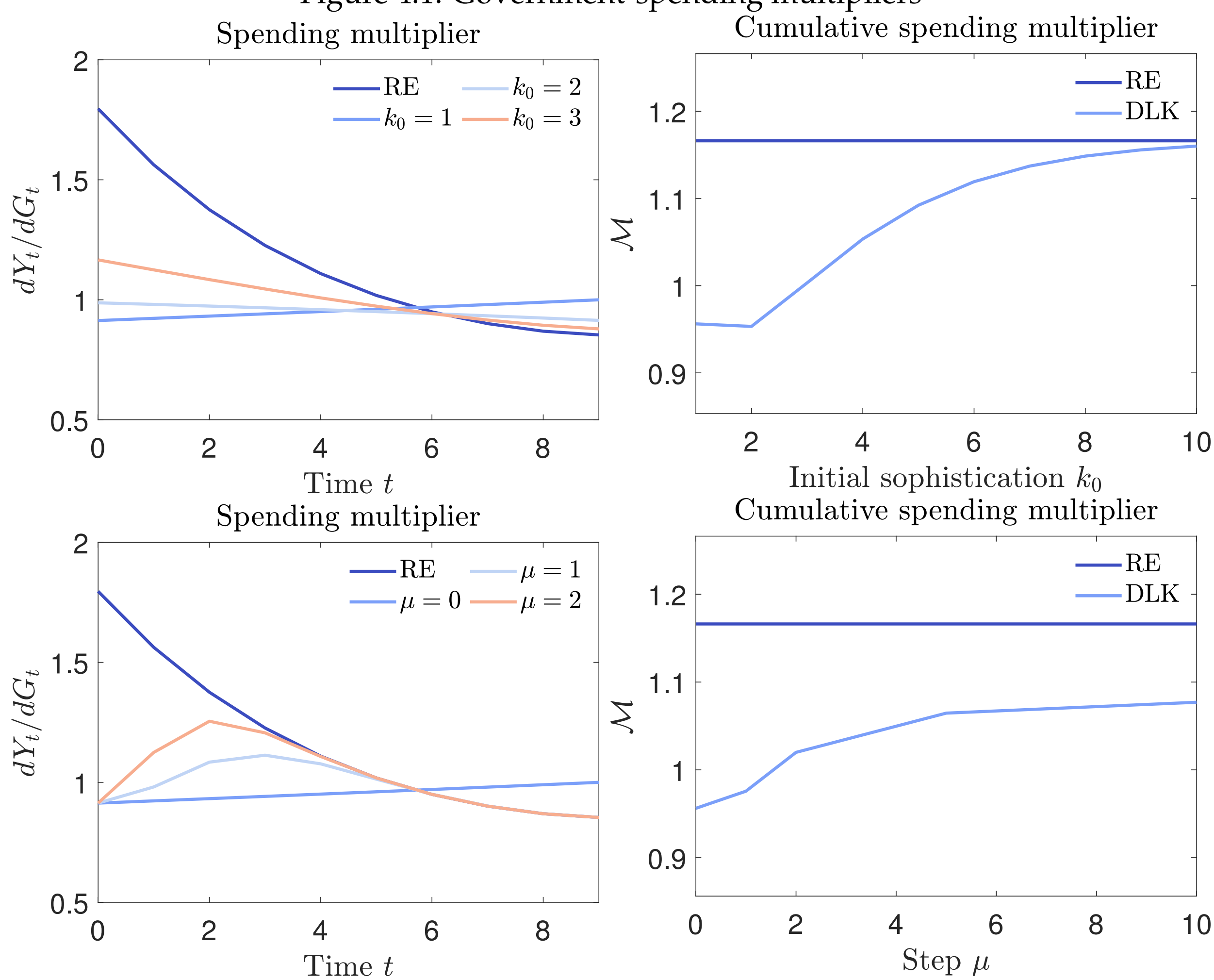
0.28, respectively. Finally, we assume that level-1 beliefs about income and inflation are given by their steady-state levels and set $\gamma_k = 0.5$ for all k.

4.1 Government-spending multipliers

This section briefly illustrates the analog to Proposition 1 for the case in which tax rates are constant and government spending rises by ΔG during the ZLB period.

4.1.1 Results

Figure 4.1: Government-spending multipliers



The first column Figure 4.1 displays the government-spending multipliers for $t = 0, 1..., 9$, $\Delta Y_t/\Delta G_t$, for various levels of $k_0$and μ. When we vary k_0 we hold μ fixed at zero which corresponds to a standard level k economy. When we change μ, we hold

k_0 fixed at one. For reference, we also display the government-spending multipliers for the case of rational expectations. In the latter case, the multiplier is initially close to 1.8 and then declines to about 0.9. Consistent with results in the NK literature, the large size of this multiplier reflects the fact that government spending induces inflation, which lowers the real interest rate during the ZLB period. Because of intertemporal substitution effects, this fall leads to a rise in households' demand for consumption and an increase in output. Other things equal, perfectly rational agents understand that these intertemporal substitution effects increase current and future output. In a virtuous cycle, the rise in future income raises people's permanent income, raising current spending and inflation. The latter effect lowers the real interest rate, strengthening the intertemporal substitution effect. The net result is a sequence of large multipliers.

Figure 4.1 shows that if $k_0 = 1$, the initial multipliers are substantially lower than in the case of rational expectations. The size of these multipliers rises for higher values of k_0. Figure 4.1 shows that when $\mu > 0$, i.e., people become more sophisticated over time, the government spending multiplier converges to its value under rational expectations. For the case of $\mu = 1$ (people become one step more sophisticated each quarter), it takes one year for the multiplier to converge to its value under rational expectations. The Figure also shows that as μ rises, i.e. people become more sophisticated more quickly, the multiplier converges more rapidly to its rational-expectations value. These observations account for the "hump-shaped" pattern of the multiplier for $\mu > 0$. Two forces are working on the dynamic level-k multiplier. As sophistication rises, the multiplier initially increases because it is converging from below to its rational expectation value. But the latter is declining over time, and convergence requires that the dynamic level-k multiplier declines.

It is useful to define the cumulative spending multiplier as[15]

$$\mathcal{M} \equiv \frac{\sum_t \Delta Y_t}{\sum_t \Delta G_t} = \sum_t \frac{\Delta G_t}{\sum_t \Delta G_t} \frac{\Delta Y_t}{\Delta G_t}.$$

The second column of Figure 4.1 shows that the cumulative multiplier increases with k_0 and μ. In the case of k_0, the cumulative multiplier convergence to its rational-expectations value because convergence pertains to sophistication at time 0. In the case of μ, the cumulative multiplier does not converge to its rational-expectations value, because convergence refers to an event after time 0.

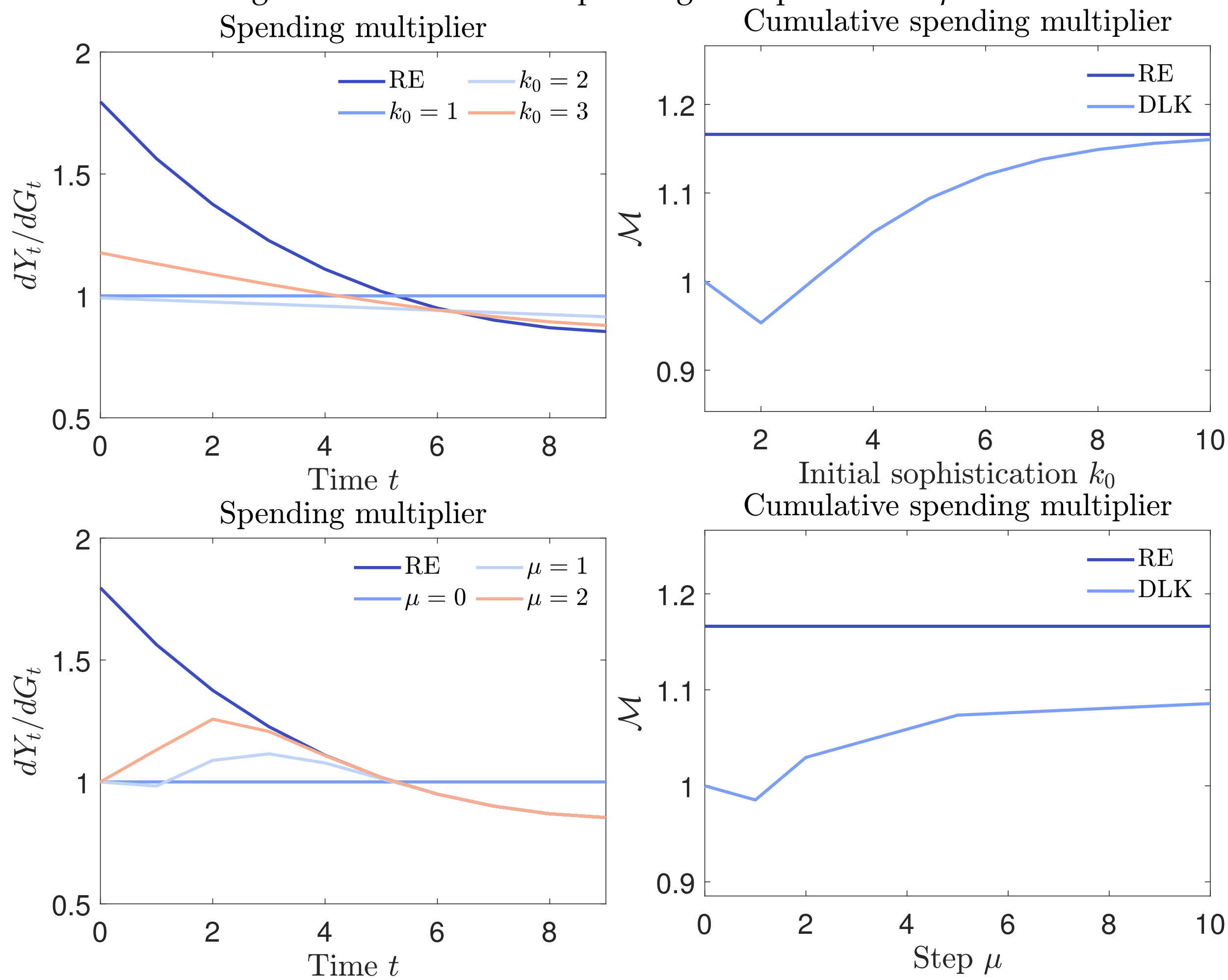
[15]Since the cumulative multiplier can be decomposed into a weighted sum of the time t multipliers, the results in Proposition 1 for the benchmark model also hold for the cumulative multiplier.

4.1.2 Robustness to η

Recall that η denotes the level-1 individuals' beliefs about the size of the government-spending multiplier, $\Delta F_t^1[Y_{t+h}]/\Delta G_{t+h}$. Proposition 1 establishes that, in the simple model, the government-spending multiplier is lower than its value under rational expectations for all $\eta < 1$. But the multiplier in the level-k economy is the same as under rational expectations when $\eta = 1$. Once we allow for inflation effects, the efficacy of government spending is reduced even in that limiting case.

Our results for $\eta = 1$ are displayed in Figure 4.2. Comparing this figure with Figure 4.1, we see that the results for the two values of η are very similar. The reason why η plays a smaller role in the extended model is as follows. The simple model focuses on the income effects of a shock to government spending in the ZLB. It abstracts from the effects of government spending on inflation. The extended model allows for both effects. So expectations regarding future inflation and its impact on real interest rates are an important determinant of aggregate demand in the extended model. The role of income expectations becomes relatively less important. Even when $\eta = 1$, the inflation effects are operative, generating a damped multiplier relative to the case of rational expectations.

Figure 4.2: Government-spending multipliers with $\eta = 1$



Taken together, the results in this section reinforce the message from the benchmark model: bounded rationality weakens the case for the efficacy of government spending as a tool for stabilizing output in the face of a shock that causes the ZLB to bind.

4.2 Tax policy

This section considers the efficacy of tax policy in the extended version of our benchmark economy. We provide two results. The first result is that Proposition 2 continues to hold so that tax policy can support the flexible-price allocation even when prices and wages are not entirely rigid. This result is summarized in 3. Second, we show through a numerical example that the tax policy remains a powerful stabilization tool for tax policies that are even simpler than those contemplated in Proposition 3.

4.2.1 Full stabilization with tax policy with sticky wages

Under rational expectations, the requisite tax policy sets consumption taxes according to

$$\tau_t^{c,*} = (1+\tau^c)\, e^{-(T-t)(\chi-\rho)} - 1. \tag{4.16}$$

Recall that in the benchmark economy, wages are fully rigid. Employment is determined entirely by the demand for labor. In the extended model, consumption taxes, $\tau_t^{c,*}$, induce distortions in labor supply which affect the equilibrium because wages aren't perfectly rigid. To support the flexible-price allocation, the government must adjust labor taxes to undo these distortions:

$$\frac{1-\tau_t^{n,*}}{1+\tau_t^{c,*}} = \frac{1-\tau^n}{1+\tau^c}. \tag{4.17}$$

Under this policy, the tax wedge on labor supply is constant over time. Critically, the government announces its policy for $\tau_t^{c,*}$ and $\tau_t^{n,*}$ as a sequence of tax rate *targets*.

We now state the analog to Proposition (2) for the extended model.

Proposition 3. *Suppose that level-*1 *people believe that the economy goes back to steady state after the ZLB period, i.e.,* $F_t^1[\mathcal{A}_{t+h}] = \mathcal{A} \equiv \{Y, C, N, L, 1, W/P\}$ for $t \geq T$ and $h \geq 0$. *Consider the log-linearized version of the model economy. Then,*

1. *For each* $k_0 \in \mathbb{N}$ *and* $\mu \in \mathbb{N}_0$*, there exists a policy* $\{\tau_t^c, \tau_t^n\}$ *which implements the flexible-price allocation.*

2. *Suppose that* $F_t^1[\mathcal{A}_{t+h}] = \mathcal{A}$ *for all* $t \geq 0$*, then the policy* $\{\tau_t^{c,*}, \tau_t^{n,*}\}$ *implements the flexible-price allocation for all* k_0 *and* μ*.*

Here $F_t^1[\mathcal{A}_{t+h}]$ denotes the beliefs of level-1 people. This proposition generalizes Proposition 2 to the extended model and demonstrates that tax policy is still very powerful even under bounded rationality in the presence of time-varying wages and prices.

4.2.2 Efficacy of tax policy when the government is incorrect about peoples' initial beliefs

Proposition 3 establishes that if level-1 beliefs about economy-wide variables coincide with their steady-state values, then the tax policy that delivers the first-best allocation does not depend on the level of sophistication in the economy. A natural question is how sensitive tax policy's efficacy is to a mismatch between the government's views about level-1 beliefs and their actual values. To address this question, we suppose that level-

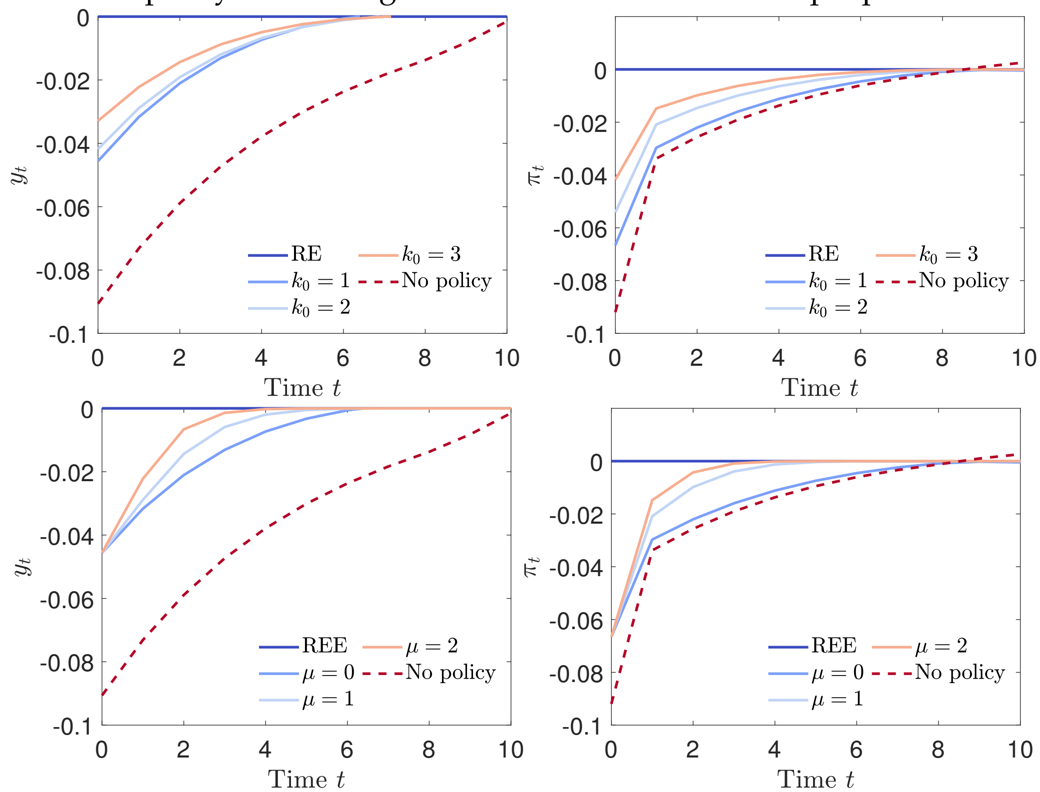
1 beliefs correspond to the rational-expectations ZLB recession without tax policy.[16] We suppose that the government enacts the policy that achieves full stabilization under rational expectations. We then analyze how the economy responds to this policy for different values of μ and k_0.

Figure 4.3 displays the dynamic path of output and inflation (in deviation from steady state) for several values of k_0 in the first row, and μ in the second row. For reference, we also display the dynamic path of this economy under two scenarios: (1) full stabilization, which coincides with the rational-expectations path for the proposed fiscal policy (dark blue line), and (2) no fiscal response (dotted red line). Recall that, with no fiscal response, the equilibrium under level-k thinking with these initial beliefs coincides with the rational-expectations equilibrium for all k.

A number of results emerge. First, while the policy does not achieve full stabilization, it does deliver substantial stimulus. The output gap is closed by more than half, even for the case of unsophisticated agents, $k_0 = 1$ and $\mu = 0$. Second, the higher the level of sophistication, the higher the efficacy of the proposed tax policy. Finally, when μ is one or two, the policy achieves near-full stabilization by the fourth quarter.

[16]We think of these initial beliefs as the worst case scenario for our tax policy exercise, since they would be the limiting result of a learning process during the ZLB. Throughout the learning process, beliefs will be generally more optimistic than in the limit. Christiano et al. (2023) show that it may take very long for a learning process to converge to these limiting beliefs during ZLB spells.

Figure 4.3: Tax policy when the government is incorrect about peoples' initial beliefs



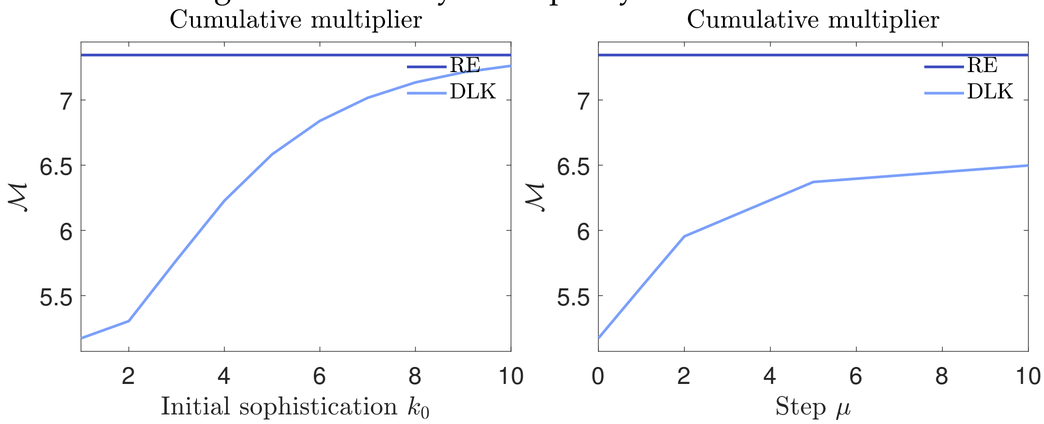
For comparability with our analysis of the fiscal multiplier, we look at the cumulative tax multiplier associated with the proposed tax policy. This multiplier is defined as the cumulative increase in output during the ZLB period induced by the policy, divided by the cumulative direct costs of the policy defined as $dS_t \equiv \hat{\tau}_t^c \times C + \hat{\tau}_t^n \times WN$, where C denotes steady-state consumption:

$$\mathcal{M} \equiv \frac{\sum_{t=0}^{T} dY_t}{\sum_{t=0}^{T} dS_t}.$$

Figure 4.4 displays the cumulative output multipliers for several values of k_0 in the first panel and μ in the second panel.[17] Consistent with our previous observations, the cumulative multipliers are large even though the proposed tax policy does not achieve the flexible-price allocation. Not surprisingly, the multipliers are increasing in people's level of sophistication, i.e., they increase with k_0 and μ.

[17] When we vary k_0 we hold μ fixed at zero and when we change μ, we hold k_0 fixed at one.

