Foreign Residents and the Future of Global Cities*

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Abstract

Global cities are attracting an increasing number of tourists and foreign residents. This surge generates capital gains for property owners but negatively impacts renters and creates potentially important production, congestion, and amenities externalities. We study the optimal policy toward local and foreign residents in a model with key features emphasized in policy debates. Using this model, we provide sufficient statistics to calculate the optimal tax/transfer policies. These policies involve implementing transfers to internalize agglomeration, congestion, and other potential externalities. Importantly, it is not optimal to restrict, tax, or subsidize home purchases by foreign residents.

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1 Introduction

In 1917, the American composer Cole Porter moved to Paris and acquired an opulent residence built in 1777 for the brother of Louis XVI. There, he hosted luminaries like F. Scott Fitzgerald and Ernest Hemmingway and composed memorable tunes like "Night and Day" and "Anything Goes."

Buying a home in a foreign country was unusual at the beginning of the 20th century but has become increasingly common in recent decades. As remote work opportunities expand (Dingel and Neiman, 2020 and Aksoy, Barrero, Bloom, Davis, Dolls, and Zarate, 2022), many more people are seeking residence in foreign destinations.

At the same time, higher incomes and reduced air travel costs have greatly increased international tourism flows. According to data compiled by the United Nations World Tourism Organization, international tourist arrivals have grown at an average annual rate of 5.9 percent between 1950 and 2018.[1]

The surge in the flow of foreign residents is transforming housing markets in many cities across the globe. These flows generate capital gains for property and land owners but negatively impact renters and create potentially important production, congestion, and amenities externalities.

Many countries have grappled with the question of how to deal with potentially large numbers of foreign residents. The policies adopted so far vary widely, ranging from laissez-faire approaches and incentive programs designed to attract foreign home buyers to special taxes and regulations designed to restrict home purchases by foreigners.[2]

[1]See Allen et al. (2020) for an insightful analysis of the effect of tourism on the welfare of the local population.

[2]France and the United States impose no restrictions on foreign home buyers. Greece, Portugal, and Spain offer tax breaks and visa programs to attract foreign buyers. Some Canadian provinces, Hong Kong, Israel, and Singapore levy special taxes on foreign property purchases. The city of Vancouver has imposed taxes on unoccupied homes. Switzerland enforces annual quotas on foreign

Determining the optimal policy regarding foreign residents is important for three reasons. First, housing is the primary asset in most household portfolios (Cocco, 2005). Second, the availability of affordable housing near the workplace influences commuting times and job choices in ways that can significantly affect worker welfare. Third, most economic activity occurs in cities (Rossi-Hansberg and Wright, 2007).

In this paper, we use a Mirrlees (1971) approach to characterize optimal policy towards local and foreign residents in a model that embeds key insights from the economic geography literature.[3] We find that it is optimal to use taxes and transfers on locals and foreigners to internalize externalities. Imposing restrictions or taxes on home purchases by foreigners is never optimal. Likewise, it is never optimal to implement programs subsidizing foreign residents' home purchases. We provide a set of sufficient statistics to evaluate the impact of an influx of foreign residents and to calculate the tax/transfer policies required to implement the optimal solution.[4]

The baseline model has two locations: the center area and the periphery. Each location has a stock of housing and offices that is fixed in the short run. Foreign residents prefer to locate in the center and have an outside option: they can always stay in their home country.

Locals can live and work in different locations by incurring commuting costs. Wages, taste shocks, location-specific amenities, and commuting times influence the locals' home and work location choices. In our benchmark model, we assume that the ownership of houses and office buildings is equally distributed in the popula-

home sales, and New Zealand has strict foreign real estate investment limitations. In Australia, foreigners are generally prohibited from purchasing established dwellings but can invest in new buildings or vacant land. The Philippines and Thailand permit foreign home ownership but prohibit land ownership.