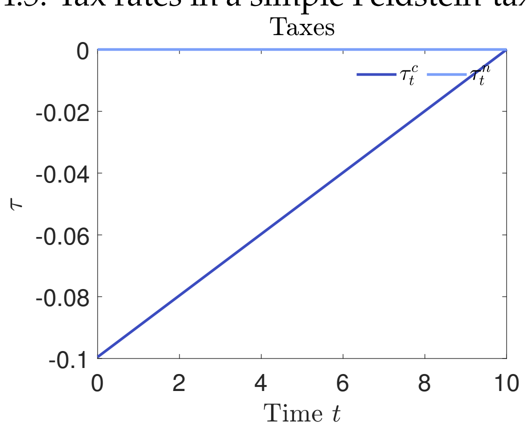
Figure 4.4: Efficacy of tax policy to initial beliefs



4.2.3 The impact of the simple Feldstein-tax policy

To achieve full stabilization, the government must adjust labor taxes to offset the distortionary impact of consumption-tax policy (see Proposition 3). Arguably, governments may be unable to undertake such a nuanced adjustment. So, we consider a simpler tax policy in which the government only changes consumption taxes but doesn't undo the associated distortions with a labor tax. Specifically, we assume that when the discount rate changes, the government decreases the consumption tax rate by 10 percent and raises the tax rate by roughly 1 percent per quarter until the discount rate returns to its pre-shock level (see Figure 4.5).

Figure 4.5: Tax rates in a simple Feldstein-tax policy

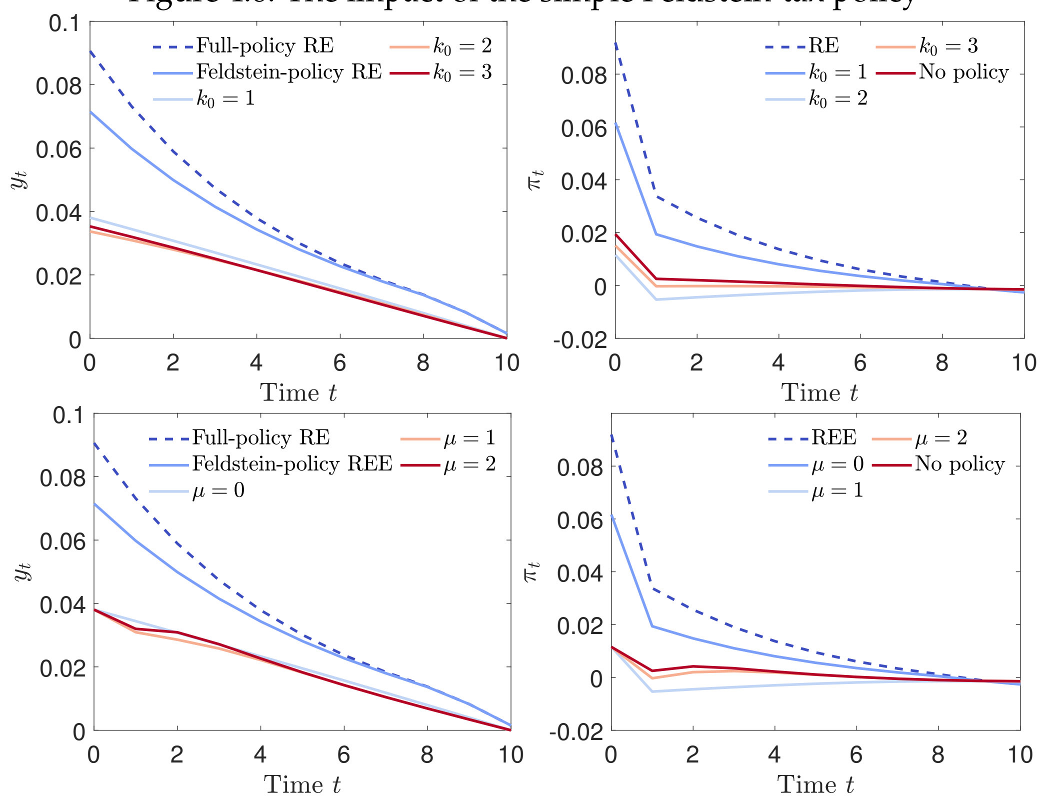


For simplicity, we assume that the government undoes the revenue implications of this policy by adjusting lump-sum taxes. We refer to this policy as a simple Feldstein-tax policy Feldstein (2003).

Recall that the equilibrium with no fiscal response depends on people's sophistication levels. So, to assess the efficacy of the simple Feldstein-tax policy, we display the difference between output and inflation with and without that policy. See Figure 4.6. For reference, we also display the difference between output and inflation under (1) the simple Feldstein-tax policy when people have rational expectations (in light blue) and (2) the full tax policy which delivers the flexible-price allocation (dotted dark blue line).

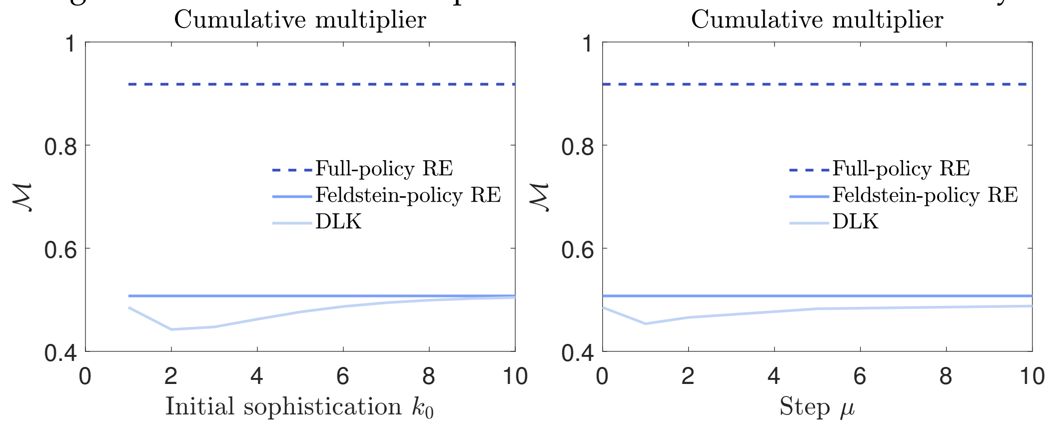
A number of results emerge. First, comparing the dotted dark and light blue lines, we see that the simple Feldstein-tax policy achieves many of the gains of the full tax policy. Second, the simple Feldstein-tax policy achieves substantial stabilization even for low levels of k_0 and μ. In that sense, there are substantial gains even for simple tax policies.

Figure 4.6: The impact of the simple Feldstein-tax policy



The first and second columns in Figure 4.7 display the cumulative tax multipliers for various levels of $k_0$and μ respectively. For reference, we also display the multipliers for the simple Feldstein-tax policy (solid blue line) and the tax policy that delivers full stabilization (dotted blue line) as detailed in Proposition 3. Note that for any k_0 and μ there are substantial stabilization benefits from the simple tax policy.

Figure 4.7: Cumulative Multipliers under a Pure Feldstein-Tax Policy



5 Communication

Proposition 3 provides a strong rationale for using tax policy to fight recessions at the ZLB. In this section, we highlight that the efficacy of the policy depends crucially on how it is communicated. We consider two communication strategies. The first is a tax policy communicated and implemented as a *sequence* of actual tax rates, as we have been working until now. The second is a tax policy that is communicated and implemented as a *rule* involving endogenous objects like inflation and the output gap. We refer to these two strategies as *sequence-based* and *rule-based* communication policies. The reason that communication matters in our setting is straightforward. Under sequence-based communication, individuals immediately know what tax rates will be in the future and incorporate those rates into their decisions. But under rule-based communication, individuals must work out the policy's future general-equilibrium effects to know what current and future tax rates will be. This difference matters in a world populated by dynamic-level- k thinkers.

For rules-based policy, we assume that the interest rate is given by a Taylor rule subject to a ZLB constraint,

$$R_t = \max\left\{\beta^{-1}\left(\frac{P_t}{P_{t-1}}\right)^{\phi_\pi} Y_t^{\phi_y}, 1\right\}. \tag{5.1}$$

Here ϕ_π is the coefficient on realized inflation and ϕ_y is the elasticity of the interest rate with respect to the output gap. As in Correia et al. (2013), we assume that the rule for consumption taxes and labor-income taxes is

$$\frac{1+\tau_t^c}{1+\tau_{t+1}^c} = \min\left\{\beta^{-1}\left(\frac{P_t}{P_{t-1}}\right)^{\phi_\pi} Y_t^{\phi_y}, 1\right\}, \tag{5.2}$$

and

$$\frac{1-\tau_t^n}{1+\tau_t^c} = \frac{1-\tau^n}{1+\tau^c}. \tag{5.3}$$

The government announces tax policy in the form of *rules*, (5.1)-(5.3). Under this tax policy, if the ZLB binds, consumption and labor tax rates change from their steady-state values. Regardless of whether ZLB binds or not, the relative price of consumption is given by:

$$R_t \frac{1+\tau_t^c}{1+\tau_{t+1}^c} = \beta^{-1}\left(\frac{P_t}{P_{t-1}}\right)^{\phi_\pi} Y_t^{\phi_y}.$$

Critically, under this announced policy, agents must predict current and future output values to forecast future tax rates. Peoples' beliefs regarding monetary and tax policies satisfy

$$F_t\left[R_t \frac{1+\tau_t^c}{1+\tau_{t+1}^c}\right] = \beta^{-1}\left(\frac{P_t}{P_{t-1}}\right)^{\phi_\pi} \left(F_t\left[Y\right]\right)^{\phi_y}$$

and

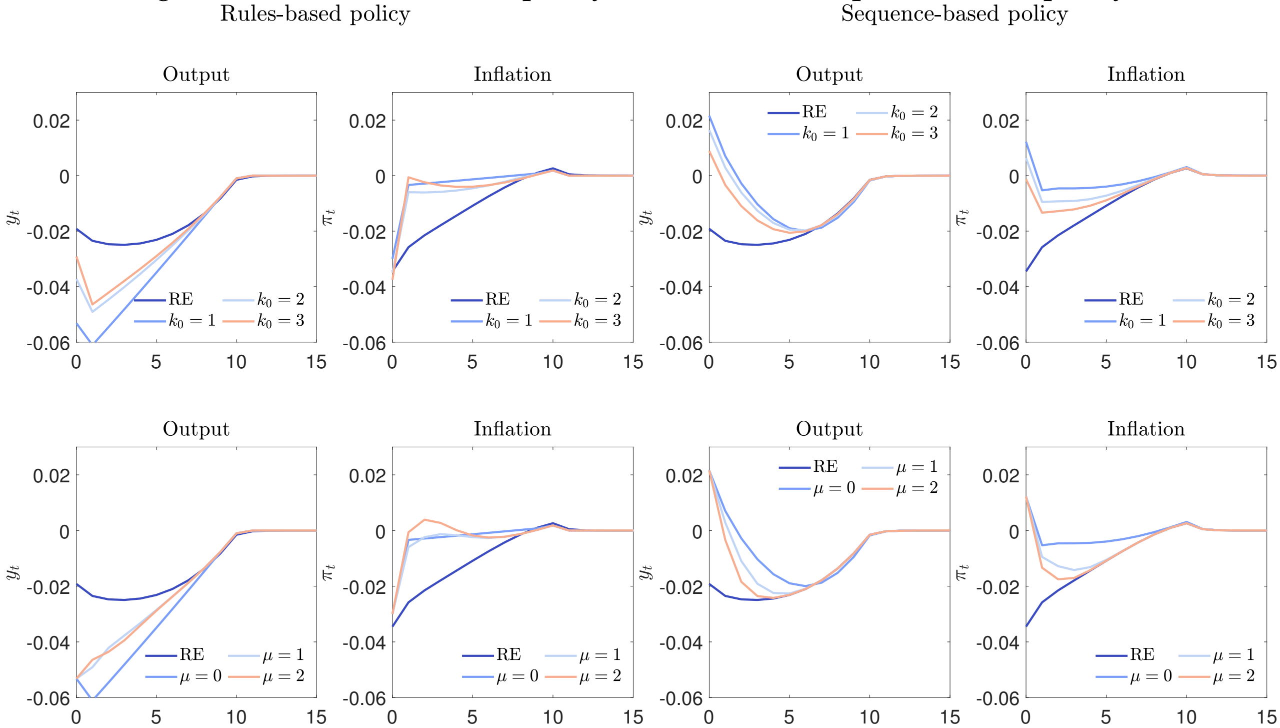
$$F_t\left[\frac{1-\tau_t^n}{1+\tau_t^c}\right] = F_t\left[\frac{1-\tau^n}{1+\tau^c}\right].$$

Unlike the tax policies we have considered so far, this rules-based policy will not implement the flexible-price allocation. However, Correia et al. (2013) show that, under rational expectations, this rules-based policy may still have a strong stimulus power. In our quantitative analysis, we follow Christiano et al. (2011) and set $\phi_\pi = 1.5$ and $\phi_y = 0.25$. In addition, we set $F_t^1\left[\mathcal{A}_{t+h}\right] = \mathcal{A}$.

To compare the equilibrium under the rules-based policy to a similar sequence-based policy, we proceed as follows. First, we solve for the equilibrium under rational expectations and obtain the implied path of consumption and labor tax rates. We then feed this path of tax rates as an announced sequence in our dynamic level-k economy and compare the levels of output and inflation obtained under the two modes of policy communication.

Figures 5.1 display our main results for different levels of k_0 and μ. Columns 1 and 2 show the log deviation of output and inflation from their steady-state values under a rules-based policy. Columns 3 and 4 display the analog results for the sequence-based policy. The first and second rows show sensitivity to varying k_0 and μ, respectively. For comparison, the dark blue lines in Figure 5.1 depict the equilibrium under rational expectations.

Figure 5.1: Communication policy: Rules versus sequence-based policy



Note that the rules-based fiscal policy has a powerful stabilizing influence on the economy under rational expectations. Without any fiscal response, the maximal drop in output exceeds seven percent. With rational expectations, the maximal decline in output would be roughly two percent under the rules-based fiscal policy. Figure 5.1 shows that under dynamic level-k thinking, rules-based fiscal policy is much less potent than under rational expectations. For example, when $k_0 = 1$ and $\mu = 0$, the maximal decline in output is slightly over six percent, i.e., three times as large as under rational expectations. For larger values of k_0, the efficacy of rules-based fiscal policy increases as people can better understand the evolution of future tax rates. When $\mu > 0$, the economy converges faster to the rational-expectations benchmark. Nevertheless, the cumulative output loss in the recession is higher than under rational expectations for all finite levels of μ.

Figure 5.1 shows that the sequence-based policy is much more powerful than the rules-based tax policy. Indeed, under the sequence-based policy, output in the dynamic level-k economy is higher than under rational expectations. Interestingly, the lower the level of sophistication in the economy, the higher the output level. We see that output is higher for low levels of k_0 and lower levels of μ. The intuition for this result is as follows. When the policy is directly announced as a sequence of tax rates, individuals immediately adjust their consumption and savings choices. However, due to their limited GE reasoning, they do not fully internalize the negative indirect effects on today's demand arising from lower future income and lower inflation associated with being at the ZLB. So the demand for consumption is higher than it would be under rational expectations, as are the equilibrium levels of output and inflation. Sequence-based communication is more effective than rules-based communication when the ZLB is binding.

6 What happens when Ricardian Equivalence fails?

In the previous sections, we assume that people fully understand the government-budget constraint and how lump-sum taxes adjust to different fiscal policies. Along with our other assumptions, this information structure implies that Ricardian equivalence holds. In this section, we investigate the consequences of abandoning this information structure. Specifically, we assume that level-1 people do not understand the government budget constraint. Higher level k people understand that constraint but, their views about the variables in the constraint don't coincide with their rational-expectations values. The net result of these assumptions is that Ricardian equivalence does not hold in the dynamic level-k economy. We show that for plausible values of k_0 and μ, our results about the relative efficacy of spending versus taxes are robust to this failure of Ricardian equivalence.

Households Given arbitrary beliefs about labor income, capital income, taxes, and inflation, the household's maximization problem implies that consumption satisfies:

$$C_t = \frac{\sum_{h\geq 0} Q_{t,t+h} F_t\left[\frac{P_{t+h}}{P_t}\right]\left\{\left(1-\tau^n_{t+h}\right)F_t\left[\frac{W_{t+h}}{P_{t+h}}\right]F_t\left[N_{t+h}\right]+F_t\left[\Omega_{t+h}\right]-F_t\left[T_{t+h}\right]\right\}+R_{t-1}B_t}{\left(1+\tau_t\right)\left[1+\sum_{h\geq 1}\left(\beta^h\frac{\xi_{t+h}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+h}F_t\left[\frac{P_{t+h}}{P_t}\right]\frac{1+\tau^c_{t+h}}{1+\tau_t}\right]^{1-\sigma}\right]}. \tag{6.1}$$

People have beliefs about the path of lump-sum taxes and other aggregate variables. They do not need to have beliefs about future government debt since that variable does not enter their decision problem directly. But it is useful to define their *implicit beliefs* regarding the path of future government debt. Using the government budget constraint (4.9), these *implicit beliefs* about B_{t+1} are given by

$$F_t\left[B_{t+1}\right]=\sum_{h=1} Q_{t,t+h}F_t\left[\frac{P_{t+h}}{P_t}\right]\left\{\tau^n_{t+h}F_t\left[\frac{W_{t+h}}{P_{t+h}}\right]F_t\left[N_{t+h}\right]+\tau^c_{t+s}\left(F_t\left[Y_{t+h}\right]-G_{t+h}\right)+F_t\left[T_{t+h}\right]-G_{t+h}\right\}. \tag{6.2}$$

Using (6.2), we can simplify equation (6.1) as follows:

$$C_t = \frac{\sum_{h\geq 0} Q_{t,t+h} F_t\left[\frac{P_{t+h}}{P_t}\right]\frac{1+\tau^c_{t+h}}{1+\tau_t}\left(F_t\left[Y_{t+h}\right]-G_{t+h}\right)+B_{t+1}-F_t\left[B_{t+1}\right]}{1+\sum_{h\geq 1}\left(\beta^s\frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+s}F_t\left[\frac{P_{t+h}}{P_t}\right]\frac{1+\tau^c_{t+h}}{1+\tau_t}\right]^{1-\sigma}}. \tag{6.3}$$

The analog equation for consumption when Ricardian equivalence holds is given by (4.7) which we repeat for convenience:

$$C_t = \frac{\sum_{h\geq 0} Q_{t,t+h}\frac{F_t[P_{t+h}]\left(1+\tau^c_{t+h}\right)}{P_t(1+\tau_t)}\left[F_t\left[Y_{t+h}\right]-G_{t+h}\right]}{1+\sum_{h\geq 1}\left(\beta^h\frac{\xi_{t+h}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+h}F_t\left[\frac{P_{t+h}}{P_t}\right]\frac{1+\tau^c_{t+h}}{1+\tau^c_t}\right]^{1-\sigma}}.$$

Note that there is an additional term $B_{t+1} - F_t\left[B_{t+1}\right]$ in (6.3). This term captures the difference between the actual value of government debt at the beginning of $t+1$ and people's implicit time t belief about the value of that debt. If $B_{t+1} - F_t\left[B_{t+1}\right] > 0$, then consumption is higher. In effect, higher-than-anticipated government spending is perceived by people as an increase in their net wealth. This failure of Ricardian equivalence arises for reasons similar to those modeled in Eusepi and Preston (2018) and Woodford and Xie (2022).

Fiscal policy As in section 4, we assume that the sequence of government spending, consumption, and labor tax rates are announced and fully internalized by individuals. Because Ricardian equivalence doesn't hold, we must make assumptions about how lump-sum taxes and debt evolve. For simplicity, we assume that government debt evolves according to

$$B_{t+1} = \left(1-\rho_B\right)B + \rho_B B_t + \rho_B\left(G_t - G - \left(\tau^c_t - \tau^c\right)C - \left(\tau^n_t - \tau^n\right)WN\right). \tag{6.4}$$

This rule is similar to the one used in Auclert et al. (2018). According to (6.4), government debt returns to steady state with persistence ρ_B and with an adjustment for government spending and distortionary taxes. Lump-sum taxes to ensure that (6.4) holds.

For example, suppose that distortionary taxes are always equal to zero so that the time t government flow budget constraint is given by $T_t = G_t + R_t B_t - B_{t+1}$. Replacing B_{t+1} with (6.4), we obtain

$$T_t = G + \left(R_t - 1\right)B_t + \left(1-\rho_B\right)\left(B_t - B\right) + \left(1-\rho_B\right)\left(G_t - G\right).$$

In steady state, lump-sum taxes are given by $T = G + \left(R-1\right)B$. So, in this case, the path of lump-sum taxes that enforces (6.4) is given by:

$$T_t = T + (R_t - 1)\, B_t - (R - 1)\, B + (1 - \rho_B)\,(B_t - B) + (1 - \rho_B)\,(G_t - G)\,.$$

An analog expression holds for the case when there distortionary taxes.

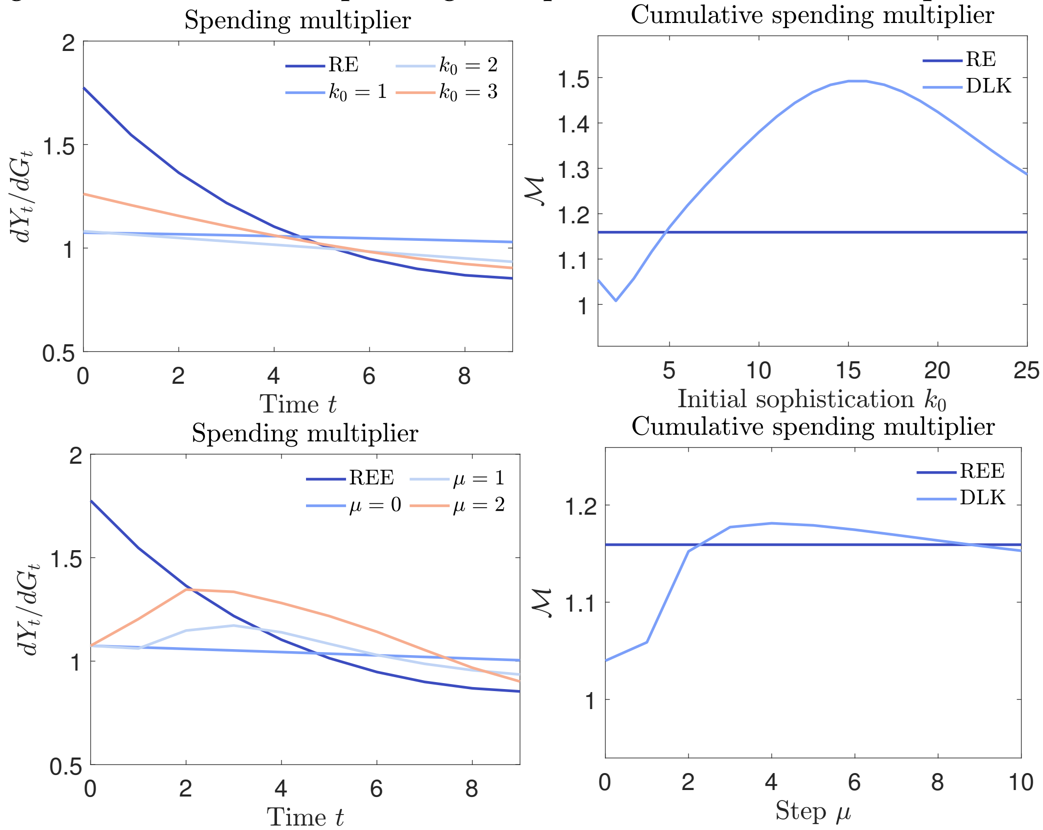
In what follows, we set the persistence of government debt to 0.9 quarterly, which is within the range of estimates in the literature, see for example, Galí et al. (2007). We also assume that the steady-state ratio of debt to GDP is 0.65, it's value at the onset of the 2008 Financial crisis.

6.1 Spending multipliers

Figure 6.1 is the analog to Figure 4.1 for the case in which Ricardian equivalence doesn't hold. The first column of Figure 6.1 displays the government-spending multipliers, $\Delta Y_t/\Delta G_t$, $t = 0, 1..., 9$, for various levels of k_0 and μ. When we change k_0 we hold μ fixed at zero. When we vary μ, we hold k_0 fixed at one. For reference, we also display the government-spending multipliers for the case of rational expectations in dark blue.

Comparing Figure 6.1 and 4.1, three key conclusions emerge. First, for low values of k_0 and μ, the values of the multipliers are similar. Second, the value of the multiplier is somewhat higher when Ricardian equivalence fails. The reason is straightforward: people spend more when they don't fully internalize the increase in lump-sum taxes associated with increased government spending. Third, for μ greater than one, the multiplier can be larger than its rational-expectations value at various times. Recall that the failure of Ricardian equivalence means that people don't entirely internalize the increase in taxes associated with increased government spending. So, they feel wealthier and spend more on consumption than people who have rational expectations. At the same time, less sophisticated people don't internalize that government spending will increase their income and inflation in the future. This force induces them to spend less than people who have rational expectations. For low levels of sophistication, the second force dominates, and the multiplier is lower when people don't have rational expectations. The opposite is true for high levels of sophistication. So k_0 close to 6 or μ close to 3, the cumulative multiplier is higher under dynamic-k level thinking than under rational expectations. As stressed above, these values of k_0 and μ are implausibly large relative to the available empirical evidence.

Figure 6.1: Government-spending multipliers without Ricardian Equivalence



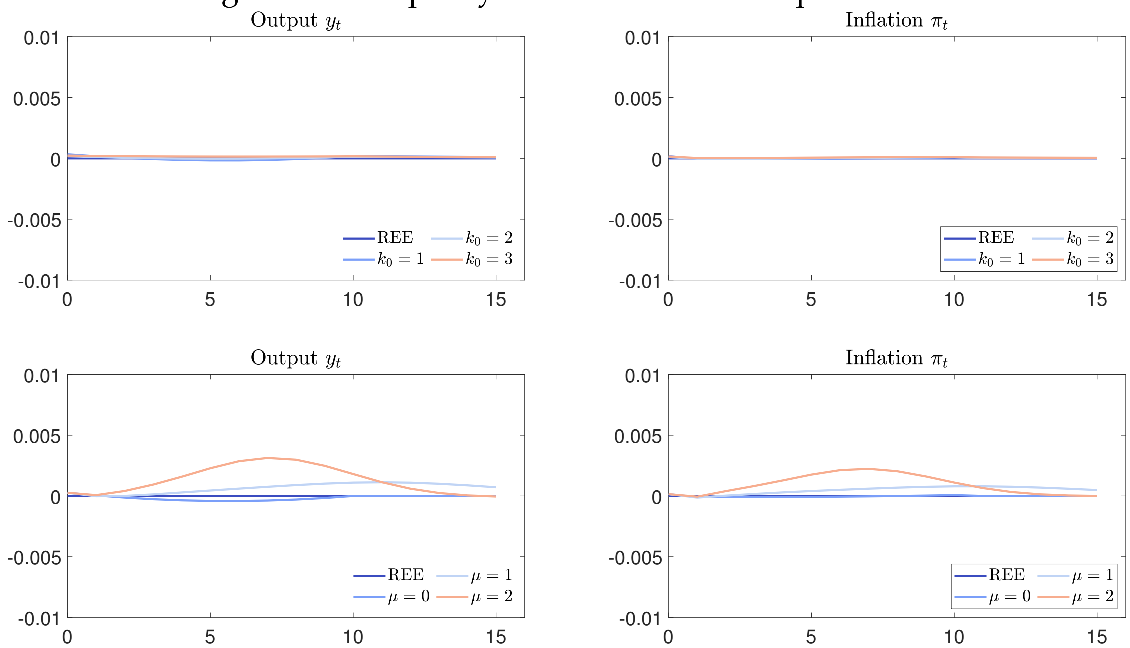
We conclude that for realistic values of k_0 and μ, our quantitative model implies that deviations from rational expectations dampen the efficacy of government spending in stimulating demand at the ZLB, even if Ricardian equivalence fails.

6.2 Tax policy

This section analyzes the efficacy of sequence-based tax policy when Ricardian equivalence doesn't hold. We assume that consumption and labor income taxes are given by (4.16) and (4.17). Recall that with Ricardian equivalence, this sequence of tax rates supported the flexible-price allocation. When Ricardian equivalence does not hold, this result doesn't obtain, i.e., when tax rate changes are deficit-financed, they also generate perceived wealth effects on aggregate demand.

Figure 6.2 displays our results for the non-Ricardian economy. The first and second column Figure 6.2 displays the log-deviation of output and inflation from steady state, respectively, for various levels of $k_0$and μ.

Figure 6.2: Tax policy without Ricardian Equivalence



Two key features are worth noting. First, the wealth effects associated with the tax policies induce an increase in aggregate demand, so output and inflation are higher than under rational expectations. Second, these wealth effects are small, so the sequence-based tax policy generates outcomes that are quantitatively close to those obtained when Ricardian equivalence holds. For example, output deviations from the flexible-price allocation are always lower than 0.4 percentage points.

The intuition for why the wealth effects are small is as follows. Under the proposed policy, the government lowers consumption taxes. But labor taxes rise to offset the distortions in labor supply. So lower consumption tax revenues are partly offset by higher labor taxes revenues. So the increase in net wealth perceived by consumption is small.

7 Conclusions

This paper addresses the question: how sensitive is the power of fiscal policy at the ZLB to the assumption of rational expectations? We do so using a standard NK model in which people have a limited understanding of the general-equilibrium effects of fiscal policy. Specifically, we assume that people form beliefs about future endogenous variables via *dynamic-level-k thinking*. This version of level-k thinking enables us to investigate how the power of fiscal policy depends on both the level of people's cognitive sophistication and how quickly they learn over time.

We conclude with the observation that, in many contexts, the assumption of rational expectations is very useful. But when people are confronted with novel circumstances, it is crucial to assess which results are robust to the assumption of rational expectations. Our analysis documents that conclusions about the efficacy of government spending as a way of stabilizing the economy when the ZLB binds are not robust to deviations from the assumption of rational expectations. In sharp contrast, conclusions about the efficacy of tax policy are robust to those deviations.

References

ADAM, K. AND A. MARCET (2011): "Internal Rationality, Imperfect Market Knowledge and Asset Prices," *Journal of Economic Theory*, 146, 1224–1252.

ANGELETOS, G.-M. AND Z. HUO (2018): "Myopia and Anchoring," Working Paper 24545, National Bureau of Economic Research.

ANGELETOS, G.-M., Z. HUO, AND K. A. SASTRY (2021): "Imperfect macroeconomic expectations: Evidence and theory," *NBER Macroeconomics Annual*, 35, 1–86.

ANGELETOS, G.-M. AND C. LIAN (2017): "Dampening General Equilibrium: From Micro to Macro," Working Paper 23379, National Bureau of Economic Research.

——— (2018): "Forward Guidance Without Common Knowledge," *American Economic Review*, 108, 2477–2512.

ANGELETOS, G.-M. AND K. A. SASTRY (2020): "Managing Expectations:Instruments vs. Targets," *The Quarterly Journal of Economics*, qjaa045.

AUCLERT, A., M. ROGNLIE, AND L. STRAUB (2018): "The Intertemporal Keynesian Cross," Working Paper 25020, National Bureau of Economic Research.

——— (2020): "Micro jumps, macro humps: Monetary policy and business cycles in an estimated HANK model," Tech. rep., National Bureau of Economic Research.

BACHMANN, R., B. BORN, O. GOLDFAYN-FRANK, G. KOCHARKOV, R. LUETTICKE, AND M. WEBER (2021): "A Temporary VAT Cut as Unconventional Fiscal Policy," Working Paper 29442, National Bureau of Economic Research.

BIANCHI-VIMERCATI, R. (2022): "Learning Unconventional Policies: Forward Guidance with Integrated Reasoning," Working paper.

BILBIIE, F. O. (2011): "Nonseparable Preferences, Frisch Labor Supply, and the Consumption Multiplier of Government Spending: One Solution to a Fiscal Policy Puzzle," *Journal of Money, Credit and Banking*, 43, 221–251.

BORDALO, P., N. GENNAIOLI, Y. MA, AND A. SHLEIFER (2020): "Overreaction in macroeconomic expectations," *American Economic Review*, 110, 2748–2782.

BORDALO, P., N. GENNAIOLI, AND A. SHLEIFER (2012): "Salience theory of choice under risk," *The Quarterly journal of economics*, 127, 1243–1285.

BOSCH-DOMENECH, A., J. G. MONTALVO, R. NAGEL, AND A. SATORRA (2002): "One, two,(three), infinity,...: Newspaper and lab beauty-contest experiments," *American Economic Review*, 92, 1687–1701.

CAMERER, C. F., T.-H. HO, AND J.-K. CHONG (2004): "A Cognitive Hierarchy Model of Games," *The Quarterly Journal of Economics*, 119, 861–898.

CHETTY, R., A. GUREN, D. MANOLI, AND A. WEBER (2011): "Are Micro and Macro Labor Supply Elasticities Consistent? A Review of Evidence on the Intensive and Extensive Margins," *American Economic Review*, 101, 471–75.

CHRISTIANO, L., M. EICHENBAUM, AND B. JOHANNSEN (2023): "The Rate of Convergence in Learning Models to Rational Expectations," Tech. rep., Mimeo.

CHRISTIANO, L., M. EICHENBAUM, AND S. REBELO (2011): "When is the Government Spending Multiplier Large?" *Journal of Political Economy*, 119, 78–121.

COIBION, O. AND Y. GORODNICHENKO (2012): "What Can Survey Forecasts Tell Us About Information Rigidities?" *Journal of Political Economy*, 120, 116–159.

——— (2015): "Information rigidity and the expectations formation process: A simple framework and new facts," *American Economic Review*, 105, 2644–78.

CORREIA, I., E. FARHI, J. P. NICOLINI, AND P. TELES (2013): "Unconventional Fiscal Policy at the Zero Bound," *American Economic Review*, 103, 1172–1211.

CRAWFORD, V. P., M. A. COSTA-GOMES, AND N. IRIBERRI (2013): "Structural Models of Nonequilibrium Strategic Thinking: Theory, Evidence, and Applications," *Journal of Economic Literature*, 51, 5–62.

D'ACUNTO, F., D. HOANG, AND M. WEBER (2020): "Managing Households' Expectations with Unconventional Policies," Tech. rep., National Bureau of Economic Research.

EGGERTSSON, G. B. (2011): "What Fiscal Policy is Effective at Zero Interest Rates?" *NBER Macroeconomics Annual*, 25, 59–112.

ERCEG, C. J., D. W. HENDERSON, AND A. T. LEVIN (2000): "Optimal Monetary Policy with Staggered Wage and Price Contracts," *Journal of monetary Economics*, 46, 281–313.

EUSEPI, S. AND B. PRESTON (2018): "Fiscal Foundations of Inflation: Imperfect Knowledge," *American Economic Review*, 108, 2551–2589.

EVANS, G. W. AND G. RAMEY (1992): "Expectation Calculation and Macroeconomic Dynamics," *The American Economic Review*, 207–224.

FARHI, E., M. PETRI, AND I. WERNING (2020): "The Fiscal Multiplier Puzzle: Liquidity Traps, Bounded Rationality, and Incomplete Markets," mimeo.

FARHI, E. AND I. WERNING (2016): "Fiscal Multipliers: Liquidity Traps and Currency Unions," in *Handbook of macroeconomics*, Elsevier, vol. 2, 2417–2492.

——— (2019): "Monetary Policy, Bounded Rationality, and Incomplete Markets," *American Economic Review*, 109, 3887–3928.

FELDSTEIN, M. (2003): "Rethinking Stabilization," Tech. rep.

GABAIX, X. (2020): "A Behavioral New Keynesian Model," *American Economic Review*, 110, 2271–2327.

GALÍ, J., J. D. LÓPEZ-SALIDO, AND J. VALLÉS (2007): "Understanding the Effects of Government Spending on Consumption," *Journal of the European Economic Association*, 5, 227–270.

GALLEGOS, J.-E. (2023): "HANK Beyond FIRE: Amplification, Forward Guidance and Belief Shocks," .

GARCÍA-SCHMIDT, M. AND M. WOODFORD (2019): "Are Low Interest Rates Deflationary? A Paradox of Perfect-Foresight Analysis," *American Economic Review*, 109, 86–120.

GUERREIRO, J. (2022): "Belief disagreement and business cycles," *Job market paper, northwestern university*.

HO, T.-H., C. CAMERER, AND K. WEIGELT (1998): "Iterated Dominance and Iterated Best Response in Experimental P-Beauty Contests"," *The American Economic Review*, 88, 947–969.

IOVINO, L. AND D. SERGEYEV (2018): "Central Bank Balance Sheet Policies Without Rational Expectations," .

NAGEL, R. (1995): "Unraveling in Guessing Games: An Experimental Study," *The American Economic Review*, 85, 1313–1326.

ROTEMBERG, J. J. AND M. WOODFORD (1997): "An Optimization-Based Econometric Framework for the Evaluation of Monetary Policy," *NBER macroeconomics annual*, 12, 297–346.

SCHMITT-GROHÉ, S. AND M. URIBE (2005): "Optimal Fiscal and Monetary Policy in a Medium-Scale Macroeconomic Model," *NBER Macroeconomics Annual*, 20, 383–425.

STAHL, D. O. AND P. W. WILSON (1995): "On Players' Models of Other Players: Theory and Experimental Evidence," *Games and Economic Behavior*, 10, 218–254.

WERNING, I. (2011): "Managing a Liquidity Trap: Monetary and Fiscal Policy," Tech. rep., National Bureau of Economic Research.

WIEDERHOLT, M. (2015): "Empirical Properties of Inflation Expectations and the Zero Lower Bound," Working paper.

WOLF, C. K. (2021): "Interest Rate Cuts vs. Stimulus Payments: A Macro Equivalence Result," Working paper.

WOODFORD, M. (2001): "Imperfect Common Knowledge and the Effects of Monetary Policy," .

——— (2011): "Simple Analytics of the Government Expenditure Multiplier," *American Economic Journal: Macroeconomics*, 3, 1–35.

——— (2018): "Monetary Policy Analysis When Planning Horizons are Finite," *NBER Macroeconomics Annual*, 33, 1–50.

WOODFORD, M. AND Y. XIE (2019): "Policy Options at the Zero Lower Bound When Foresight is Limited," in *AEA Papers and Proceedings*, vol. 109, 433–37.

——— (2022): "Fiscal and monetary stabilization policy at the zero lower bound: Consequences of limited foresight," Tech. rep.

A Appendix to Section 3

A.1 Proof of proposition 1

We begin by establish that these properties hold for the outcome of any standard level-k thinking economy, i.e., with no step $\mu = 0$. Let Y_t^k denote the level of output in an economy with constant sophistication k. We can solve for the government-spending multiplier recursively as follows:

$$\frac{\Delta Y_t^k}{\Delta G_t} = 1 + \Omega_t \sum_{h=1}^{T-t-1} \left[\frac{\Delta Y_{t+h}^{k-1}}{\Delta G_{t+h}} - 1 \right] \frac{\Delta G_{t+h}}{\Delta G_t},$$

where the level-1 government-spending multiplier is given by

$$\frac{\Delta Y_t^1}{\Delta G_t} = 1 + \Omega_t \sum_{h=1}^{T-t-1} [\eta - 1] \frac{\Delta G_{t+h}}{\Delta G_t}$$

Suppose that $0 \leq \eta < 1$. Note that since $\Delta G_{t+h}/\Delta G_t > 0$, then $\Delta Y_t^1/\Delta G_t \leq 1$ for all t. By induction, suppose that $\Delta Y_t^{k-1}/\Delta G_t \leq 1$ for all t, then

$$\frac{\Delta Y_t^k}{\Delta G_t} = 1 + \Omega_t \sum_{h=1}^{T-t-1} \left[\underbrace{\frac{\Delta Y_{t+h}^{k-1}}{\Delta G_{t+h}} - 1}_{\leq 0} \right] \frac{\Delta G_{t+h}}{\Delta G_t} \leq 1,$$

for all t. Furthermore, if $1 - \Omega_t \sum_{h=1}^{T-t-1} \frac{\Delta G_{t+h}}{\Delta G_t} \geq 0$ for all t and $\eta \in [0, 1]$, then

$$\frac{\Delta Y_t^1}{\Delta G_t} = \left\{ 1 - \Omega_t \sum_{h=1}^{T-t-1} \frac{\Delta G_{t+h}}{\Delta G_t} \right\} + \eta \left\{ \Omega_t \sum_{h=1}^{T-t-1} \frac{\Delta G_{t+h}}{\Delta G_t} \right\} \geq \eta$$

for all t. It follows that

$$\frac{\Delta Y_t^2}{\Delta G_t} = 1 + \Omega_t \sum_{h=1}^{T-t-1} \left[\frac{\Delta Y_{t+h}^1}{\Delta G_{t+h}} - 1 \right] \frac{\Delta G_{t+h}}{\Delta G_t} \geq 1 - \Omega_t \sum_{h=1}^{T-t-1} [\eta - 1] \frac{\Delta G_{t+h}}{\Delta G_t} = \frac{\Delta Y_t^1}{\Delta G_t}.$$

By induction, suppose that $\Delta Y_t^k/\Delta G_t \geq \Delta Y_t^{k-1}/\Delta G_t$, then

$$\frac{\Delta Y_t^{k+1}}{\Delta G_t} = 1 + \Omega_t \sum_{h=1}^{T-t-1} \left[\frac{\Delta Y_{t+h}^k}{\Delta G_{t+h}} - 1 \right] \frac{\Delta G_{t+h}}{\Delta G_t} \geq 1 + \Omega_t \sum_{h=1}^{T-t-1} \left[\frac{\Delta Y_{t+h}^{k-1}}{\Delta G_{t+h}} - 1 \right] \frac{\Delta G_{t+h}}{\Delta G_t} = \frac{\Delta Y_t^k}{\Delta G_t}.$$

Then the (1) holds for the standard level-k economy.

Now, suppose that $\eta = 1$, then

$$\frac{\Delta Y_t^1}{\Delta G_t} = 1 + \Omega_t \sum_{h=1}^{T-t-1} [\eta - 1] \frac{\Delta G_{t+h}}{\Delta G_t} = 1.$$

It then follows that if $\Delta Y_t^{k-1} / \Delta G_t = 1$ for all t, then

$$\frac{\Delta Y_t^k}{\Delta G_t} = 1 + \Omega_t \sum_{h=1}^{T-t-1} [1 - 1] \frac{\Delta G_{t+h}}{\Delta G_t} = 1.$$

Note that for any initial sophistication level k_0 and step μ, the spending multiplier is given by

$$\frac{\Delta Y_t}{\Delta G_t} = \frac{\Delta Y_t^{k_0 + \mu t}}{\Delta G_t}.$$

It immediately follows that the results apply for any $k_0 \in \mathbb{N}$ and $\mu \in \mathbb{N}_0$.

A.2 Proof of proposition 2

(1) As we show in the main text, for any level of cognitive sophistication, setting

$$1 + \tau_{T-1} = (1 + \tau)\, e^{-(\chi - \rho)} \tag{A.1}$$

implements $Y_{T-1} = 1$ for all k.