[3]Important contributions to this literature include Alonso (1964), Mills (1967), Muth (1969), Lucas and Rossi-Hansberg (2002), Rossi-Hansberg (2005), Desmet and Rossi-Hansberg (2013), Ahlfeldt et al. (2015), and Allen et al. (2015).

[4]We do not analyze the possibility of multiple equilibria. See Owens, Rossi-Hansberg, and Sarte (2020) for an analysis of how policy can also be used to implement a particular equilibrium.

tion. We revisit this assumption in Section 5, where we consider a model in which property ownership is unequally distributed.

We begin by examining the competitive equilibrium and analyzing the impact of a marginal increase in the number of foreign residents on social welfare. We identify two effects of this increase. The first relates to the agglomeration or production externality emphasized by Jacobs (1969), Lucas (1988, 2001), and Lucas and Rossi-Hansberg (2002). This effect can be negative if the arrival of foreigners leads to the relocation of local work effort from high- to low-productivity locations. The second effect pertains to the housing capital gains that accrue to the locals. An influx of foreign residents increases the demand for houses in the city center, increasing rents. So, locals make capital gains by selling houses to foreigners. We call this effect the foreign-residents surplus, and it is always positive.[5]

Next, we study the policy toward local and foreign residents that maximizes the welfare of the local population. We assume that the planner operates within a Mirrleesian environment. We set no a priori restrictions on the instruments available to the planner. Instead, we assume the planner faces information constraints: it cannot observe taste shocks that influence where the locals choose to live and work. We characterize this second-best optimum. It is optimal to distort location decisions by giving higher transfers to locals working in the city center to foster agglomeration externalities. In this baseline model, the entry of foreign residents should not be restricted, and their housing choices should not be distorted. So, foreigners' house purchases are neither taxed nor subsidized.

We then expand our model to incorporate four additional features to address issues discussed in policy circles. We introduce congestion externalities by assuming that commuting costs increase with the number of commuters. We show that, with

[5] This effect is analogous to the immigration surplus, e.g., Borjas (1995) and Guerreiro, Rebelo, and Teles (2020). Alternatively, we can also think of house sales as exports and interpret the benefits as gains from trade. We pursue this interpretation in appendix F.

endogenous commuting time, the sufficient statistics that describe the optimal transfers to locals also take into account the correction of congestion externalities and the interaction of congestion and agglomeration externalities. This interaction arises because increased commuting time reduces agglomeration externalities.

In the extended model, we also allow for remote work, i.e., workers can work from home. In this setting, locals can continue working for firms located in the city center, without having to commute. Remote workers do not contribute to agglomeration externalities, but they also do not commute and so do not contribute to congestion. The optimal transfers take into account the trade-off between the negative relative contribution of remote workers to the agglomeration externality and their positive relative contribution to the congestion externality.

We also assume that foreigners value authenticity, that is, they derive utility from having locals live and work in the city center. At first sight, one might think that this feature would not affect the social optimum. After all, the planner does not include the utility of foreigners in the social welfare function. However, it is optimal to internalize this externality by providing transfers to locals who live and work in the city center. The rationale for this policy is that the externality affects the participation constraint of foreigners and influences their decision to relocate.

Finally, we assume that foreigners may directly impact the value that locals attach to amenities in the city center. We show that these amenity externalities do not affect the statistics for the optimal transfers to locals but introduce a reason to distort the entry of foreigners. If these externalities are negative, it is optimal to correct them by imposing a lump-sum tax on foreigners, similar to the per-diem or per-night tax levied by an increasing number of cities. As in the baseline model, the optimal policy does not tax foreigners' house purchases.

We discuss an important extension to our baseline model: we assume that property ownership is unequally distributed. In this situation, once externalities are corrected, it is feasible to implement transfers to redistribute the capital gains so that ex-ante (before taste shocks are realized) all locals benefit from the influx of foreigners. This form of heterogeneity does not affect the optimal policy towards foreign residents described above.