For any k_0 and μ, let $k_t \equiv k_0 + \mu t$. The equilibrium level of output at time t is a function only of current and future consumption taxes plus beliefs about future output:

$$Y_t = \left(\frac{1 + \tau^c}{1 + \tau_t^c} \right)^{\sigma} \frac{(1 - \beta) \sum_{h=1}^{T-t-1} \left(\frac{1+\tau_{t+h}^c}{1+\tau^c} \right) F^{k_t} [Y_{t+h}] + 1}{(1 - \beta) \sum_{h=1}^{T-t-1} e^{\sigma(\chi - \rho)h} \left[\frac{1+\tau_{t+h}^c}{1+\tau^c} \right]^{1-\sigma} + e^{(T-t)\sigma(\chi - \rho)}}.$$

Note that $E^{k_t} [Y_{t+h}]$ is independent of τ_t for all h. It follows that we can construct a policy as follows:

Set τ_{T-1} to the value implied by (A.1). Then, proceed recursively from that date. For each $t \leq T - 2$, fix τ_{t+s} for $s \geq 1$. These imply a path for $E^{k_t} [Y_{t+h}]$ for $h \geq 1$. Let us choose

τ_t so that

$$\left(\frac{1+\tau^c}{1+\tau_t^c}\right)^{\sigma} \frac{(1-\beta)\sum_{h=1}^{T-t-1}\left(\frac{1+\tau_{t+h}^c}{1+\tau^c}\right)F^{k_t}\left[Y_{t+h}\right]+1}{(1-\beta)\sum_{h=1}^{T-t-1}e^{\sigma(\chi-\rho)h}\left[\frac{1+\tau_{t+h}^c}{1+\tau^c}\right]^{1-\sigma}+e^{(T-t)\sigma(\chi-\rho)}}=1$$

or, equivalently,

$$1+\tau_t^c=(1+\tau^c)\left(\frac{(1-\beta)\sum_{h=1}^{T-t-1}\left(\frac{1+\tau_{t+h}^c}{1+\tau^c}\right)F^{k_t}\left[Y_{t+h}\right]+1}{(1-\beta)\sum_{h=1}^{T-t-1}e^{\sigma(\chi-\rho)h}\left[\frac{1+\tau_{t+h}^c}{1+\tau^c}\right]^{1-\sigma}+e^{(T-t)\sigma(\chi-\rho)}}\right)^{1/\sigma}.$$

This implies that

$$Y_t=1$$

for all t.

(2) Suppose that $F_t^1\left[Y_{t+h}\right]=1$. Then, the equilibrium in a standard level-k thinking economy is given by

$$Y_t^1=\left(\frac{1+\tau^c}{1+\tau_t^{c,*}}\right)^{\sigma} \frac{(1-\beta)\sum_{h=1}^{T-t-1}\left(\frac{1+\tau_{t+h}^{c,*}}{1+\tau^c}\right)+1}{(1-\beta)\sum_{h=1}^{T-t-1}e^{\sigma(\chi-\rho)h}\left[\frac{1+\tau_{t+h}^{c,*}}{1+\tau^c}\right]^{1-\sigma}+e^{(T-t)\sigma(\chi-\rho)}}=1.$$

It follows that $F_t^k\left[Y_{t+h}\right]=1$ and $Y_{t+h}^k=1$ for all standard level-k thinking economies. Note that for any initial sophistication level k_0 and step μ, the level of output is given by

$$Y_t=Y_t^{k_0+\mu t}.$$

It immediately follows that the results apply for any $k_0\in\mathbb{N}$ and $\mu\in\mathbb{N}_0$.

B Appendix to Section 4

B.1 Consumption function

The household's optimal consumption plan satisfies:

$$C_t = \frac{\sum_{s\geq 0} Q_{t,t+s} F_t\left[\frac{P_{t+s}}{P_t}\right]\left\{\left(1-\tau^n_{t+s}\right) F_t\left[\frac{W_{t+s}}{P_{t+s}}\right] F_t\left[N_{t+s}\right] + F_t\left[\Omega_{t+s}\right] - F_t\left[T_{t+s}\right]\right\} + R_{t-1}B_t}{\left(1+\tau_t\right)\left[1+\sum_{s\geq 1}\left(\beta^s \frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+s} F_t\left[\frac{P_{t+s}}{P_t}\right]\frac{1+\tau^c_{t+s}}{1+\tau^c_t}\right]^{1-\sigma}\right]}.$$

Given their beliefs for output, the household's expectations for lump-sum taxes are given by 4.6. Replacing beliefs for lump-sum taxes, we obtain:

$$C_t = \frac{\sum_{s\geq 0} Q_{t,t+s} F_t\left[\frac{P_{t+s}}{P_t}\right]\left\{F_t\left[\frac{W_{t+s}}{P_{t+s}}\right] F_t\left[N_{t+s}\right] + \tau^c_{t+s} F_t\left[C_{t+s}\right] + F_t\left[\Omega_{t+s}\right] - F_t\left[G_{t+s}\right]\right\}}{P_t\left(1+\tau^c_t\right)\left[1+\sum_{s\geq 1}\left(\beta^s \frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+s} F_t\left[\frac{P_{t+s}}{P_t}\right]\frac{1+\tau^c_{t+s}}{1+\tau^c_t}\right]^{1-\sigma}\right]}.$$

Using the fact that

$$F_t\left[Y_{t+s}\right] = F_t\left[\frac{W_{t+s}}{P_{t+s}}\right] F_t\left[N_{t+s}\right] + F_t\left[\Omega_{t+s}\right]$$

and

$$F_t\left[C_{t+s}\right] = F_t\left[Y_{t+s}\right] - G_{t+s}$$

we can write the consumption function as

$$C_t = \frac{\sum_{s\geq 0} Q_{t,t+s} F_t\left[P_{t+s}\right]\left\{F_t\left[Y_{t+s}\right] - G_{t+s} + \tau^c_{t+s}\left(F_t\left[Y_{t+s}\right] - G_{t+s}\right)\right\}}{P_t\left(1+\tau^c_t\right)\left[1+\sum_{s\geq 1}\left(\beta^s \frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+s}\frac{P^e_{t+s}\left(1+\tau^c_{t+s}\right)}{P_t\left(1+\tau^c_t\right)}\right]^{1-\sigma}\right]},$$

or equivalently

$$C_t = \frac{\sum_{s\geq 0} Q_{t,t+s} F_t\left[\frac{P_{t+s}}{P_t}\right]\frac{1+\tau^c_{t+s}}{1+\tau^c_t}\left[F_t\left[Y_{t+s}\right] - G_{t+s}\right]}{1+\sum_{s\geq 1}\left(\beta^s \frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+s} F_t\left[\frac{P_{t+s}}{P_t}\right]\frac{1+\tau^c_{t+s}}{1+\tau^c_t}\right]^{1-\sigma}},$$

B.2 Unions and wage setting

In this appendix we solve the problem of the union and derive the wage equation 4.8. The problem of a union that gets to reset its wage is

$$\max_{w_{u,t},\{\widetilde{n}_{u,t+s}\}} \sum_{s\geq 0} (\beta\lambda)^s \left\{ u'\left(F_t\left[C_{t+s}\right]\right) \frac{1-\tau^n_{t+s}}{1+\tau^c_{t+s}} \frac{w_{u,t}\widetilde{n}_{u,t+s}}{F_t\left[P_{t+s}\right]} - v'\left(F_t\left[L_{t+s}\right]\right)\widetilde{n}_{u,t+s} \right\}$$

subject to the constraint

$$\widetilde{n}_{u,t+s} = \left(\frac{w_{u,t}}{F_t\left[W_{t+s}\right]}\right)^{-\theta} N^e_{t+s}.$$

Because every union represents an infinitesimal number of workers in each household, the union does not directly affect aggregate consumption, C_t, hours worked by the household, L_t, the composite labor input, N_t, aggregate wages, W_t, and prices, P_t. As discussed in the main text, we assume that the union has rational expectations with respect to the exogenous variables, but is boundedly rational with respect to future endogenous variables.

The optimal reset wage W^*_t solves the following first order condition:

$$\sum_{s\geq 0} (\beta\lambda)^s \theta v'\left(F_t\left[L_{t+s}\right]\right) \left(\frac{W^*_t}{F_t\left[W_{t+s}\right]}\right)^{-\theta} F_t\left[N_{t+s}\right]$$
$$= \sum_{s\geq 0} (\beta\lambda)^s (\theta-1)\, u'\left(F_t\left[C_{t+s}\right]\right) \frac{1-\tau^n_{t+s}}{1+\tau^c_{t+s}} \frac{W^*_t \left(\frac{W^*_t}{F_t\left[W_{t+s}\right]}\right)^{-\theta} F_t\left[N_{t+s}\right]}{F_t\left[P_{t+s}\right]}$$

which can be equivalently written as follows:

$$\frac{W^*_t}{P_t} = \frac{\theta}{\theta-1} \frac{\sum_{s=0}^{\infty} (\beta\lambda)^s \xi_{t+s} \left(F_t\left[\frac{P_{t+s}}{P_t}\right]\right)^{\theta} \left(F_t\left[\frac{W_{t+s}}{P_{t+s}}\right]\right)^{\theta} F_t\left[N_{t+s}\right] v'\left(F_t\left[L_{t+s}\right]\right)}{\sum_{s=0}^{\infty} (\beta\lambda)^s \xi_{t+s} \left(F_t\left[\frac{P_{t+s}}{P_t}\right]\right)^{\theta-1} \left(F_t\left[\frac{W_{t+s}}{P_{t+s}}\right]\right)^{\theta} F_t\left[N_{t+s}\right] u'\left(F_t\left[C_{t+s}\right]\right) \frac{1-\tau^n_{t+s}}{1+\tau^c_{t+s}}}.$$

B.3 Equilibrium conditions and the linearized system

Given beliefs, a temporary equilibrium denotes a solution to the following system of equations:

1. The consumption function

$$C_t = \frac{\sum_{s\geq 0} Q_{t,t+s} F_t \left[\frac{P_{t+s}}{P_t}\right] \frac{1+\tau^c_{t+s}}{1+\tau^c_t} \left[F_t \left[Y_{t+s}\right] - G_{t+s}\right]}{1 + \sum_{s\geq 1} \left(\beta^s \frac{\xi_{t+s}}{\xi_t}\right)^{\sigma} \left[Q_{t,t+s} F_t \left[\frac{P_{t+s}}{P_t}\right] \frac{1+\tau^c_{t+s}}{1+\tau^c_t}\right]^{1-\sigma}},$$

where we have imposed market clearing, $C_t = Y_t - G_t$.

2. Unions optimal wage setting

$$\frac{W_t^*}{P_t} = \frac{\theta}{\theta - 1} \frac{\sum_{s=0}^{\infty} (\beta\lambda)^s \xi_{t+s} \left(F_t \left[\frac{P_{t+s}}{P_t}\right]\right)^{\theta} \left(F_t \left[\frac{W_{t+s}}{P_{t+s}}\right]\right)^{\theta} F_t \left[N_{t+s}\right] v' \left(F_t \left[L_{t+s}\right]\right)}{\sum_{s=0}^{\infty} (\beta\lambda)^s \xi_{t+s} \left(F_t \left[\frac{P_{t+s}}{P_t}\right]\right)^{\theta - 1} \left(F_t \left[\frac{W_{t+s}}{P_{t+s}}\right]\right)^{\theta} F_t \left[N_{t+s}\right] u' \left(F_t \left[C_{t+s}\right]\right) \frac{1-\tau^n_{t+s}}{1+\tau^c_{t+s}}}.$$

and the aggregate wage is

$$W_t = \left[\lambda W_{t-1}^{1-\theta} + (1-\lambda) \left(W_t^*\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}.$$

3. Real wages are equal to the marginal productivity of labor

$$\frac{W_t}{P_t} = (1-\alpha) A \left(\frac{\overline{K}}{N_t}\right)^{\alpha}.$$

4. Output is given by

$$Y_t = A \overline{K}^{\alpha} N_t^{1-\alpha},$$

where

$$L_t = \mu_t N_t$$

$$\mu_t = \int_0^1 \left(\frac{w_{u,t}}{W_t}\right)^{-\theta} du = \lambda \mu_{t-1} \left(\frac{W_{t-1}}{W_t}\right)^{-\theta} + (1-\lambda) \left(\frac{W_t^*}{W_t}\right)^{-\theta}$$

where $\mu_{-1} = 1$.

5. Market clearing

$$C_t + G_t = Y_t.$$

For each quantity and price X_t we denote their log-linear deviation from steady state by $x_t \equiv \log X_t - \log X$. For taxes we denote their log-linear deviation by $\hat{\tau}^c_t = \log(1+\tau^c_t) - \log(1+\tau^c)$ and $\hat{\tau}^n_t = -\{\log(1-\tau^n_t) - \log(1-\tau^n)\}$. Finally, $\log \xi_{t+1}/\xi_t = \chi_t$, where $\chi_t = \chi > 0$ for $t \leq T-1$ and $\chi_t = 0$ for $t \geq T$. The log-linear system can be written as follows.

Consumption is given by

$$c_t = \frac{(1-\beta)}{\beta}\sum_{s=1}^{\infty}\beta^s \frac{Y}{C}\left\{F_t\left[y_{t+s}\right] - \frac{G}{Y}g_{t+s}\right\} - \sigma\sum_{s=0}^{\infty}\beta^s\left\{r_{t+s} - F_t\left[\pi_{t+s+1}\right] - \left(\hat{\tau}^c_{t+s+1} - \hat{\tau}^c_{t+s}\right) + \chi_{t+s}\right\}. \tag{B.1}$$

Wage inflation $\pi^w_t = w_t - w_{t-1}$ is given by

$$\begin{aligned}\pi^w_t =& \frac{(1-\lambda)(1-\beta\lambda)}{\lambda}\sum_{s\geq 0}^{\infty}(\beta\lambda)^s\left\{\varphi F_t\left[n_{t+s}\right] + \sigma^{-1}F_t\left[c_{t+s}\right] + \hat{\tau}^n_{t+s} + \hat{\tau}^c_{t+s} + \alpha F_t\left[n_{t+s}\right]\right\} \\ & + \frac{1-\lambda}{\lambda}\sum_{s\geq 1}^{\infty}(\beta\lambda)^s F_t\left[\pi^w_{t+s}\right].\end{aligned} \tag{B.2}$$

Below, we show how to derive these two equations below.

Price inflation $\pi_t = p_t - p_{t-1}$ is given by

$$\pi_t = \pi^w_t + \alpha\Delta n_t. \tag{B.3}$$

Finally, output is given by

$$y_t = (1-\alpha)\, n_t, \tag{B.4}$$

and the market clearing condition is

$$\frac{C}{Y}c_t + \frac{G}{Y}g_t = y_t. \tag{B.5}$$

To first order, $n_t = l_t$.

Log-linearized wage inflation Wage setting is given by

$$\frac{W^*_t}{P_t} = \frac{\theta}{\theta-1}\frac{\sum_{s=0}^{\infty}(\beta\lambda)^s\xi_{t+s}\left(F_t\left[\frac{P_{t+s}}{P_t}\right]\right)^{\theta}\left(F_t\left[\frac{W_{t+s}}{P_{t+s}}\right]\right)^{\theta}F_t\left[N_{t+s}\right]v'\left(F_t\left[L_{t+s}\right]\right)}{\sum_{s=0}^{\infty}(\beta\lambda)^s\xi_{t+s}\left(F_t\left[\frac{P_{t+s}}{P_t}\right]\right)^{\theta-1}\left(F_t\left[\frac{W_{t+s}}{P_{t+s}}\right]\right)^{\theta}F_t\left[N_{t+s}\right]u'\left(F_t\left[C_{t+s}\right]\right)\frac{1-\tau^n_{t+s}}{1+\tau^c_{t+s}}}.$$

and the aggregate wage is

$$W_t = \left[\lambda W^{1-\theta}_{t-1} + (1-\lambda)\left(W^*_t\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}.$$

Log-linearizing the wage setting condition we obtain

$$w_t^* - p_t = (1-\beta\lambda) \sum_{s\geq 0}^{\infty} (\beta\lambda)^s \left\{ \varphi F_t [n_{t+s}] + \sigma^{-1} F_t [c_{t+s}] + \hat{\tau}^n_{t+s} + \hat{\tau}^c_{t+s} \right\}$$
$$+ \sum_{s=1}^{\infty} (\beta\lambda)^s F_t [p_{t+s}] - \sum_{s=0}^{\infty} (\beta\lambda)^{s+1} F_t [p_{t+s}],$$

$$\Leftrightarrow w_t^* - w_t = (1-\beta\lambda) \sum_{s\geq 0}^{\infty} (\beta\lambda)^s \left\{ \varphi F_t [n_{t+s}] + \sigma^{-1} F_t [c_{t+s}] + \hat{\tau}^n_{t+s} + \hat{\tau}^c_{t+s} \right\} + \sum_{s=1}^{\infty} (\beta\lambda)^s F_t [p_{t+s}]$$
$$- \sum_{s=0}^{\infty} (\beta\lambda)^{s+1} F_t [p_{t+s}] + p_t - w_t$$

or equivalently,

$$w_t^* - w_t = (1-\beta\lambda) \sum_{s\geq 0}^{\infty} (\beta\lambda)^s \left\{ \varphi F_t [n_{t+s}] + \sigma^{-1} F_t [c_{t+s}] + \hat{\tau}^n_{t+s} + \hat{\tau}^c_{t+s} - (F_t [w_{t+s}] - F_t [p_{t+s}]) \right\}$$
$$+ \sum_{s\geq 1}^{\infty} (\beta\lambda)^s F_t [\pi^w_{t+s}]$$

since $F_t [w_{t+s}] - F_t [p_{t+s}] = -\alpha F_t [n_{t+s}]$ then

$$w_t^* - w_t = (1-\beta\lambda) \sum_{s\geq 0}^{\infty} (\beta\lambda)^s \left\{ \varphi F_t [n_{t+s}] + \sigma^{-1} F_t [c_{t+s}] + \hat{\tau}^n_{t+s} + \hat{\tau}^c_{t+s} + \alpha F_t [n_{t+s}] \right\}$$
$$+ \sum_{s\geq 1}^{\infty} (\beta\lambda)^s F_t [\pi^w_{t+s}].$$

Log-linearizing the aggregate wage condition we obtain

$$w_t = \lambda w_{t-1} + (1-\lambda) w_t^*.$$

Now, define $\pi_t^w = w_t - w_{t-1}$, we can use the equation above to show that

$$\lambda \pi_t^w = (1-\lambda)(w_t^* - w_t) \Leftrightarrow \pi_t^w = \frac{1-\lambda}{\lambda}(w_t^* - w_t).$$

Replacing $w_t^* - w_t$ we find that

$$\pi_t^w = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda} \sum_{s\geq 0}^{\infty} (\beta\lambda)^s \left\{ \varphi F_t [n_{t+s}] + \sigma^{-1} F_t [c_{t+s}] + \hat{\tau}_{t+s}^n + \hat{\tau}_{t+s}^c + \alpha F_t [n_{t+s}] \right\}$$
$$+ \frac{1-\lambda}{\lambda} \sum_{s\geq 1}^{\infty} (\beta\lambda)^s F_t [\pi_{t+s}^w] .$$

B.4 Cognitive hierarchies

For simplicity, we show how to obtain equation (4.15) in a simplified linear model without inflation. It is easy to extend these results to the model with inflation. Given their beliefs, individual's consumption is given by

$$c_t = (1-\beta) \sum_{s=0}^{\infty} \beta^s \frac{Y}{C} F_t [y_{t+s}] - \sigma \sum_{s=0}^{\infty} \beta^{s+1} \left\{ r_{t+s} - \left(\hat{\tau}_{t+s+1}^c - \hat{\tau}_{t+s}^c \right) + \chi_{t+s} \right\} . \quad \text{(B.6)}$$

Following Camerer et al. (2004), we assume that level-k individuals think that other people are distributed over lower levels of cognitive ability according to the distribution $f_k(j)$ for $0 \leq j \leq k-1$. The reasoning process underlying the generalized level-k model is analogous to the process in the standard level-k model. As in Farhi and Werning (2019), we assume that contemporaneous output, y_t, is observed.

To analyze this economy we must introduce the concept of a level-0 person. This type of person continues to act as they did before the discount rate shock, i.e. $y_t^0 = 0$. It is always possible to specify beliefs $\left\{ F_t^0 [y_{t+s}] \right\}$ that support such an action.