Our model provides some insights into the implications of an inflow of foreign residents for optimal long-run city design. By the long run, we mean a time frame where offices can be converted into houses and vice versa in both the city center and the periphery. In our model, it is optimal to convert offices into houses in the city center to meet the increased demand for housing. However, the optimal solution for the periphery is ambiguous. On the one hand, more locals reside in the periphery, raising the marginal value of houses in that area. On the other hand, more people work in the periphery, increasing the value of offices.

Our paper relates to the analysis of the impact of foreign home buyers on welfare by Favilukis and Van Nieuwerburgh (2021). These authors develop a quantitative model with two locations and assume that foreign residents buy disproportionally more houses in the city center. Favilukis and Van Nieuwerburgh (2021) argue that the influx of foreign residents reduces local welfare. Their model does not feature the externalities we emphasize. The driver of overall welfare losses is redistribution across people with heterogeneous levels of home ownership. We discuss these issues in Section 5. In that section, we show that policies that distort foreigners' entry are never optimal because they decrease the gains associated with the foreign residents surplus. Instead, the optimal policy features appropriate redistribution of these capital gains.

Our analysis also relates to recent work on using transfers to internalize agglomeration externalities. Prominent examples include Fu and Gregory (2019), Fajgelbaum and Gaubert (2020), and Rossi-Hansberg et al. (2019).[6]

[6]In general, the optimal policy depends on the distribution of the location-taste shocks. Davis and Gregory (2021) argue that the distribution of these shocks cannot be identified using location-choice data. Rossi-Hansberg et al. (2019) show that the optimal policy in their environment is not significantly affected by the form of the taste-shock distribution.

The structure of the paper is as follows. Section 2 introduces the baseline model, characterizes the competitive equilibrium, and assesses the impact of a marginal increase in the number of foreign residents on social welfare. Section 3 outlines the Mirrleesian optimal policy in the baseline model. In Section 4, we extend the model to include traffic congestion, remote work, amenity effects, and the possibility that foreign residents value the authenticity of the city center. Section 5 examines a variant of the model that incorporates unequal property holdings. Section 6 discusses how the influx of foreign residents affects long-run city design. Section 7 concludes.

2 Competitive equilibrium in the baseline model

There are two locations in the model: the center and the periphery. Both locations produce a single tradable good by combining labor and a type of capital that we refer to as office buildings.

The index ℓ takes the value c or p depending on whether a person lives in the center or the periphery. Similarly, the index j takes the value c or p depending on whether a person works in the center or the periphery.

Each local person i draws a taste shock, $\xi_{i,\ell,j}$, with respect to living in location ℓ and working in location j. Following McFadden (1973), we assume that this shock is governed by a Gumbel($0,\eta^{-1}$) distribution.[7] These shocks eliminate corner solutions with respect to location choices and make the analysis tractable because the maximum of n i.i.d. Gumbel variables follows a Gumbel distribution.

Locals who live in location ℓ and work in location j derive utility from housing services ($h_{\ell,j}$) and from consuming a single tradable good ($c_{\ell,j}$). They exogenously supply one unit of labor, which they allocate to working and commuting.

In this version of the model, local people have an equal endowment of houses

[7]The mean of this distribution is not zero, but this value does not influence the comparative evaluations individuals make between different locations.

and office buildings. We relax this assumption in Section 5.

Location choices The utility that local person i derives from living in location ℓ and working in location j has two components:[8]

$$\xi_{i,\ell,j} + u_{\ell,j}. \tag{1}$$

The first is the taste shock, $\xi_{i,\ell,j}$. The second is given by

$$u_{\ell,j} \equiv \overline{u}_\ell + c_{\ell,j} + v\left(h_{\ell,j}\right). \tag{2}$$

We refer to $u_{\ell,j}$ as "common utility" because it is common to all who live in location ℓ and work in location j. The variable $c_{\ell,j}$ denotes consumption, $h_{\ell,j}$ housing services, and $\overline{u}_\ell$ the utility that locals derive from the amenities in location ℓ. We assume that $v(h)$ is strictly increasing and concave.

Person i maximizes utility subject to the budget constraint

$$c_{\ell,j} + r_\ell h_{\ell,j} = w_j\left(1 - t_{\ell,j}\right) + T. \tag{3}$$

The variable $t_{\ell,j}$ denotes the time it takes to commute from a home in location ℓ to work at an office in location j. Commuting costs are zero for those who live and work in the same location ($t_{\ell,\ell} = 0$). The variable r_ℓ denotes the cost of renting a unit of housing in location ℓ, and T denotes the housing and office rents, which are given by

$$T \equiv \sum_\ell r_\ell \overline{H}_\ell + \sum_j r_j^K \overline{K}_j. \tag{4}$$

The variable r_j^K denotes the rental rate of office buildings in location j. In Section 5, we consider a version of the model in which people are heterogeneous with respect to their ownership of offices and houses.

[8] To simplify the notation, we omit subscript i for variables that do not depend on i for the allocations we discuss.

The first-order conditions for this problem are

$$v'\left(h_{\ell,j}\right) = r_\ell,$$

$$c_{\ell,j} = w_j\left(1 - t_{\ell,j}\right) + T - r_\ell h_{\ell,j}.$$

All locals living in location ℓ have the same housing consumption, i.e., $h_{\ell,j} = h_\ell$ for all j. The resulting common utility is

$$u_{\ell,j} = \overline{u}_\ell + w_j\left(1 - t_{\ell,j}\right) + T - r_\ell h_{\ell,j} + v\left(h_{\ell,j}\right). \tag{5}$$

A person lives in ℓ and works in j if

$$u_{\ell,j} + \xi_{i,\ell,j} = \max_{\ell',j'}\{u_{\ell',j'} + \xi_{i,\ell',j'}\}.$$

Following standard steps, the share of people who live in ℓ and work in j can be computed as follows:[9]

$$\pi_{\ell,j} = \frac{e^{\eta u_{\ell,j}}}{\sum_{\ell',j'} e^{\eta u_{\ell',j'}}}. \tag{6}$$

Foreign residents To simplify, we assume that foreign residents are not subject to taste shocks and prefer to live in the city center.[10] Their problem is to choose consumption (c_f) and housing in the center (h_f) so as to maximize their utility,

$$\overline{u}_f + c_f + v\left(h_f\right),$$

where $\overline{u}_f$ is the value foreign residents attach to the amenities in the center. Foreigners bring a fixed endowment of the tradable good (y_f) that they use to pay for consumption, housing services, and any potential taxes. We assume that these taxes are zero in the competitive equilibrium. The foreign residents' budget constraint is:

$$c_f + r_c h_f = y_f.$$

[9]The explicit derivation of this formula can be found in the appendix.

[10]In Appendix E, we discuss the case in which foreigners live both in the city center and the periphery.

The first-order conditions for this problem are

$$v'\left(h_f\right) = r_c,$$

$$c_f = y_f - r_c h_f.$$

These conditions imply that foreign residents choose the same housing consumption as locals who live in the center.

Foreigners can stay in their own country and receive utility u_f^*. They only migrate if their participation constraint is satisfied:

$$\overline{u}_f + c_f + v\left(h_f\right) \geq u_f^*, \tag{7}$$

Firms' problem Each location has a measure one of identical firms. Firms in location j produce output (y_j) by combining offices (k_j) and labor (l_j) according to a Cobb-Douglas production function:

$$y_j = A\left(L_j\right) k_j^{\alpha} l_j^{1-\alpha}.$$

The function $A(L_j)$ represents an agglomeration or production externality. Locations with more workers tend to be more productive because there are more opportunities for workers to learn from each other. We assume that the function $A(L_j)$ takes the form:

$$A\left(L_j\right) = L_j^{\gamma}.$$

The parameter $\gamma \geq 0$ controls the strength of the agglomeration externality. If $\gamma = 0$, there are no production externalities. The higher is γ, the stronger are these externalities.