Level-1 individuals believe that the economy is populated by level-1 people so $F_t^1 [y_{t+s}] = y_t^0$. Given current output y_t,

$$c_t^1 (y_t) = (1-\beta) \sum_{s=0}^{\infty} \beta^s \frac{Y}{C} F_t^1 [y_{t+s}] - \sigma \sum_{s=0}^{\infty} \beta^{s+1} \left\{ r_{t+s} - \left(\hat{\tau}_{t+s+1}^c - \hat{\tau}_{t+s}^c \right) + \chi_{t+s} \right\} . \quad \text{(B.7)}$$

Suppose that the economy is populated entirely by level-1 individuals. Solving (B.7) for y_t^1 yields,

$$y_t^1 = (1-\beta) \sum_{s=1}^{\infty} \beta^{s-1} F_t^1 [y_{t+s}] - \sigma \frac{C}{Y} \sum_{s=0}^{\infty} \beta^s \left\{ r_{t+s} - \left(\hat{\tau}_{t+s+1}^c - \hat{\tau}_{t+s}^c \right) + \chi_{t+s} \right\} .$$

Level-2 individuals believe that a fraction $f_2(j)$ of the population is level $j = 0, 1$ and work through the problem of level-0 and level-1 people. So they believe that y_t^2 is the solution to

$$y_t^{e,2} = \sum_{j=0}^{1} f_2(j)\, c_t^j \left(y_t^{e,2} \right).$$

More generally, level-k people believe that output is the solution to

$$F_t^k [y_{t+h}] \equiv \sum_{j=0}^{k-1} f_k(j)\, c_{t+g}^j \left(F_t^k [y_{t+h}] \right). \tag{B.8}$$

Since output is contemporaneously observed, people with different cognitive levels expect different consumption levels for people who are less sophisticated than themselves. Technically, this means that level-k people think that level-j people behave according to

$$c_t^j(y_t) = (1-\beta) \sum_{s=0}^{\infty} \beta^s \frac{Y}{C} F_t^j [y_{t+s}] - \sigma \sum_{s=0}^{\infty} \beta^{s+1} \left\{ r_{t+s} - \left(\hat{\tau}_{t+s+1}^c - \hat{\tau}_{t+s}^c \right) + \chi_{t+s} \right\}, \tag{B.9}$$

for $j \geq 1$. For simplicity, we assume that this equation holds for level-0 people.[18]

Using conditions (B.8) and (B.9), the beliefs of level-k individuals can be written as

$$F_t^k [y_{t+h}] = \sum_{j=0}^{k-1} f_k(j)\, y_{t+h}^j,$$

where

$$y_t^j = (1-\beta) \sum_{s=1}^{\infty} \beta^{s-1} F_t^j [y_{t+s}] - \sigma \frac{C}{Y} \sum_{s=0}^{\infty} \beta^s \left\{ r_{t+s} - \left(\hat{\tau}_{t+s+1}^c - \hat{\tau}_{t+s}^c \right) + \chi_{t+s} \right\}.$$

Camerer et al. (2004) assume that the distributions $f_k(\cdot)$ are consistent with the physical distribution of cognitive levels in the economy. In contrast, we maintain the representative agent assumption, so that everyone shares the same level k. We assume that agents of different cognitive levels agree on the relative proportions of lower cognitive levels. The distributions $f_k(\cdot)$ are such that for any $k_1 < k_2$ and $s, s' < k_1$

$$\frac{f_{k_1}(s)}{f_{k_1}(s')} = \frac{f_{k_2}(s)}{f_{k_2}(s')}. \tag{B.10}$$

[18] This assumption is convenient because we suppose that people see contemporaneous aggregate output when making consumption decisions. In a continuous-time version of our economy, consumption would effectively not depend on output.

Let $\gamma_k \equiv f_k(k-1)$ for all k. Then assumption (B.10) implies that $f_k(j) = (1-\gamma_k) f_{k-1}(j)$ for $j \leq k-2$. We can write the expectation of level-k individuals as follows:

$$F_t^k[y_{t+h}] = (1-\gamma_k) \sum_{j=0}^{k-2} f_{k-1}(j)\, y_{t+h}^j + \gamma_k y_{t+h}^{k-1} = (1-\gamma_k) F_t^{k-1}[y_{t+h}] + \gamma_k y_{t+h}^{k-1}. \tag{B.11}$$

Intuitively, the beliefs of a level-k thinker are given by a weighted average of the beliefs of level $k-1$ agents and the equilibrium that would arise if everyone in the economy was a level-$(k-1)$ thinker. Standard level-k thinking corresponds to the case of $\gamma_k = 1$. By varying γ_k, we can control the intensity of updating across level-k iterations.

B.5 Proof of proposition 3

Part 1 The proof strategy is as follows. First, we show that if level-1 people believe that the economy will stay at steady state for $t \geq T$, then all level-k beliefs and corresponding equilibria feature output, consumption, labor and wage inflation remaining at their steady-state levels from $t \geq T$, and price inflation is zero for $t \geq T+1$. Second, we note that beliefs about future output, inflation, consumption, and labor are a function only of future tax rates and policies. Finally, for a given level k, we recursively construct a sequence of policies $\left\{\hat{\tau}_t^{c,k}, \hat{\tau}_t^{n,k}\right\}$ which implements the flexible-price allocation and always features zero inflation for all t.

(1) Suppose that $F_t^1[y_{t+s}] = F_t^1[c_{t+s}] = F_t^1[n_{t+s}] = 0$ and $F_t^1\left[\pi_{t+s}^w\right] = F_t^1[\pi_{t+s}] = 0$ if $t \geq T$. Then, setting $g_t = \hat{\tau}_t^c = \hat{\tau}^n = r_t = 0$ for all $t \geq T$, implies that consumption, output, and labor for $t \geq T$ are given by

$$c_t = \frac{(1-\beta)}{\beta} \sum_{s=1}^{\infty} \beta^s \frac{Y}{C} F_t^1[y_{t+s}] = 0,$$

$$y_t = \frac{C}{Y} c_t = 0,$$

and

$$n_t = \frac{y_t}{1-\alpha} = 0,$$

respectively. Then, wage inflation for $t \geq T$ is given by

$$\pi_t^w = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda} \left\{\varphi n_t + \sigma^{-1} c_t + \alpha n_t\right\} = 0.$$

Finally, this implies that price inflation is

$$\pi_t = \pi_t^w + \alpha \Delta n_t = 0$$

for $t \geq T+1$, and $\pi_T = -\alpha n_{T-1}$. This then shows the initial beliefs $F_t^1 [y_{t+s}] = F_t^1 [c_{t+s}] = F_t^1 [n_{t+s}] = F_t^1 \left[\pi_{t+s}^w\right] = F_t^1 [\pi_{t+s}] = 0$ are consistent with what happens in equilibrium. This result implies that in the actual economy people believe $F_t [y_{t+s}] = F_t [c_{t+s}] = F_t [n_{t+s}] = F_t \left[\pi_{t+s}^w\right] = F_t [\pi_{t+s}] = 0$ for $t \geq T$.

(2) Recall that the temporary equilibrium for time t solves the system of equations (B.1)-(B.5). This equilibrium does not depend on policies before time t. So, for each t, level-k beliefs are unaffected by past policies, $\{\hat{\tau}_s^c, \hat{\tau}_s^n\}_{s=0}^{t-1}$.

(3) For $t = T-1$, the level-k equilibrium levels of consumption and wage inflation solve

$$c_{T-1}^k = -\sigma \left\{-F_t [\pi_T] + \hat{\tau}_{T-1}^c + \chi - \rho\right\},$$

and

$$\pi_{T-1}^w = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda} \left\{\varphi n_{T-1} + \sigma^{-1} c_{T-1} + \hat{\tau}_{T-1}^n + \hat{\tau}_{T-1}^c + \alpha n_{T-1}\right\}.$$

Note that by setting $\hat{\tau}_{t+s}^c = \rho + F_t [\pi_T] - \chi$, then $c_{T-1}^k = 0$. Since consumption remains at its steady-state level, then $y_{T-1} = n_{T-1} = 0$. Setting $\hat{\tau}_{T-1}^n = -\hat{\tau}_{T-1}^c$, implies that $\pi_{T-1}^w = 0$. Furthermore, since $\pi_T = -\alpha n_{T-1}$ then this policy also implies that $\pi_T = 0$.

We now proceed recursively. At time t, fix the future policies $\left\{\hat{\tau}_{t+s}^c, \hat{\tau}_{t+s}^n\right\}_{s\geq 1}$ and the implied beliefs $\left\{F_t [y_{t+s}], F_t [c_{t+s}], F_t [n_{t+s}], F_t \left[\pi_{t+s}^w\right], F_t [\pi_{t+s}]\right\}_{t,s\geq 1}$. Consumption at time t is given by we set $\hat{\tau}_t^{c,k}$ so that

$$c_t = \frac{(1-\beta)}{\beta} \sum_{s=1}^{\infty} \beta^s \frac{Y}{C} F_t [y_{t+s}] - \sigma \sum_{s=0}^{\infty} \beta^s \left\{r_{t+s} - F_t [\pi_{t+s+1}] - \left(\hat{\tau}_{t+s+1}^c - \hat{\tau}_{t+s}^c\right) + \chi_{t+s}\right\}.$$

We set $\hat{\tau}_t^c$ such that $c_t^k = 0$, which implies

$$\begin{aligned}\hat{\tau}_t^c = \frac{(1-\beta)}{\beta\sigma} \sum_{s=1}^{T-t-1} \left[\beta^s \frac{Y}{C} F_t [y_{t+s}]\right] - \left\{-F_t [\pi_{t+1}] - \hat{\tau}_{t+1}^c + \chi - \rho\right\} \\ - \sum_{s=1}^{\infty} \beta^s \left\{-F_t [\pi_{t+s+1}] - \left(\hat{\tau}_{t+s+1}^c - \hat{\tau}_{t+s}^c\right) + \chi_{t+s} - \rho\right\}.\end{aligned}$$

Since $c_t = 0$, it follows from (B.4) and (B.5) that $n_t = y_t = 0$. Wage inflation is given by

$$\pi_t^w = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda} \sum_{s\geq 0}^{\infty} (\beta\lambda)^s \left\{ \varphi F_t [n_{t+s}] + \sigma^{-1} F_t [c_{t+s}] + \hat{\tau}_{t+s}^n + \hat{\tau}_{t+s}^c + \alpha F_t [n_{t+s}] \right\} + \frac{1-\lambda}{\lambda} \sum_{s\geq 1}^{\infty} (\beta\lambda)^s F_t [\pi_{t+s}^w].$$

We set $\hat{\tau}_t^n$ such that $\pi_t^w = 0$, which implies

$$\hat{\tau}_t^n = -\hat{\tau}_t^c - \sum_{s=1}^{\infty} (\beta\lambda)^s \left\{ \varphi F_t [n_{t+s}] + \sigma^{-1} F_t [c_{t+s}] + \hat{\tau}_{t+s}^n + \hat{\tau}_{t+s}^c + \alpha F_t [n_{t+s}] \right\} - \frac{1}{1-\beta\lambda} \sum_{s\geq 1}^{\infty} (\beta\lambda)^s F_t [\pi_{t+s}^w].$$

These policies implement an allocation in which $n_t^k = 0$ and $\pi_t^{w,k} = 0$ for all t. It follows (B.3) from then $\pi_t^k = 0$ for all t.

Part 2 Suppose that beliefs are anchored at the initial steady state. Consider setting taxes on consumption and labor such that

$$\tau_t^c = (1+\tau^c) e^{-(T-t)(\chi-\rho)} - 1.$$

$$\frac{1-\tau_t^n}{1+\tau_t^c} = \frac{1-\tau^n}{1+\tau^c}.$$

Then, consumption is given by

$$C_t = \frac{\sum_{s\geq 1} Q_{t,t+s} \frac{1+\tau_{t+s}^c}{1+\tau_t^c} \{Y - G\}}{\sum_{s\geq 1} \left(\beta^s \frac{\xi_{t+s}}{\xi_t}\right)^\sigma \left[Q_{t,t+s} \frac{1+\tau_{t+s}^c}{1+\tau_t^c}\right]^{1-\sigma}} = \frac{\sum_{s\geq 1} Q_{t,t+s} \frac{1+\tau_{t+s}^c}{1+\tau_t^c}}{\sum_{s\geq 1} \beta^s \frac{\xi_{t+s}}{\xi_t}} \{Y - G\} = C.$$

This implies that

$$Y_t = C_t + G = C + G = Y,$$

and then

$$N_t = \left(\frac{Y}{A\overline{K}^\alpha}\right)^{\frac{1}{1-\alpha}} = N.$$

The reset wage is:

$$\frac{W_t^*}{P_t} = \frac{\theta}{\theta-1} \frac{\sum_{s=0}^{\infty} (\beta\lambda)^s \xi_{t+s} \left(\frac{W}{P}\right)^{\theta} N v'(L)}{\sum_{s=0}^{\infty} (\beta\lambda)^s \xi_{t+s} \left(\frac{W}{P}\right)^{\theta} N u'(C) \frac{1-\tau_{t+s}^n}{1+\tau_{t+s}^c}}$$
$$= \frac{\theta}{\theta-1} \frac{1+\tau^c}{1-\tau^n} \frac{v'(L)}{u'(C)} = \frac{W}{P}.$$

Then, from the first-order condition of the firm we see that W_t/P_t is constant

$$\frac{W_t}{P_t} = (1-\alpha) A \left(\frac{\overline{K}}{N}\right)^{\alpha} = \frac{W}{P}$$

which, combined with

$$\frac{W_t}{P_t} = \left[\lambda \left(\frac{W_{t-1}}{P_{t-1}} \frac{P_{t-1}}{P_t}\right)^{1-\theta} + (1-\lambda) \left(\frac{W_t^*}{P_t}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

implies that $P_t = P_{t-1}$ for all t. Finally, this implies that $N_t = L_t = N$. This result shows that, under the proposed policy, the beliefs become self-confirming. It follows that the result extends to any dynamic level-k thinking economy.

C Appendix to Section 5

C.1 Equilibrium conditions and the linearized system

Given beliefs, a temporary equilibrium denotes a solution to the following system of equations:

1. The consumption function

$$C_t = \frac{\sum_{s\geq 0} F_t \left[Q_{t,t+s} \frac{P_{t+s}}{P_t} \frac{1+\tau_{t+s}^c}{1+\tau_t^c}\right] \left[F_t \left[Y_{t+s}\right] - G_{t+s}\right]}{1 + \sum_{s\geq 1} \left(\beta^s \frac{\xi_{t+s}}{\xi_t}\right)^{\sigma} F_t \left[Q_{t,t+s} \frac{P_{t+s}}{P_t} \frac{1+\tau_{t+s}^c}{1+\tau_t^c}\right]^{1-\sigma}},$$

where we have imposed market clearing, $C_t = Y_t - G_t$.

2. Unions optimal wage setting

$$\frac{W_t^*}{P_t} = \frac{\theta}{\theta-1} \frac{\sum_{s=0}^{\infty} (\beta\lambda)^s \xi_{t+s} \left(F_t \left[\frac{P_{t+s}}{P_t} \right] \right)^{\theta} \left(F_t \left[\frac{W_{t+s}}{P_{t+s}} \right] \right)^{\theta} F_t \left[N_{t+s} \right] v' \left(F_t \left[L_{t+s} \right] \right)}{\sum_{s=0}^{\infty} (\beta\lambda)^s \xi_{t+s} \left(F_t \left[\frac{P_{t+s}}{P_t} \right] \right)^{\theta-1} \left(F_t \left[\frac{W_{t+s}}{P_{t+s}} \right] \right)^{\theta} F_t \left[N_{t+s} \right] u' \left(F_t \left[C_{t+s} \right] \right) F_t \left[\frac{1-\tau_{t+s}^n}{1+\tau_{t+s}^c} \right]}.$$

and the aggregate wage is

$$W_t = \left[\lambda W_{t-1}^{1-\theta} + (1-\lambda) \left(W_t^* \right)^{1-\theta} \right]^{\frac{1}{1-\theta}}.$$

3. Real wages are equal to the marginal productivity of labor

$$\frac{W_t}{P_t} = (1-\alpha) A \left(\frac{\overline{K}}{N_t} \right)^{\alpha}.$$

4. Output is given by

$$Y_t = A \overline{K}^{\alpha} N_t^{1-\alpha},$$

where

$$L_t = \mu_t N_t$$

$$\mu_t = \int_0^1 \left(\frac{w_{u,t}}{W_t} \right)^{-\theta} du = \lambda \mu_{t-1} \left(\frac{W_{t-1}}{W_t} \right)^{-\theta} + (1-\lambda) \left(\frac{W_t^*}{W_t} \right)^{-\theta}$$

where $\mu_{-1} = 1$.

5. Monetary and fiscal policies:

$$\frac{1-\tau_t^n}{1+\tau_t^c} = \frac{1-\tau^n}{1+\tau^c}. \tag{C.1}$$

$$R_t \frac{1+\tau_t^c}{1+\tau_{t+1}^c} = \beta^{-1} \left(\frac{P_t}{P_{t-1}} \right)^{\phi_\pi} Y_t^{\phi_y}.$$

6. Market clearing

$$C_t + G_t = Y_t.$$

For each quantity and price X_t we denote their log-linear deviation from steady state by $x_t \equiv \log X_t - \log X$. For taxes we denote their log-linear deviation by $\hat{\tau}_t^c = \log(1+\tau_t^c) - \log(1+\tau^c)$ and $\hat{\tau}_t^n = -\{\log(1-\tau_t^n) - \log(1-\tau^n)\}$. Finally, $\log \xi_{t+1}/\xi_t = \chi_t$, where $\chi_t = \chi > 0$ for $t \leq T-1$ and $\chi_t = 0$ for $t \geq T$. The log-linear system can be written as follows.

First note that

$$\hat{\tau}_t^n = -\hat{\tau}_t^c$$

and

$$r_t + \hat{\tau}_t^c - \hat{\tau}_{t+1}^c = \phi_\pi \pi_t + \phi_y y_t$$

Consumption is given by

$$c_t = \frac{(1-\beta)}{\beta} \sum_{s=1}^{\infty} \beta^s \frac{Y}{C} \left\{ F_t [y_{t+s}] - \frac{G}{Y} g_{t+s} \right\} - \sigma \sum_{s=0}^{\infty} \beta^s \left\{ \phi_\pi F_t [\pi_{t+s}] + \phi_y F_t [y_{t+s}] - F_t [\pi_{t+s+1}] + \chi_{t+s} \right\}. \tag{C.2}$$

Wage inflation $\pi_t^w = w_t - w_{t-1}$ is given by

$$\pi_t^w = \frac{(1-\lambda)(1-\beta\lambda)}{\lambda} \sum_{s \geq 0}^{\infty} (\beta\lambda)^s \left\{ \varphi F_t [n_{t+s}] + \sigma^{-1} F_t [c_{t+s}] + \alpha F_t [n_{t+s}] \right\} + \frac{1-\lambda}{\lambda} \sum_{s \geq 1}^{\infty} (\beta\lambda)^s F_t [\pi_{t+s}^w]. \tag{C.3}$$

Below, we show how to derive these two equations below.

Price inflation $\pi_t = p_t - p_{t-1}$ is given by

$$\pi_t = \pi_t^w + \alpha \Delta n_t. \tag{C.4}$$

Finally, output is given by

$$y_t = (1-\alpha) n_t, \tag{C.5}$$

and the market clearing condition is

$$\frac{C}{Y} c_t + \frac{G}{Y} g_t = y_t. \tag{C.6}$$

To first order, $n_t = l_t$.

D Bounded rationality – alternative models

In the benchmark model, we assume that people are standard level-k thinkers. However, our results do not depend crucially on the specific assumptions underlying this model of bounded rationality. In this appendix, we show that the main results of our model continue to hold under alternative models of bounded rationality. We first derive the benchmark model under a generalization level-k thinking model called "Cognitive Hierarchies" based on Camerer et al. (2004). Second, we show that our results are also robust to assuming that people have *reflective expectations* as in García-Schmidt and Woodford (2019). Finally, we also show that our results hold under the *shallow reasoning* model of Angeletos and Sastry (2020). For simplicity, we show this for the benchmark model without inflation, but these same principles hold more generally. Furthermore, we keep peoples' cognitive ability constant over time to keep the models close to those developed in Camerer et al. (2004), García-Schmidt and Woodford (2019), and Angeletos and Sastry (2020). It would be straightforward to extend these models to include learning in real time.

D.1 Generalized level-k thinking – Cognitive Hierarchies

In this section, we show that our results for the standard level-k thinking in the benchmark model go through in the generalized level-k thinking model. We restrict our analysis to the case in which policies are announced as targets, since we already discuss the implications of this model under rules in the main text.

While in standard level-k thinking, an individual with ability k believes that everyone else is level $k-1$, the generalized model allows individuals to conjecture that the population is distributed across all lower cognitive levels. Formally, we assume that individuals with ability k believe that a fraction $f_k(j)$ of the population is level $j = 0, 1, ..., k-1$. The reasoning process is initialized with some equilibrium if the economy is populated by level-0 agents, Y_t^0. For technical reasons, it is useful to define the beliefs $\left\{F_t^0[Y_{t+s}]\right\}$ which justify $Y_t^0 = \mathcal{Y}_t\left(\left\{F_t^0[Y_{t+s}]\right\}_{s\geq 1}\right)$ for all t.

Level-1 agents believe that everyone is level 0, i.e., $f_1(0) = 1$, and so they believe that output is given by:

$$F_t^1[Y_{t+s}] = Y_{t+s}^0.$$

The equilibrium in an economy where all individuals are level-1 is given by

$$Y_t^1 = \mathcal{Y}_t\left(\left\{F_t^1[Y_{t+s}]\right\}_{s\geq 1}\right).$$

Level-2 people believe that a fraction $f_2(0)$ and $f_2(1)$ are level 0 and 1, respectively. Under the assumptions discussed in section 3.2, we can write their beliefs as

$$F_t^2[Y_t] = \sum_{j=0}^{1} f_2(j)\, Y_t^j.$$

More generally, the level-k beliefs can be constructed recursively

$$F_t^k \left[Y_{t+s} \right] = \sum_{j=0}^{1} f_2 \left(j \right) Y_{t+s}^k.$$

We assume that agents of different cognitive levels agree on the relative proportions of lower cognitive levels. Let $\gamma_k \equiv f_k \left(k-1 \right)$ for all k. Then assumption (4.14) implies that $f_k \left(j \right) = \left(1 - \gamma_k \right) f_{k-1} \left(j \right)$ for $j \leq k-2$. We can write the expectation of level-k individuals as follows:

$$F_t^k \left[Y_{t+s} \right] = \left(1 - \gamma_k \right) F_t^{k-1} \left[Y_{t+s} \right] + \gamma_k Y_{t+s}^{k-1}. \tag{D.1}$$

Intuitively, the beliefs of a level-k thinker are given by a weighted average of the beliefs of level $k-1$ agents and the temporary equilibrium that would arise under those beliefs. Standard level-k thinking corresponds to the case of $\gamma_k = 1$. By varying γ_k, we can control the intensity of learning across level-k iterations.