The problem of a representative firm in location j is to maximize profits:

$$A\left(L_j\right) k_j^{\alpha} l_j^{1-\alpha} - w_j l_j - r_j^K k_j.$$

The first-order condition for the firms' problem are:[11]

$$w_j = (1-\alpha)\, A\left(L_j\right)\left(\frac{k_j}{l_j}\right)^{\alpha}, \tag{8}$$

$$r_j^K = \alpha A\left(L_j\right)\left(\frac{k_j}{l_j}\right)^{\alpha-1}. \tag{9}$$

Equilibrium conditions The goods market clearing condition is:

$$\sum_{\ell,j}\pi_{\ell,j}c_{\ell,j} + N_f c_f = \sum_j A\left(L_j\right) L_j^{1-\alpha}\overline{K}_j^{\alpha} + N_f y_f. \tag{10}$$

On the left-hand side of this equation, we have the sum of the locals' consumption across all living and working locations ($\sum_{\ell,j}\pi_{\ell,j}c_{\ell,j}$) and the total consumption by foreign residents, $N_f c_f$, where N_f denotes the total amount of foreign residents. On the right-hand side we have the production in the center and periphery $\left(\sum_j A\left(L_j\right) L_j^{1-\alpha}\overline{K}_j^{\alpha}\right)$ and the endowment of goods brought by the foreigners $N_f y_f$.

The labor market clearing condition is

$$l_j = L_j,$$

where L_j is the amount of labor available in location j. This variable is equal to the time supplied by all the people who work at location j net of commuting costs

$$L_j = \sum_{\ell}\pi_{\ell,j}\left(1 - t_{\ell,j}\right).$$

The market clearing condition for office buildings in location j is

$$k_j = \overline{K}_j.$$

Finally, the housing market clearing conditions for the center and the periphery are

$$\pi_{c,c}h_{c,c} + \pi_{c,p}h_{c,p} + N_f h_f = \overline{H}_c, \tag{11}$$

$$\pi_{p,c}h_{p,c} + \pi_{p,p}h_{p,p} = \overline{H}_p, \tag{12}$$

[11] Since the technology is constant returns to scale, profits are zero.

We also define the total population living in location ℓ as

$$\Pi_\ell \equiv \sum_j \pi_{\ell,j}. \tag{13}$$

The welfare impact of an increase in foreign residents We define social welfare as the sum of the utility of all local people.

$$\mathcal{W} \equiv \int_0^1 \max \left\{ u_{\ell,j} + \xi_{i,\ell,j} \right\} di \tag{14}$$

In the appendix, we show that social welfare is given by:

$$\mathcal{W} = \frac{\log \left(\sum_{\ell,j} e^{\eta u_{\ell,j}} \right)}{\eta} + \frac{1}{\eta} \int_0^\infty [-\log(y)\, e^{-y}] dy,$$

where $\int_0^\infty [-\log(y)\, e^{-y}] dy$ is the Euler-Mascheroni constant.

We assume that the foreign residents' participation constraint, (7), is satisfied, and that the function $v(h)$ takes the form $v(h) = h^{1-\sigma} / (1-\sigma)$. The following proposition provides sufficient statistics to evaluate the impact of an influx of foreign residents on social welfare.

Proposition 1. *The change in social welfare from a marginal increase in the number of foreign residents can be decomposed into the sum of two terms:*

$$d\mathcal{W} = \mathcal{FS} + \mathcal{PE}.$$

The first term is the foreign-residents surplus, $\mathcal{FS}$, and it is given by:

$$\mathcal{FS} \equiv \sigma \frac{N_f}{\Pi_c + N_f} r_c h_f \left(d\Pi_c + dN_f \right).$$

The second term is the production or agglomeration externality term, $\mathcal{PE}$, and it is given by:

$$\mathcal{PE} \equiv \gamma \times \mathbb{COV} \left[\frac{Y_j}{L_j} \left(1 - t_{\ell,j}\right), \frac{d\pi_{\ell,j}}{\pi_{\ell,j}} \right] = \gamma \sum_{\ell,j} \pi_{\ell,j} \frac{Y_j}{L_j} \left(1 - t_{\ell,j}\right) \frac{d\pi_{\ell,j}}{\pi_{\ell,j}}.$$

See appendix A.3 for the proof.

The foreign resident surplus equals the capital gains realized on the houses sold to foreigners. Foreigners replace some of the locals who live in the center ($d\Pi_c < 0, dN_f > 0$). In addition, people in the center reduce housing consumption, making space for additional foreign residents. As a result, the number of people who live in the center increases ($d\Pi_c + dN_f > 0$). Since everyone living in the center consumes the same amount of housing, per capita housing consumption falls. Rents rise, resulting in an increase in rental income obtained from foreigners. This effect is the foreign resident surplus.

The foreign resident surplus is similar to the immigration surplus discussed in the immigration literature (e.g., Borjas, 1995 and Guerreiro, Rebelo, and Teles, 2020). This surplus results from an increase in the income accruing to non-labor factors such as land.

The interpretation of the production or agglomeration externality, $\mathcal{PE}$, is as follows. Labor is better allocated to locations with high average labor-productivity because the production-externality effect becomes more relevant. If, on average, people leave higher productivity locations, the covariance is negative, and there is a welfare loss.

Three pertinent comments about this component of the change in welfare are as follows. If foreigners choose the same distribution of locations as locals and $\sigma = 1$, then $d\pi_{j,\ell} = 0$ and $\mathbb{COV} = 0$ (see Appendix E). So, there is no welfare loss from the production externality. Second, the production externalities would be more important in a model with multiple peripheries because workers who move from the center would scatter across different peripheries. Third, the ability of locals to work from home reduces production externalities.

3 Mirrleesian optimal policy

Our analysis of the impact of foreign residents on the competitive equilibrium suggests two questions. First, is it optimal to restrict home purchases by foreigners when the foreign resident surplus is lower than the production externality? Second, is it optimal to tax foreign home purchases to internalize the production externality when $\mathbb{COV} < 0$? We show that the answer to both questions is no.

In the spirit of Mirrlees (1971), we make no ex-ante restrictions on the set of instruments that the government can design. Instead, we work directly with the informational constraints that arise because agent types are unobservable. We assume that the planner can distinguish between locals and foreigners but does not observe idiosyncratic taste shocks. The planner only has information about individuals' residential and work locations. In other words, we assume that the planner can condition the allocations for locals on their location decisions but not on their idiosyncratic taste shocks.

The incentive constraints are that location decisions must be privately optimal given the allocations chosen by the planner. In other words, incentive compatibility requires that two local people who live in the same location and work in the same location have the same level of consumption and housing and so the same common utility, $u_{\ell,j}$. It follows that person i chooses to live in location ℓ and work in location j if

$$u_{\ell,j} + \xi_{i,\ell,j} = \max\{u_{\ell',j'} + \xi_{i,\ell',j'}\}.$$

In the appendix, we show that these incentive compatibility constraints imply that

$$\pi_{\ell,j} = \frac{e^{\eta u_{\ell,j}}}{\sum_{\ell',j'} e^{\eta u_{\ell',j'}}}. \tag{15}$$

We can compute the Mirrleesian optimal allocations as follows. The planner maximizes the welfare function (14), subject to the resource constraints for goods, (10), where $L_j \equiv \sum_\ell \pi_{\ell,j}(1 - t_{\ell,j})$, the resource constraint for houses in each location, (11) and (12), the location-decisions constraints, (15), and the foreign-resident participation constraint, (7).