While the standard level-k thinking model assumes that everyone is level k, the generalized level-k thinking model also allows for heterogeneity cognitive abilities. We let $f \left(k \right)$ for $k = 0, 1, ...$ denote the share of individuals who are level k in the economy. The observed equilibrium path is thus given by

$$Y_t = \sum_{k=0}^{\infty} f \left(k \right) Y_t^k. \tag{D.2}$$

D.1.1 Government-spending multipliers

We continue to define the level-k multiplier as $\Delta Y_t^k / \Delta G_t$ which is given by

$$\frac{\Delta Y_t^k}{\Delta G_t} = 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[\frac{\Delta F_t^k \left[Y_{t+s} \right]}{\Delta G_{t+s}} - 1 \right] \frac{\Delta G_{t+s}}{\Delta G_t},$$

where

$$\frac{\Delta F_t^k \left[Y_t \right]}{\Delta G_{t+s}} = \left(1 - \gamma_k \right) \frac{\Delta F_t^{k-1} \left[Y_{t+s} \right]}{\Delta G_{t+s}} + \gamma_k \frac{\Delta Y_{t+s}^{k-1}}{\Delta G_{t+s}}$$

for $k \geq 2$.The observed government-spending multiplier is given by:

$$\frac{\Delta Y_t}{\Delta G_t} = \sum_{k=0}^{\infty} f \left(k \right) \frac{\Delta Y_t^k}{\Delta G_t}.$$

Suppose that $\Delta F_t^1 \left[Y_{t+s} \right] / \Delta G_{t+s} = \Delta Y_{t+s}^0 / \Delta G_{t+s} = \eta$, this implies that

$$\frac{\Delta Y_t^1}{\Delta G_t} = 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[\eta - 1 \right] \frac{\Delta G_{t+s}}{\Delta G_t}.$$

If $\eta < 1$, then $\Delta Y_t^1 / \Delta G_t \leq 1$ which implies that$\Delta F_t^2 \left[Y_{t+s} \right] / \Delta G_{t+s} \leq 1$. For any k, if $\Delta F_t^k \left[Y_{t+s} \right] / \Delta G_{t+s} \leq 1$ then $\Delta Y_t^k / \Delta G_t \leq 1$, which implies that $\Delta F_t^{k+1} \left[Y_{t+s} \right] / \Delta G_{t+s} \leq 1$. As a result, for any $f(k)$,

$$\frac{\Delta Y_t}{\Delta G_t} = \sum_{k=0}^{\infty} f(k) \frac{\Delta Y_t^k}{\Delta G_t} \leq 1.$$

If $\eta = 1$, then $\Delta Y_t^1 / \Delta G_t = 1$ which implies that$\Delta F_t^2 \left[Y_{t+s} \right] / \Delta G_{t+s} = 1$. For any k, if $\Delta F_t^k \left[Y_{t+s} \right] / \Delta G_{t+s} = 1$ then $\Delta Y_t^k / \Delta G_t = 1$ for all k, which implies that $\Delta F_t^{k+1} \left[Y_{t+s} \right] / \Delta G_{t+s} = 1$. As a result, for any $f(k)$,

$$\frac{\Delta Y_t}{\Delta G_t} = \sum_{k=0}^{\infty} f(k) \frac{\Delta Y_t^k}{\Delta G_t} = 1,$$

for all $f(k)$.

If $\eta > 1$, then $\Delta Y_t^1 / \Delta G_t \geq 1$ which implies that$\Delta F_t^2 \left[Y_{t+s} \right] / \Delta G_{t+s} \geq 1$. For any k, if $\Delta F_t^k \left[Y_{t+s} \right] / \Delta G_{t+s} \geq 1$ then $\Delta Y_t^k / \Delta G_t \geq 1$, which implies that $\Delta F_t^{k+1} \left[Y_{t+s} \right] / \Delta G_{t+s} \geq 1$. As a result, for any $f(k)$,

$$\frac{\Delta Y_t}{\Delta G_t} = \sum_{k=0}^{\infty} f(k) \frac{\Delta Y_t^k}{\Delta G_t} \geq 1.$$

Suppose that $1 - \Omega_t \sum_{s=1}^{T-t-1} \frac{\Delta G_{t+s}}{\Delta G_t} > 0$. Note that:

$$\frac{\Delta Y_t^1}{\Delta G_t} = \left\{ 1 - \Omega_t \sum_{s=1}^{T-t-1} \frac{\Delta G_{t+s}}{\Delta G_t} \right\} + \eta \left\{ \Omega_t \sum_{s=1}^{T-t-1} \frac{\Delta G_{t+s}}{\Delta G_t} \right\}.$$

If $\eta < 1$, then $\Delta Y_t^1 / \Delta G_t \geq \eta$ and $\Delta F_t^2 \left[Y_{t+s} \right] / \Delta G_{t+s} \geq \Delta F_t^1 \left[Y_{t+s} \right] / \Delta G_{t+s} = \eta$. This immediately implies that $\Delta Y_t^2 / \Delta G_t \geq \Delta Y_t^1 / \Delta G_t$. We now show that $\Delta F_t^k \left[Y_{t+s} \right] / \Delta G_{t+s}$ and $\Delta Y_t^k / \Delta G_t$ are increasing in k. To see this, suppose that $\Delta Y_t^j / \Delta G_t \geq \Delta Y_t^{j-1} / \Delta G_t$ for all $j \leq k$ then this implies that $\Delta F_t^{k+1} \left[Y_{t+s} \right] / \Delta G_{t+s} \geq \Delta F_t^k \left[Y_{t+s} \right] / \Delta G_{t+s}$. Furthermore,

$$\frac{\Delta Y_t^{k+1}}{\Delta G_t} = 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[\frac{\Delta F_t^{k+1} \left[Y_{t+s} \right]}{\Delta G_{t+s}} - 1 \right] \frac{\Delta G_{t+s}}{\Delta G_t} \geq 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[\frac{\Delta F_t^k \left[Y_{t+s} \right]}{\Delta G_{t+s}} - 1 \right] \frac{\Delta G_{t+s}}{\Delta G_t} = \frac{\Delta Y_t^k}{\Delta G_t}.$$

This shows that $\Delta Y_t^k / \Delta G_t$ is increasing in individual cognitive ability k. But the equilibrium spending multiplier depends on the full distribution $f(k)$. The analog statement to proposition 1 requires assumptions on the distribution $f(k)$. When comparing to economies, we say that one economy is strictly more sophisticated than another if its distribution of cognitive abilities first-order dominates the distribution of the second one. Formally, consider two economies with distributions $f^A(k)$ and $f^B(k)$. Suppose that $\sum_{s=0}^{k} f^A(s) \leq \sum_{s=0}^{k} f^B(s)$ for all k. Then, the government-spending multiplier is higher in economy B than economy A.

If $\eta > 1$, then $\Delta Y_t^1 / \Delta G_t \leq \eta$ and $\Delta F_t^2 [Y_{t+s}] / \Delta G_{t+s} \leq \Delta F_t^1 [Y_{t+s}] / \Delta G_{t+s} = \eta$. This immediately implies that $\Delta Y_t^2 / \Delta G_t \leq \Delta Y_t^1 / \Delta G_t$. We now show that $\Delta F_t^k [Y_{t+s}] / \Delta G_{t+s}$ and $\Delta Y_t^k / \Delta G_t$ are decreasing in k. To see this, suppose that $\Delta Y_t^j / \Delta G_t \leq \Delta Y_t^{j-1} / \Delta G_t$ for all $j \leq k$ then this implies that $\Delta F_t^{k+1} [Y_{t+s}] / \Delta G_{t+s} \leq \Delta F_t^k [Y_{t+s}] / \Delta G_{t+s}$. Furthermore,

$$\frac{\Delta Y_t^{k+1}}{\Delta G_t} = 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[\frac{\Delta F_t^{k+1} [Y_{t+s}]}{\Delta G_{t+s}} - 1 \right] \frac{\Delta G_{t+s}}{\Delta G_t} \leq 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[\frac{\Delta F_t^{k} [Y_{t+s}]}{\Delta G_{t+s}} - 1 \right] \frac{\Delta G_{t+s}}{\Delta G_t} = \frac{\Delta Y_t^k}{\Delta G_t}.$$

This shows that $\Delta Y_t^k / \Delta G_t$ is increasing in individual cognitive ability k. But the equilibrium spending multiplier depends on the full distribution $f(k)$. The analog statement to proposition 1 requires assumptions on the distribution $f(k)$. When comparing to economies, we say that one economy is strictly more sophisticated than another if its distribution of cognitive abilities first-order dominates the distribution of the second one. Formally, consider two economies with distributions $f^A(k)$ and $f^B(k)$. Suppose that $\sum_{s=0}^{k} f^A(s) \leq \sum_{s=0}^{k} f^B(s)$ for all k. Then, the government-spending multiplier is lower in economy B than economy A.

D.1.2 Consumption-tax policy

The equilibrium in this economy is given by

$$Y_t = \left(\frac{1+\tau^c}{1+\tau_t^c} \right)^{\sigma} \frac{(1-\beta) \sum_{s=1}^{T-t-1} \left(\frac{1+\tau_{t+s}^c}{1+\tau^c} \right) \sum_{k=0}^{\infty} f(k) F_t^k [Y_{t+s}] + 1}{(1-\beta) \sum_{s=1}^{T-t-1} e^{\sigma(\chi-\rho)s} \left[\frac{1+\tau_{t+s}^c}{1+\tau^c} \right]^{1-\sigma} + e^{(T-t)\sigma(\chi-\rho)}}.$$

As before, beliefs about future output $F_t^k [Y_{t+s}]$ for any k is only a function of future tax policy, which implies that the analog construction of tax policy τ_t^c implements $Y_t = 1$. Note, however, that this policy may now imply consumption heterogeneity across different cognitive levels, because they may have different beliefs about future output. As it turns out, this is not the case if $F_t^1 [Y_{t+s}] = 1$. We show this next.

Suppose now that $F_t^1 [Y_{t+s}] = 1$. Then, announcing the tax policy $\tau_t^{c,*}$ implies that

$Y_t^1 = 1$. It then follows that $Y_t^{e,k} = Y_t^k = 1$ for all k. As a result,

$$Y_t = 1$$

for any $f(k)$. This shows that proposition 2 continues to hold.

D.2 Reflective expectations

García-Schmidt and Woodford (2019) describe a different process of belief formation which they call *reflective expectations.* This process allows cognitive ability to vary continuously but is otherwise similar in spirit to level k. Indexing beliefs by the cognitive ability n, García-Schmidt and Woodford (2019) assume that beliefs evolve according to

$$\frac{dF_t^n [Y_{t+s}]}{dn} = Y_{t+s}^n - F_t^n [Y_{t+s}],$$

for $n \geq 0$ and starting from the initial expectations $F_t^0 [Y_{t+s}]$, where Y_t^n denotes the equilibrium in an economy with level-n people.We use supercript k to denote equilibria and beliefs under level-k thinking and superscript n to denote equilibria and beliefs under reflective expectations.

García-Schmidt and Woodford (2019) show that the beliefs of a level-n individual with reflective expectations are equivalent to a convex combination of standard level-k beliefs determined by a Poisson distribution with mean n, i.e.,

$$F_t^n [Y_{t+s}] = \sum_{k=1}^{\infty} \frac{n^{k-1} e^{-n}}{(k-1)!} F_t^k [Y_{t+s}^e], \tag{D.3}$$

where $Y_t^{e,k}$ denote the beliefs that standard level-k thinkers have, which we develop in section 3. Equation (D.3) can be used to analyze the relationship between the equilibrium properties of standard level-k thinking and reflective expectations economies.

D.2.1 Government-spending multipliers

For the case of the government-spending multiplier, the beliefs of a level n individual can be computed from the beliefs under level-k thinking as follows:

$$\frac{\Delta F_t^n [Y_{t+s}]}{\Delta G_{t+s}} = \sum_{k=1}^{\infty} \frac{n^{k-1} e^{-n}}{(k-1)!} \frac{\Delta F_t^k [Y_{t+s}^e]}{\Delta G_{t+s}}.$$

Suppose $\eta < 1$. Since $\Delta Y_t^k / \Delta G_t \leq 1$ for all k, then $\Delta F_t^n \left[Y_{t+s} \right] / \Delta G_{t+s} \leq 1$ for all n. Also, since the level-k multiplier increases with k, then so does the level-n belief over the multiplier. Suppose $\eta = 1$. Since $\Delta Y_t^k / \Delta G_t = 1$ for all k, then $\Delta F_t^n \left[Y_{t+s} \right] / \Delta G_{t+s} = 1$ for all n. Suppose $\eta > 1$. Since $\Delta Y_t^k / \Delta G_t \geq 1$ for all k, then $\Delta F_t^n \left[Y_{t+s} \right] / \Delta G_{t+s} \geq 1$ for all n. Also, since the level-k multiplier decreases with k, then so does the level-n belief over the multiplier.

The equilibrium spending multiplier under reflective expectations is given by:

$$\frac{\Delta Y_t^n}{\Delta G_t} = 1 + \Omega_t \sum_{s=1}^{T-t-1} \left[\frac{\Delta F_t^n \left[Y_{t+s} \right]}{\Delta G_{t+s}} - 1 \right] \frac{\Delta G_{t+s}}{\Delta G_t}.$$

This relationship follows directly from Lemma 1. If $\eta < 1$ then since $\Delta F_t^n \left[Y_{t+s} \right] / \Delta G_{t+s} \leq 1$ for all t, then $\Delta Y_t^n / \Delta G_t \leq 1$ for all t. Also, since the $\Delta F_t^n \left[Y_{t+s} \right] / \Delta G_{t+s}$ is increasing with n, then $\Delta Y_t^n / \Delta G_t$ is increasing in n. If $\eta = 1$ then since $\Delta F_t^n \left[Y_{t+s} \right] / \Delta G_{t+s} = 1$ for all t, then $\Delta Y_t^n / \Delta G_t = 1$ for all t. If $\eta > 1$ then since $\Delta F_t^n \left[Y_{t+s} \right] / \Delta G_{t+s} \geq 1$ for all t, then $\Delta Y_t^n / \Delta G_t \geq 1$ for all t. Also, since the $\Delta F_t^n \left[Y_{t+s} \right] / \Delta G_{t+s}$ is decreasing with n, then $\Delta Y_t^n / \Delta G_t$ is decreasing in n.

D.2.2 Consumption-tax policy

The temporary equilibrium with reflective expectations is given by:

$$Y_t^n = \left(\frac{1 + \tau^c}{1 + \tau_t^c} \right)^\sigma \frac{(1 - \beta) \sum_{s=1}^{T-t-1} \left(\frac{1 + \tau_{t+s}^c}{1 + \tau^c} \right) F_t^n \left[Y_{t+s} \right] + 1}{(1 - \beta) \sum_{s=1}^{T-t-1} e^{\sigma(\chi - \rho)s} \left[\frac{1 + \tau_{t+s}^c}{1 + \tau^c} \right]^{1-\sigma} + e^{(T-t)\sigma(\chi - \rho)}},$$

where

$$\frac{dF_t^n \left[Y_{t+s} \right]}{dn} = Y_{t+s}^n - F_t^n \left[Y_{t+s} \right].$$

As it turns out, the results of Proposition 2 extend to the model with reflective expectations. We prove this result below.

Set τ_{T-1}^c to the value implied by (A.1). Then, proceed recursively from that date. For each $t \leq T - 2$, fix τ_{t+s}^c for $s \geq 1$. These imply a path for $Y_{t+s}^{e,n}$ for $s \geq 1$. Let us choose τ_t^c so that

$$\left(\frac{1 + \tau^c}{1 + \tau_t^c} \right)^\sigma \frac{(1 - \beta) \sum_{s=1}^{T-t-1} \left(\frac{1 + \tau_{t+s}^c}{1 + \tau^c} \right) F_t^n \left[Y_{t+s} \right] + 1}{(1 - \beta) \sum_{s=1}^{T-t-1} e^{\sigma(\chi - \rho)s} \left[\frac{1 + \tau_{t+s}^c}{1 + \tau^c} \right]^{1-\sigma} + e^{(T-t)\sigma(\chi - \rho)}} = 1$$

or, equivalently,

$$1+\tau_t^c=(1+\tau^c)\left(\frac{(1-\beta)\sum_{s=1}^{T-t-1}\left(\frac{1+\tau_{t+s}^c}{1+\tau^c}\right)F_t^n\left[Y_{t+s}\right]+1}{(1-\beta)\sum_{s=1}^{T-t-1}e^{\sigma(\chi-\rho)s}\left[\frac{1+\tau_{t+s}^c}{1+\tau^c}\right]^{1-\sigma}+e^{(T-t)\sigma(\chi-\rho)}}\right)^{1/\sigma}.$$

This implies that

$$Y_t^n=1$$

for all t.

Suppose that $Y_t^{e,0}=1$ and

$$\tau_t^c=\tau_t^{c,*}=(1+\tau^c)\,e^{-(T-t)(\chi-\rho)}-1.$$

Then,

$$Y_t^0=\left(\frac{1+\tau^c}{1+\tau_t^{c,*}}\right)^{\sigma}\frac{(1-\beta)\sum_{s=1}^{T-t-1}\left(\frac{1+\tau_{t+s}^{c,*}}{1+\tau^c}\right)+1}{(1-\beta)\sum_{s=1}^{T-t-1}e^{\sigma(\chi-\rho)s}\left[\frac{1+\tau_{t+s}^c}{1+\tau^c}\right]^{1-\sigma}+e^{(T-t)\sigma(\chi-\rho)}}=1$$

and

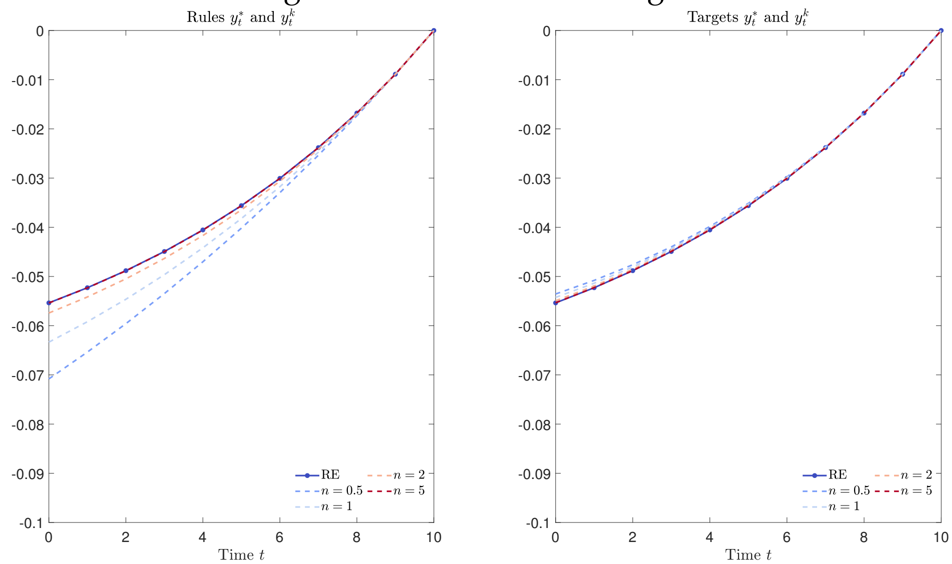
$$\frac{dF_t^n\left[Y_{t+s}\right]}{dn}|_{n=0}=Y_t^0-F_t^0\left[Y_{t+s}\right]=1-1=0,$$

which implies that $dY_t^n/dn=0$ for all n and then $Y_t^n=Y_t^0=1$ for all n.

Rules versus targets Figure D.1 shows the reflective equilibria for different levels of n both for rules-based communication and targets-communication in the left and right panels, respectively. Consistent with the results for the generalized level-k model, output contracts more sharply for lower levels of cognitive ability. As highlighted by Angeletos and Sastry (2020), the peculiar oscillatory feature that is present under standard level-k thinking does not arise under reflective expectations. We see that as cognitive ability rises, output converges to that under rational expectations. Also in line with the results in the baseline model, we see that, with targets, output contracts less with lower levels of cognitive sophistication and the level of output also converges to the rational-expectations equilibrium as n increases.

This confirms the claim in the paper that all the results in the benchmark model extend to the reflective expectations model.

Figure D.1: Rules versus targets



D.3 Shallow reasoning

Angeletos and Sastry (2020) describe a different process of belief formation which they refer to as *shallow reasoning*. In this model it is assumed that everyone is rational and attentive, knows that everyone else is rational but believe that only a fraction λ are attentive to changes in the economic environment. For simplicity, we work with the linearized equilibrium relation. The consumption of individual i can be written as follows:

$$c_{i,t} = (1-\beta) \sum_{s=0}^{T-1-(t-s)} \beta^s \frac{Y}{C} \left[\mathbb{E}_i y_{t+s} - g_{t+s}\right] - \sigma\beta \sum_{s=0}^{T-1-t} \beta^s \left\{r_{t+s} - \Delta\hat{\tau}^c_{t+s+1} + \chi_{t+s}\right\},$$

where $\mathbb{E}_i\left[y_t\right]$ denotes individual i's expectation of output. Lower-case letters denote log-deviations from steady-state values, except for $g_t = G_t/Y$. Market clearing requires $y_t = \frac{C}{Y}\int c_{i,t} di + g_t$. Individual i fully understands that other individuals have the same policy function, conditional on their beliefs. Using the market clearing condition we can write

$$y_t = g_t + (1-\beta) \sum_{s=1}^{T-1-t} \beta^{s-1} \left[\overline{\mathbb{E}} y_{t+s} - g_{t+s}\right] - \frac{C}{Y}\sigma \sum_{s=0}^{T-1-t} \beta^s \left\{r_{t+s} - \Delta\hat{\tau}^c_{t+s+1} + \chi_{t+s}\right\},$$

where $\overline{\mathbb{E}}\left[y_t\right] \equiv \int_0^1 \mathbb{E}_i\left[y_t\right] di$ denotes the average expectation in the economy. Let

$$\Psi_t \equiv g_t - (1-\beta) \sum_{s=1}^{T-1-t} \beta^{s-1} g_{t+s} - \frac{C}{Y}\sigma\beta \sum_{s=0}^{T-1-t} \beta^s \left\{r_{t+s} - \Delta\hat{\tau}^c_{t+s+1} + \chi_{t+s}\right\}$$

We can write

$$\boldsymbol{y} = (1-\beta)\,\boldsymbol{M}\overline{\mathbb{E}}\left[\boldsymbol{y}\right] + \boldsymbol{\Psi}$$

where

$$\boldsymbol{y} \equiv \begin{bmatrix} y_0 \\ y_1 \\ \dots \\ y_{T-1} \end{bmatrix}, \quad \boldsymbol{M} \equiv \begin{bmatrix} 0 & 1 & \beta & \dots & \beta^{T-1} \\ 0 & 0 & 1 & \dots & \beta^{T-2} \\ \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & \dots & 0 \end{bmatrix}, \quad \boldsymbol{\Psi} \equiv \begin{bmatrix} \Psi_0 \\ \Psi_1 \\ \dots \\ \Psi_{T-1} \end{bmatrix}.$$

This implies that

$$\overline{\mathbb{E}}\left[\boldsymbol{y}\right] = (1-\beta)\,\boldsymbol{M}\overline{\mathbb{E}}^2\left[\boldsymbol{y}\right] + \overline{\mathbb{E}}\left[\boldsymbol{\Psi}\right],$$

where $\overline{\mathbb{E}}^h\left[\cdot\right] \equiv \overline{\mathbb{E}}\left[\overline{\mathbb{E}}^{h-1}\left[\cdot\right]\right]$. Note that the law of iterated expectations does not apply for the average expectation. Then, iterating on this relation and using the fact that $\boldsymbol{M}^h$ converges to a zero matrix as h goes to infinity, we obtain

$$\overline{\mathbb{E}}\left[\boldsymbol{y}\right] = \sum_{h=1}^{\infty}\left\{(1-\beta)\,\boldsymbol{M}\right\}^{h-1}\overline{\mathbb{E}}^h\left[\boldsymbol{\Psi}\right].$$

Following Angeletos and Sastry (2020), the behavioral assumptions imply that $\overline{\mathbb{E}}^h\left[\boldsymbol{\Psi}\right] = \lambda^h\boldsymbol{\Psi}$, and so

$$\overline{\mathbb{E}}\left[\boldsymbol{y}\right] = \lambda\left[\boldsymbol{I} - (1-\beta)\,\boldsymbol{M}\lambda\right]^{-1}\boldsymbol{\Psi} = \lambda\boldsymbol{y},$$

where the last equality follows from the fact that

$$\begin{aligned} \boldsymbol{y} &= (1-\beta)\,\boldsymbol{M}\overline{\mathbb{E}}\left[\boldsymbol{y}\right] + \boldsymbol{\Psi} = (1-\beta)\,\boldsymbol{M}\lambda\left[\boldsymbol{I} - (1-\beta)\,\boldsymbol{M}\lambda\right]^{-1}\boldsymbol{\Psi} + \boldsymbol{\Psi} \\ &= \left[\boldsymbol{I} - (1-\beta)\,\boldsymbol{M}\lambda\right]^{-1}\boldsymbol{\Psi} \end{aligned}$$

As a result, we can write the equilibrium relation as:

$$y_t = g_t + (1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}\left[\lambda y_{t+s} - g_{t+s}\right] - \frac{C}{Y}\sigma\beta\sum_{s=0}^{T-1-t}\beta^s\left\{r_{t+s} - \Delta\hat{\tau}^c_{t+s+1} + \chi_{t+s}\right\}. \tag{D.4}$$

D.3.1 Government-spending multipliers

Using the equilibrium relation (D.4), we find that the fiscal spending multiplier solves the following recursion:

$$\frac{\Delta Y_t}{\Delta G_t} = 1 + (1-\beta)\sum_{s=1}^{T-1-t} \beta^{s-1}\left[\lambda \frac{\Delta Y_{t+s}}{\Delta G_{t+s}} - 1\right]\frac{\Delta G_{t+s}}{\Delta G_t}. \tag{D.5}$$

For consistency with earlier results, the multiplier is expressed in terms of levels of Y_t and G_t. As in the benchmark model, the date $T-1$ fiscal multiplier is the same as the rational-expectations fiscal multiplier:

$$\frac{\Delta Y_{T-1}}{\Delta G_{T-1}} = 1.$$

This then implies that

$$\frac{\Delta Y_{T-2}}{\Delta G_{T-2}} = 1 - (1-\beta)\left[1-\lambda\right]\frac{\Delta G_{T-1}}{\Delta G_{T-2}}.$$

Since $\lambda < 1$, then $\Delta Y_{T-2}/\Delta G_{T-2} < 1$. As $\lambda \to 1$ then $\Delta Y_{T-2}/\Delta G_{T-2} \to 1$ which coincides with the rational-expectations multiplier. We can also see that the fiscal multiplier is monotonically increasing in λ,

$$\frac{d\frac{\Delta Y_{T-2}}{\Delta G_{T-2}}}{d\lambda} = (1-\beta)\frac{\Delta G_{T-1}}{\Delta G_{T-2}} > 0,$$

so as λ increases the multiplier gets closer to the rational-expectations multiplier. Via standard inductive arguments these properties extend to all time t multipliers. To see this result, note that for $\lambda < 1$, if $\Delta Y_{t+s}/\Delta G_{t+s} \leq 1$ for all $s \geq 1$ then

$$\frac{\Delta Y_t}{\Delta G_t} = 1 + (1-\beta)\sum_{s=1}^{T-1-t} \beta^{s-1}\left[\lambda \frac{\Delta Y_{t+s}}{\Delta G_{t+s}} - 1\right]\frac{\Delta G_{t+s}}{\Delta G_t} < 1.$$

Furthermore,

$$\lim_{\lambda\to 1}\frac{\Delta Y_t}{\Delta G_t} = 1 + (1-\beta)\sum_{s=1}^{T-1-t} \beta^{s-1}\left[\lim_{\lambda\to 1}\frac{\Delta Y_{t+s}}{\Delta G_{t+s}} - 1\right]\frac{\Delta G_{t+s}}{\Delta G_t} = 1$$

as long as $\lim_{\lambda\to 1}\frac{\Delta Y_{t+s}}{\Delta G_{t+s}} = 1$. This result shows that all time t spending multipliers converge to the rational-expectations multipliers as λ goes to one. Furthermore, under the assumption that

$$1-(1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}\frac{\Delta G_{t+s}}{\Delta G_t}>0, \tag{D.6}$$

we find that $\Delta Y_t/\Delta G_t > 0$ for all t. Differentiating (D.5) with respect to λ, we obtain:

$$\frac{d\frac{\Delta Y_t}{\Delta G_t}}{d\lambda}=(1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}\left[\frac{\Delta Y_{t+s}}{\Delta G_{t+s}}+\lambda\frac{d\frac{\Delta Y_{t+s}}{\Delta G_{t+s}}}{d\lambda}\right]\frac{\Delta G_{t+s}}{\Delta G_t}.$$

Under assumption (D.6), we know that $\Delta Y_{t+s}/\Delta G_{t+s} > 0$. Then, if

$$\frac{d\frac{\Delta Y_{t+s}}{\Delta G_{t+s}}}{d\lambda}>0,$$

then $d\frac{\Delta Y_t}{\Delta G_t}/d\lambda > 0$. Since we have shown that $d\frac{\Delta Y_{T-2}}{\Delta G_{T-2}}/d\lambda > 0$, then it is true that $d\frac{\Delta Y_t}{\Delta G_t}/d\lambda > 0$ for all t. This confirms that the shallow reasoning spending multiplier is increasing in the sophistication parameter λ.

Finally, suppose that $\Delta G_t = \zeta^t \Delta G_0$ for $\zeta > 0$, then

$$\frac{\Delta Y_{T-2}}{\Delta G_{T-2}}=1-(1-\beta)\left[1-\lambda\right]\zeta \Rightarrow d\frac{\Delta Y_{T-2}}{\Delta G_{T-2}}/d\zeta<0$$

and

$$\frac{\Delta Y_t}{\Delta G_t}=1+(1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}\left[\lambda\frac{\Delta Y_{t+s}}{\Delta G_{t+s}}-1\right]\zeta^s. \tag{D.7}$$

$$d\frac{\Delta Y_t}{\Delta G_t}/d\zeta=(1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}\lambda\frac{d\frac{\Delta Y_{t+s}}{\Delta G_{t+s}}}{d\zeta}\zeta^s+(1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}\left[\underbrace{\lambda\frac{\Delta Y_{t+s}}{\Delta G_{t+s}}-1}_{<0}\right]s\zeta^{s-1}<0$$

as long as $d\frac{\Delta Y_{t+s}}{\Delta G_{t+s}}/d\zeta < 0$. As a result, the spending multiplier is decreasing in the persistence of government spending.

D.3.2 Consumption-tax policy

Suppose that $g_t = 0$ for all t and for simplicity suppose that $Y = C$. Interest rates are at the ZLB for $t \le T-1$, and go back to steady-state levels for $t \ge T$:

$$r_t=\log R_t-\rho=\begin{cases}-\rho & \text{if } t\le T-1\\ 0 & \text{if } t\ge T.\end{cases}$$

Then, we find that for $t \geq T$ output is back to steady state $y_t = 0$. However, for $t \leq T-1$ output solves the fixed-point system of equations of $\{y_t\}_{t=0}^{T-1}$:

$$y_t = (1-\beta) \sum_{s=1}^{T-1-t} \beta^{s-1} \lambda y_{t+s} - \sigma \sum_{s=0}^{T-1-t} \beta^s \left\{ (\chi - \rho) - \left(\hat{\tau}^c_{t+s+1} - \hat{\tau}^c_{t+s} \right) \right\}. \tag{D.8}$$

Then, consider the policy that implements full stabilization under rational expectations:

$$1 + \tau^c_t = (1 + \tau^c)\, e^{-(T-t)(\chi - \rho)}$$

which implies that

$$\frac{1+\tau^c_t}{1+\tau^c_{t+1}} = e^{-(\chi-\rho)} \Rightarrow \hat{\tau}^c_{t+1} - \hat{\tau}^c_t = \chi - \rho.$$

Replacing these consumption taxes in the equilibrium relation (D.8), we obtain

$$y_t = (1-\beta) \sum_{s=1}^{T-1-t} \beta^{s-1} \lambda y_{t+s},$$

which implies that $y_t = 0$ for all t is a shallow reasoning equilibrium under this policy. In sum, the same policy that implements the flexible-price allocation under rational expectations also implements the flexible-price allocation irrespective of the degree of rationality λ.

Rules versus targets Consider now the case in which policy is designed as rules, i.e., such that interest rates and consumption taxes are set so that

$$r_t = \max\left\{ \phi_y y_t, -\rho \right\},$$

and

$$\hat{\tau}^c_{t+1} - \hat{\tau}^c_t = \min\left\{ \phi_y y_t + \rho, 0 \right\}$$

which implies that:

$$r_t + \hat{\tau}^c_{t+1} - \hat{\tau}^c_t = \phi_y y_t.$$

The shallow reasoning equilibrium is a solution to the fixed point system of equations given by:

$$y_t = -\frac{\sigma\chi}{1+\sigma\phi_y} \frac{1-\beta^{T-t}}{1-\beta} - \left(\beta - \frac{1}{1+\sigma\phi_y} \right) \sum_{s=1}^{T-1-t} \beta^{s-1} \lambda y_{t+s}.$$

As before, if $\lambda = 1$, then $y_t = -\frac{\chi}{\phi_y}\left[1 - \left(1+\sigma\phi_y\right)^{-(T-t)}\right] = y_t^* < 0$ which is the rational-expectations equilibrium. Furthermore, note that for $t = T-1$:

$$y_{T-1} = -\frac{\sigma\chi}{1+\sigma\phi_y} = y_{T-1}^* < 0$$

for any λ. Next, we show that, if $\beta > \left(1+\sigma\phi_y\right)^{-1}$, for $\lambda < 1$, $y_t < y_t^*$ for all $t \leq T-2$.

Output at time $t = T-2$ is given by

$$\begin{aligned} y_{T-2} &= -\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^2}{1-\beta} - \left(\beta - \frac{1}{1+\sigma\phi_y}\right)\lambda y_{T-1} \\ &< -\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^2}{1-\beta} - \left(\beta - \frac{1}{1+\sigma\phi_y}\right) y_{T-1}^* = y_{T-2}^*, \end{aligned}$$

which shows that $y_{T-2} < y_{T-2}^*$. Furthermore, we also find that $\lambda y_{T-2} > y_{T-2}^*$, which follows from the fact that:

$$\begin{aligned} \lambda y_{T-2} - y_{T-2}^* &= -\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^2}{1-\beta}(\lambda-1) - \left(\beta - \frac{1}{1+\sigma\phi_y}\right)\left(\lambda^2 y_{T-1} - y_{T-1}^*\right) \\ &= (\lambda-1)\left\{-\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^2}{1-\beta} - \left(\beta - \frac{1}{1+\sigma\phi_y}\right)(\lambda+1)\, y_{T-1}^*\right\} \\ &> (\lambda-1)\left\{-\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^2}{1-\beta} - \left(\beta - \frac{1}{1+\sigma\phi_y}\right) y_{T-1}^*\right\} = (\lambda-1)\, y_{T-2}^* > 0. \end{aligned}$$

Therefore, we find that $y_{T-2} < y_{T-2}^*$, but $\lambda y_{T-2} > y_{T-2}^*$, i.e., $y_{T-2} \in (\lambda^{-1}y_{T-2}^*, y_{T-2}^*)$. For any t, suppose that $y_{t+s} \in (\lambda^{-1}y_{t+s}^*, y_{t+s}^*]$ for all $s = 1, ..., T-t-1$, then

$$\begin{aligned} y_t &= -\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^{T-t}}{1-\beta} - \left(\beta - \frac{1}{1+\sigma\phi_y}\right)\sum_{s=1}^{T-1-t}\beta^{s-1}\lambda y_{t+s} \\ &< -\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^{T-t}}{1-\beta} - \left(\beta - \frac{1}{1+\sigma\phi_y}\right)\sum_{s=1}^{T-1-t}\beta^{s-1}\lambda\lambda^{-1} y_{t+s}^* = y_t^*. \end{aligned}$$

Furthermore, we also find that $\lambda y_t > y_t^*$, which follows from the fact that

$$\begin{aligned}
\lambda y_t - y_t^* &= -\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^{T-t}}{1-\beta}(\lambda-1) - \left(\beta - \frac{1}{1+\sigma\phi_y}\right)\sum_{s=1}^{T-1-t}\beta^{s-1}\left(\lambda^2 y_{t+s} - y_{t+s}^*\right) \\
&> -\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^{T-t}}{1-\beta}(\lambda-1) - \left(\beta - \frac{1}{1+\sigma\phi_y}\right)\sum_{s=1}^{T-1-t}\beta^{s-1}\left(\lambda^2 y_{t+s}^* - y_{t+s}^*\right) \\
&= (\lambda-1)\left[-\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^{T-t}}{1-\beta} - \left(\beta - \frac{1}{1+\sigma\phi_y}\right)\sum_{s=1}^{T-1-t}\beta^{s-1}(\lambda+1)\, y_{t+s}^*\right] \\
&> (\lambda-1)\left[-\frac{\sigma\chi}{1+\sigma\phi_y}\frac{1-\beta^{T-t}}{1-\beta} - \left(\beta - \frac{1}{1+\sigma\phi_y}\right)\sum_{s=1}^{T-1-t}\beta^{s-1} y_{t+s}^*\right] \\
&> (\lambda-1)\, y_t^* > 0.
\end{aligned}$$

Then, by induction, we find that $y_t \in (\lambda^{-1}y_t^*, y_t^*]$, which shows that the stabilizing power of fiscal policy under rules becomes weaker.

Suppose now, that the policy is communicated as targets. We show that under targets-based communication $y_t \geq y_t^*$ for all t. First, using (D.8) we find that:

$$\lim_{\lambda\to 0} y_t = -\sigma \sum_{s=0}^{T-1-t}\beta^s\left\{(\chi-\rho) - \left(\hat{\tau}_{t+s+1}^{c,r} - \hat{\tau}_{t+s}^{c,r}\right)\right\} < 0,$$

and

$$\frac{dy_{T-2}}{d\lambda} = (1-\beta)\, y_{T-1}^* < 0 \Rightarrow y_{T-2} < 0,$$

for all λ. Now, note that

$$\frac{dy_t}{d\lambda} = (1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1} y_{t+s} + (1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}\lambda\frac{dy_{t+s}}{d\lambda}.$$

So, as long as $y_{t+s} \leq 0$ and $dy_{t+s}/d\lambda \leq 0$ for all $s \geq 1$, with one strict inequality, then we find that $dy_t/d\lambda < 0$ and $y_t < 0$. Furthermore, to show that $y_t > y_t^*$, note that

$$y_t - y_t^* = (1-\beta)\sum_{s=1}^{T-1-t}\beta^{s-1}\left\{\lambda y_{t+s} - y_{t+s}^*\right\}.$$

As before, this implies that $y_{T-1} = y_{T-1}^*$. Now, evaluating time $t = T-1$, we see that

$$y_{T-2} - y_{T-2}^* = (1-\beta)\{\lambda - 1\}\, y_{T-1}^* > 0 \Rightarrow y_{T-2} > y_{T-2}^*.$$

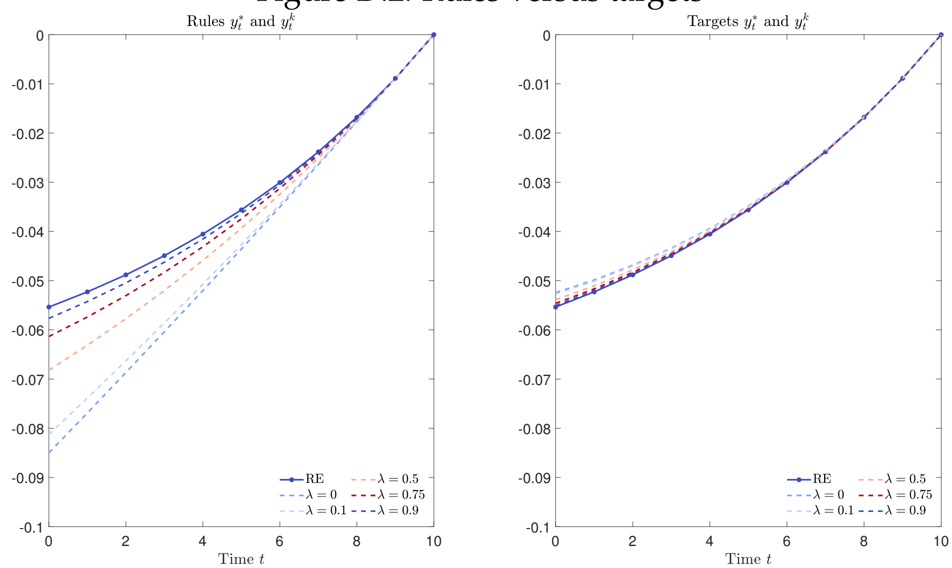
This result serves as the base for the inductive argument. Suppose that $0 > y_{t+2} > y_{t+s}^*$

for all s, then

$$y_t - y_t^* = \sum_{s=1}^{T-1-t} \beta^{s-1} \left\{ \lambda y_{t+s} - y_{t+s}^* \right\} > 0.$$

Figure D.2 shows the equilibrium path for log-output in the economy with shallow reasoning for different levels of λ. As highlighted by Angeletos and Sastry (2020), the peculiar oscillatory feature that is present under simple level-k thinking does not arise under reflective expectations. We see that as cognitive ability rises, output converges to that under rational expectations. Also in line with the results in the baseline model, we see

Figure D.2: Rules versus targets



that, with targets, output contracts less with lower levels of cognitive sophistication and the level of output also converges to the rational-expectations equilibrium as λ increases.

This confirms the claim in the paper that all the results in the benchmark model extend to the shallow reasoning model.

E A model with sticky prices

In this appendix, we present an alternative New Keynesian model with sticky prices instead of sticky wages and show that our main results continue to hold for this alternative specification. We assume that households have the same utility function as the one in our benchmark model, see (3.1).

The final good is produced using a continuum of intermediate inputs $y_{u,t}$ for $u \in [0,1]$

according to the technology:

$$Y_t = \left[\int_0^1 y_{u,t}^{\frac{\theta-1}{\theta}} du\right]^{\frac{\theta}{\theta-1}}.$$

Each variety u is produced by a monopolistic firm using the technology:

$$y_{u,t} = An_{u,t}^{1-\alpha}.$$

The good market clearing condition is still given by (3.3). We assume that the government has access to the same monetary and fiscal instruments as in section 4.