We solve this policy problem in two steps. First, we take the number of foreigners N_f as given and solve for the remaining quantities. Then, we characterize the necessary conditions for the optimal number of foreign residents N_f.

3.1 Optimal policy for a fixed number of foreign residents

We now describe the optimal policy assuming that a given number of foreigners, N_f, enter the country. We define transfers to individuals living in location ℓ and working in location j as

$$T_{\ell,j} \equiv c_{\ell,j} + \hat{r}_{\ell,j} h_{\ell,j} - w_j(1 - t_{\ell,j}), \tag{16}$$

Where $\hat{r}_{\ell,j}$ the effective rent paid by individuals who live in ℓ and work in j, is given by $\hat{r}_{\ell,j} = v'(h_{\ell,j})$. Differences in prices across individuals may arise if the government taxes housing purchases, i.e., $\hat{r}_{\ell,j}$ denotes the after-tax price. We compute wages and office rents using equations (8) and (9), respectively, replacing $l_j = L_j$ and $k_j = \overline{K}_j$. The following proposition provides sufficient statistics to calculate the tax/transfer policies required to implement the optimal solution.

Proposition 2. *In the optimal solution, all locals living in the same location pay the same rent $\hat{r}_{\ell,j} = r_\ell$, i.e., the planner does not distort the locals' house purchases. Transfers to locals have two key features:*

1. *Absent externalities, rents on houses and offices and proceeds from the entry fee paid by foreigners are equally distributed among locals.*

2. *The planner corrects the production externality by giving higher (lower) transfers to location pairs with higher (lower) than average labor income.*

The total transfers implemented by the planner are the sum of two terms:

$$T_{\ell,j} = \Xi + \Xi^{\mathcal{PE}}_{\ell,j}, \tag{17}$$

where

1. *the common term is*

$$\Xi \equiv \underbrace{\sum_{\ell} r_\ell \overline{H}_\ell + \sum_j r_j^K \overline{K}_j}_{\text{Rents on houses and offices}} + \overbrace{\Theta_f}^{\text{Taxes on foreigners}},$$

and $\Theta_f \equiv N_f (y_f - c_f - r_c h_f)$ *denotes the proceeds from taxes on foreigners.*

2. *the production-externality correction term is*

$$\Xi^{\mathcal{PE}}_{\ell,j} \equiv \gamma \times \left\{ Y_j \frac{1 - t_{\ell,j}}{L_j} - \sum_{\ell,j} \pi_{\ell,j} Y_j \frac{1 - t_{\ell,j}}{L_j} \right\}.$$

In the optimum, all locals in a location ℓ face the same housing price r_ℓ. In other words, the marginal rates of substitution between houses and consumption are equalized between all locals. This result is in the spirit of the optimality of uniform commodity taxation (Atkinson and Stiglitz, 1976) and production efficiency (Diamond and Mirrlees, 1971).

With quasi-linear preferences, welfare is independent of the distribution of consumption. Only aggregate consumption matters. So, the planner can engineer any distribution of consumption to discipline location decisions without affecting aggregate consumption.[12] This result implies that optimal transfers are driven only by efficiency considerations. Redistribution considerations play no role.

In the absence of externalities, private location decisions are socially optimal, so the planner does not have an incentive to distort these decisions. The optimal transfers simply redistribute the rents on houses and offices and the proceeds from taxing foreigners equally across the local population. In the presence of agglomeration externalities, location decisions turn out to be suboptimal from a social standpoint (see

[12]This result also implies that second-best aggregate quantities actually coincide with the first-best ones.

also Fajgelbaum et al., 2019 and Rossi-Hansberg et al., 2019). It follows that the planner needs to implement transfers that incentivize people to move to the locations where their positive contribution to agglomeration externalities is highest.