Final goods firms The representative final goods producer maximizes profits

$$P_t Y_t - \int_0^1 p_{u,t} y_{u,t} du,$$

which implies that demand for the intermediate input is given by

$$y_{u,t} = \left(\frac{p_{u,t}}{P_t}\right)^{-\theta} Y_t.$$

The aggregate price level satisfies:

$$P_t = \left[\int_0^1 p_{u,t}^{1-\theta} du\right]^{\frac{1}{1-\theta}}.$$

Intermediate goods producers Each intermediate good u is produced by a monopolist. Producers set prices subject to Calvo frictions. At time t, a fraction $1-\lambda$ can reset their price. As is standard, it is optimal for producers to choose the same reset price, P_t^*. The optimal reset price is the solution to:

$$\max_{P_t^*} \sum_{s=0}^{\infty} \lambda^s Q_{t,t+s} F_t\left[\frac{P_{t+s}}{P_t}\right] \left\{ \left(\frac{P_t^*}{F_t\left[P_{t+s}\right]}\right)^{1-\theta} F_t\left[Y_{t+s}\right] - F_t\left[\frac{W_{t+s}}{P_{t+s}}\right] \frac{1}{A^{\frac{1}{1-\alpha}}} \left(\frac{P_t^*}{F_t\left[P_{t+s}\right]}\right)^{-\frac{\theta}{1-\alpha}} \left(F_t\left[Y_{t+s}\right]\right)^{\frac{1}{1-\alpha}} \right\}.$$

We assume that the monopolist has rational expectations with respect to exogenous variables, but is boundedly rational with respect to endogenous variables. In particular, we assume that the firm forms beliefs about future aggregate prices, P_t^e, wages, W_t^e,and output Y_t^e using level-k thinking.

The first-order condition implies that:

$$\frac{P_t^*}{P_t} = \left\{ \frac{\theta}{(\theta-1)(1-\alpha)} \frac{\sum_{s=0}^{\infty} \lambda^s Q_{t,t+s} F_t \left[\frac{P_{t+s}}{P_t}\right] F_t \left[\frac{W_{t+s}}{P_{t+s}}\right] \frac{1}{A^{\frac{1}{1-\alpha}}} \left(F_t \left[\frac{P_{t+s}}{P_t}\right]\right)^{\frac{\theta}{1-\alpha}} \left(F_t \left[Y_{t+s}\right]\right)^{\frac{1}{1-\alpha}}}{\sum_{s=0}^{\infty} \lambda^s Q_{t,t+s} F_t \left[\frac{P_{t+s}}{P_t}\right] \left(F_t \left[\frac{P_{t+s}}{P_t}\right]\right)^{\theta-1} F_t \left[Y_{t+s}\right]} \right\}^{\frac{1-\alpha}{1-\alpha(1-\theta)}} . \tag{E.1}$$

Let lower case letters denote the log-deviation of a variable from its steady-state value, $x_t \equiv \log X_t - \log X$. Using (E.1) we obtain

$$p_t^* - p_t = \zeta (1-\lambda\beta) \sum_{s=0}^{\infty} (\beta\lambda)^s \left\{ F_t \left[w_{t+s} - p_{t+s}\right] + \frac{\alpha}{1-\alpha} F_t \left[y_{t+s}\right] \right\} + \sum_{s=1}^{\infty} (\beta\lambda)^s F_t \left[\pi_{t+s}\right], \tag{E.2}$$

where $\zeta \equiv \frac{1-\alpha}{1-\alpha(1-\theta)}$.

The price level is given by

$$P_t = \left[\lambda P_{t-1}^{1-\theta} + (1-\lambda)\left(P_t^*\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

so

$$p_t = \lambda p_{t-1} + (1-\lambda) p_t^* \Leftrightarrow \pi_t = \frac{1-\lambda}{\lambda} \left(p_t^* - p_t\right). \tag{E.3}$$

Combining (E.2) and (E.3) we obtain:

$$\pi_t = \kappa \sum_{s=0}^{\infty} (\beta\lambda)^s \left\{ F_t \left[w_{t+s} - p_{t+s}\right] + \frac{\alpha}{1-\alpha} F_t \left[y_{t+s}\right] \right\} + \frac{1-\lambda}{\lambda} \sum_{s=1}^{\infty} (\beta\lambda)^s F_t \left[\pi_{t+s}\right], \tag{E.4}$$

where $\kappa \equiv \zeta \frac{(1-\lambda)(1-\lambda\beta)}{\lambda}$.

Household The household chooses consumption and labor to maximize:

$$\max \sum_{s=0}^{\infty} \beta^s \xi_{t+s} \left[u\left(\widetilde{C}_{t+s}\right) - v\left(\widetilde{N}_{t+s}\right)\right]$$

$$\sum_{s=0}^{\infty} Q_{t,t+s} F_t \left[\frac{P_{t+s}}{P_t}\right] \left(1+\tau_{t+s}^c\right) \widetilde{C}_{t+s} = \sum_{s=0}^{\infty} Q_{t,t+s} F_t \left[\frac{P_{t+s}}{P_t}\right] \left[\left(1-\tau_{t+s}^n\right) F_t \left[\frac{W_{t+s}}{P_{t+s}}\right] \widetilde{N}_{t+s} + F_t \left[\Omega_{t+s}\right] - F_t \left[T_{t+s}\right.\right.$$

The solution to this problem implies

$$C_t = \frac{\sum_{s\geq 0} Q_{t,t+s} F_t\left[\frac{P_{t+s}}{P_t}\right]\left\{\left(1-\tau^n_{t+s}\right)F_t\left[\frac{W_{t+s}}{P_{t+s}}\right]\widetilde{N}_{t+s} + F_t\left[\Omega_{t+s}\right] - F_t\left[T_{t+s}\right]\right\} + R_{t-1}b_{i,t}}{\left(1+\tau_t\right)\left[1+\sum_{s\geq 1}\left(\beta^s\frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+s}F_t\left[\frac{P_{t+s}}{P_t}\right]\frac{\left(1+\tau^c_{t+s}\right)}{\left(1+\tau^c_t\right)}\right]^{1-\sigma}\right]},$$

where

$$\widetilde{N}^{\varphi}_{t+s} = \frac{1-\tau^n_{t+s}}{1+\tau^c_{t+s}}F_t\left[\frac{W_{t+s}}{P_{t+1}}\right]\left(\beta^s\frac{\xi_{t+s}}{\xi_t}\right)^{-1}Q_{t,t+s}F_t\left[\frac{P_{t+s}}{P_t}\right]\frac{1+\tau^c_{t+s}}{1+\tau^c_t}C_t^{-\sigma^{-1}}. \tag{E.5}$$

Using people's beliefs about the government budget constraint, (4.6), and the aggregate resource constraint, (3.3), we obtain

$$C_t = \frac{\sum_{s\geq 1} Q_{t,t+s}F_t\left[\frac{P_{t+s}}{P_t}\right]\frac{1+\tau^c_{t+s}}{1+\tau^c_t}\left\{\left(\frac{1-\tau^n_{t+s}}{1-\tau^c_{t+s}}\right)F_t\left[\frac{W_{t+s}}{P_{t+s}}\right]\left\{\widetilde{N}_{t+s} - F_t\left[N_{t+s}\right]\right\} + F_t\left[Y_{t+s}\right] - G_{t+s}\right\}}{\sum_{s\geq 1}\left(\beta^s\frac{\xi_{t+s}}{\xi_t}\right)^{\sigma}\left[Q_{t,t+s}F_t\left[\frac{P_{t+s}}{P_t}\right]\frac{1+\tau^c_{t+s}}{1+\tau^c_t}\right]^{1-\sigma}}. \tag{E.6}$$

Log-linearizing equations E.5and E.6 yields:

$$\begin{aligned}\widetilde{n}_{t+s} &= -\varphi^{-1}\left(\hat{\tau}^n_{t+s} + \hat{\tau}^c_{t+s}\right) + \varphi^{-1}F_t\left[w_{t+s} - p_{t+s}\right] \\ &\quad - \varphi^{-1}\sum_{m=0}^{s-1}\left(r_{t+m} - \pi^e_{t+m+1} - \Delta\hat{\tau}^c_{t+m} + \chi_{t+m}\right) - \left(\varphi\sigma\right)^{-1}c_t\end{aligned} \tag{E.7}$$

and

$$\begin{aligned}c_t &= \frac{1-\beta}{\beta}\sum_{s\geq 1}\beta^s\frac{Y}{C}\left\{F_t\left[y_{t+s}\right] - g_{t+s} - \omega_N F_t\left[n_{t+s}\right]\right\} + \frac{1-\beta}{\beta}\sum_{s\geq 1}\beta^s\frac{Y}{C}\omega_N\widetilde{n}_{t+s} \\ &\quad - \sigma\sum_{m=0}^{\infty}\beta^s\left\{r_{t+s} - F_t\left[\pi_{t+s+1}\right] - \Delta\hat{\tau}^c_{t+s} + \sigma\chi_{t+s}\right\}\end{aligned} \tag{E.8}$$

where $\omega_N = \left(\frac{1-\tau^n}{1-\tau^c}\right)\frac{W}{P}\frac{N}{Y}$. Replacing (E.7) in (E.8), we obtain:

$$\begin{aligned}c_t &= \psi\sum_{s\geq 1}\beta^s\frac{Y}{C}\left\{F_t\left[y_{t+s}\right] - g_{t+s} - \omega_N F_t\left[n_{t+s}\right] + \varphi^{-1}\left\{F_t\left[w_{t+s} - p_{t+s}\right] - \left(\hat{\tau}^n_{t+s} + \hat{\tau}^c_{t+s}\right)\right\}\right\} \\ &\quad - \sigma\sum_{m=0}^{\infty}\beta^s\left\{r_{t+s} - F_t\left[\pi_{t+s+1}\right] - \Delta\hat{\tau}^c_{t+s} + \sigma\chi_{t+s}\right\}\end{aligned}$$

where $\psi \equiv \frac{\sigma}{\sigma+\frac{Y}{C}\omega_N\varphi^{-1}}\frac{1-\beta}{\beta}$.

Equilibrium In equilibrium, labor-market clearing, $N_t = \int n_{u,t} du$, implies that:

$$N_t = \int n_{u,t} du = \int \left(\frac{y_{u,t}}{A}\right)^{\frac{1}{1-\alpha}} du = \int \left(\frac{Y_t}{A}\right)^{\frac{1}{1-\alpha}} \left(\frac{p_{u,t}}{P_t}\right)^{-\frac{\theta}{1-\alpha}} du$$

which implies that

$$Y_t = \mu_t^{\alpha-1} A N_t^{1-\alpha} = C_t + G_t,$$

where $\mu_t = \int \left(\frac{p_{u,t}}{P_t}\right)^{-\frac{\theta}{1-\alpha}}$ denotes the standard price distortion. Starting from an non-distorted steady state implies $\mu_{-1} = 1$ and to first order the price distortion is zero.

The temporary equilibrium conditions are as follows.

1. Consumption is given by

$$c_t = \psi \sum_{s\geq 1} \beta^s \frac{Y}{C} \left\{ F_t \left[y_{t+s}\right] - g_{t+s} - \omega_N F_t \left[n_{t+s}\right] + \varphi^{-1} \left\{ F_t \left[w_{t+s} - p_{t+s}\right] - \left(\hat{\tau}^n_{t+s} + \hat{\tau}^c_{t+s}\right) \right\} \right\}$$
$$- \sigma \sum_{m=0}^{\infty} \beta^s \left\{ r_{t+s} - F_t \left[\pi_{t+s+1}\right] - \Delta\hat{\tau}^c_{t+s} + \sigma \chi_{t+s} \right\}. \tag{E.9}$$

2. Inflation is given by

$$\pi_t = \kappa \sum_{s=0}^{\infty} (\beta\lambda)^s \left\{ F_t \left[w_{t+s} - p_{t+s}\right] + \frac{\alpha}{1-\alpha} F_t \left[y_{t+s}\right] \right\} + \frac{1-\lambda}{\lambda} \sum_{s=1}^{\infty} (\beta\lambda)^s F_t \left[\pi_{t+s}\right]. \tag{E.10}$$

3. Output is given by

$$y_t = (1-\alpha)\, n_t. \tag{E.11}$$

4. Market clearing implies

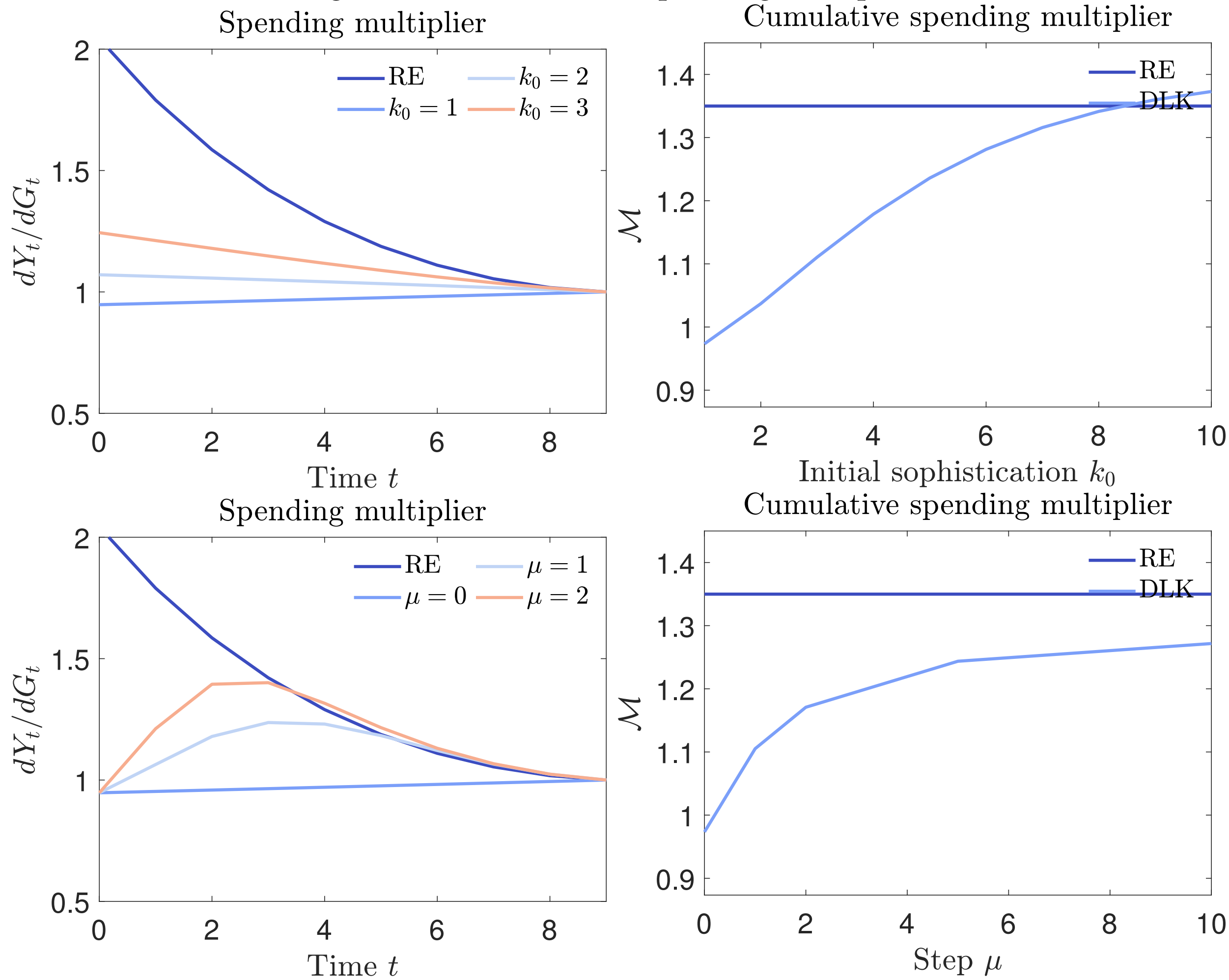
$$y_t = \frac{C}{Y} c_t + g_t. \tag{E.12}$$

Note that we assume that the beliefs that firms have about the real wage are consistent with household labor supply. An equilibrium is a solution to this system along with a specification of belief formation corresponding to level-k thinking.

E.1 Government-spending multipliers

In this section we briefly illustrate the analog to Proposition 1 for the case in which tax rates are constant and government spending rises by ΔG during the ZLB period.

Figure E.1: Government-spending multipliers



Comparing figures 4.1 and E.1, we see that the implications of level-k thinking for the government multiplier are essentially the same, regardless of whether Calvo frictions apply to wages and prices.

E.2 Consumption-tax policy

Proposition 3 continues to hold for the economy in which prices, rather than wages, are subject to Calvo frictions.

Proof. (Part 1) The proof strategy is as follows. Fix a k. First, we show that if the level-1 believe that the economy will stay at steady state for $t \geq T$, then this implies that all level-k beliefs and equilibrium feature output, consumption, labor and wage inflation remaining at their steady-state levels from $t \geq T$, and price inflation becoming zero from $t \geq T+1$ on. Second, we note that beliefs about future output, inflation, consumption, and labor are a function only of future tax rates and policies. (3) Finally, we recursively construct a sequence of policies $\{\hat{\tau}_t^c, \hat{\tau}^n\}$ which implements the flexible-price allocation and always features zero inflation. □

(1) Suppose that $F_t^1[y_{t+s}] = F_t^1[c_{t+s}] = F_t^1[n_{t+s}] = 0$ and $F_t^1[\pi_{t+s}] = 0$ if $t \geq T$. Then, the policies $g_t = \hat{\tau}_t^c = \hat{\tau}^n = r_t = 0$ for all $t \geq T$ imply that consumption, output, and labor for $t \geq T$ are given by

$$c_t = \psi \sum_{s\geq 1} \beta^s \frac{Y}{C} \left\{ F_t[y_{t+s}] - \omega_N F_t[n_{t+s}] + \varphi^{-1} F_t[w_{t+s} - p_{t+s}] \right\} = 0.$$

$$y_t = \frac{C}{Y} c_t = 0,$$

and

$$n_t = \frac{y_t}{1-\alpha} = 0,$$

respectively. Finally, inflation is given by

$$\pi_t = \kappa \sum_{s=0}^{\infty} (\beta\lambda)^s \left\{ \varphi F_t[n_{t+s}] + \sigma^{-1} F_t[c_{t+s}] + \frac{\alpha}{1-\alpha} F_t[y_{t+s}] \right\} + \frac{1-\lambda}{\lambda} \sum_{s=1}^{\infty} (\beta\lambda)^s F_t[\pi_{t+s}] = 0.$$

This then shows that starting from the initial beliefs $F_t[y_{t+s}] = F_t[c_{t+s}] = F_t[n_{t+s}] = 0$ and $F_t[\pi_{t+s}] = 0$ implies that the same holds for all k.

(2) Note that the temporary equilibrium for time t, which solves the system of equations (E.9)-(E.12) does not depend on policies before time t. This implies that for each t, beliefs at time t are unaffected by policies $\{\hat{\tau}_s^c, \hat{\tau}_s^n\}_{s=0}^{t-1}$.

(3) We now proceed recursively. At time t, given policies $\{\hat{\tau}_{t+s}^c, \hat{\tau}_{t+s}^n\}_{s\geq 1}$ and beliefs, we set the consumption tax $\hat{\tau}_t^c$ so that

$$\hat{\tau}_t^c = \frac{\psi}{\sigma} \sum_{s\geq 1} \beta^s \frac{Y}{C} \left\{ F_t[y_{t+s}] - \omega_N F_t[n_{t+s}] + \varphi^{-1} \left\{ F_t^k[w_{t+s} - p_{t+s}] - (\hat{\tau}_{t+s}^n + \hat{\tau}_{t+s}^c) \right\} \right\}$$
$$- \left\{ -F_t[\pi_{t+1}] - \hat{\tau}_{t+1}^c + \chi - \rho \right\} - \sum_{s=1}^{\infty} \beta^s \left\{ r_{t+s} - F_t[\pi_{t+s+1}] - \Delta\hat{\tau}_{t+s}^c + \chi_{t+s} \right\},$$

which implies that $c_t^k = 0$. It then follows that $n_t^k = y_t^k = 0$. Then, setting $\hat{\tau}_{t+s}^{n,k}$ such that

$$\hat{\tau}_t^n = -\hat{\tau}_t^c - \sum_{s=1}^{\infty} (\beta\lambda)^s \left\{ (\hat{\tau}_{t+s}^n + \hat{\tau}_{t+s}^c) + \varphi F_t[n_{t+s}] + \sigma^{-1} F_t[c_{t+s}] + \frac{\alpha}{1-\alpha} F_t[y_{t+s}] \right\} - \frac{1-\lambda}{\lambda\kappa} \sum_{s=1}^{\infty} (\beta\lambda)^s F_t[\pi$$

which implies that $\pi_t = 0$.

Proof. (Part 2) Under this assumption, the consumption function still implies that $C_t = C$, which implies that $N_t = N$ and $Y_t = Y$, i.e., both consumption, labor, and output in the level-1 economy stay at their steady-state levels. Using the fact that $(1 - \tau_t^n) / (1 + \tau_t^c) = (1 - \tau^n) / (1 + \tau^c)$, this implies that the relative wage W_t / P_t remains at its pre-shock steady state as well. Finally, this implies that $p_t^* = p_t$ and so inflation is always zero. The same argument then holds for $k > 1$. □