The transfer needed to correct the externality depends only on the comparison of labor income for a given pair of living- and work-location decisions and the average labor income in the overall economy. The planner gives a relatively higher subsidy for people who choose location pairs with higher than average contributions to production and gives a relatively lower subsidy to people who choose location pairs with lower than average contributions to production.

Foreigners' house purchases are taxed if the marginal rate of substitution between houses and consumption is higher for foreigners than for locals. We define the house tax (or wedge) as

$$\tau_h \equiv \frac{v'(h_f)}{v'(h_{c,\ell})} - 1. \tag{18}$$

Foreigners pay an entry fee if their income exceeds their expenditure on consumption and housing goods. We define this fee as

$$T_f \equiv y_f - c_f - (1 + \tau_h) r_c h_f. \tag{19}$$

So, the total proceeds from taxing foreigners are $\Theta_f = N_f \tau_h r_c h_f + N_f T_f$. The following proposition summarizes the optimal treatment of foreigners in this model.

Proposition 3. *In the optimal allocation, foreigners' house purchases are not taxed, and there is an optimal entry fee that sets foreigners' utility equal to their outside option:*

1. *No taxes on foreigners' house purchases, $\tau_h = 0$.*

2. *There is an optimal entry fee on foreigners which satisfies*

$$T_f = \overline{u}_f + y_f - r_c h_f + v(h_f) - u_f^*.$$

It is optimal for the planner to set the marginal rates of substitution between houses and consumption equal for all individuals who reside in the same location. This result means that foreigners' home purchases should not be distorted, i.e., the tax on foreign home purchases is zero. So, foreigners and locals in the city center pay the same house prices, r_c. This result follows from standard public finance principles: it is better to use a discriminatory lump-sum tax than to distort the purchases of goods.

Second, it is optimal to levy a lump-sum entry fee on foreigners that sets their utility equal to their outside option, i.e., makes foreigners indifferent about moving. The reason for this result is as follows. Since the welfare function only takes into account the utility of locals, it is optimal for the planner to tax the foreigners' gains from entry and rebate these proceeds to the locals. The intuition for these results is also related to the literature on the optimal trade tariff. We now elaborate further on this relation.

Relation to the optimal tariff literature We can interpret the sales of houses to foreigners as exports paid for in units of the tradable consumption good. So, it is natural to relate our findings to standard results in the trade literature.

The home country is a monopolist on the sale of houses. So, through the lens of the trade literature, our model implies that the optimal trade tariff is zero. This conclusion apparently contradicts the classical result that it is optimal to manipulate the terms of trade.

This apparent contradiction emerges because, unlike in the standard trade literature, we impose no exogenous restrictions on the policy instruments available to the home country. In particular, the government can impose a lump-sum tax on foreigners in our model.

In Appendix F, we use a standard model of international trade to discuss the connection with our results. Using that trade model, we show that when this type of lump-sum instrument is available, the optimal policy is to set the trade tariff to zero and instead charge a right-to-trade fee. This fee extracts the foreigners' gains from trade.

This setup is analogous to a monopolist who uses a two-part tariff: it sets the price equal to the marginal cost and charges a fixed fee that extracts all the consumer surplus. Similarly, in our model, it is optimal not to tax house purchases by foreigners and instead impose a lump-sum tax on foreigners.

3.2 The optimal number of foreign residents

We now discuss the policies that optimize the number of foreign residents. At the optimum, foreigners' utility always equals their outside option. Foreigners are indifferent about moving. This result implies that the implementation of the optimum is consistent with the free mobility of foreigners into the country. Consequently, it is not optimal to impose binding quotas or other restrictions on the number of foreigners who can enter the home country. The intuition for this result is that it is always better to control the inflow of foreign residents through an entry fee rather than a quota system. The entry fee generates additional tax revenue that can be redistributed toward locals. We summarize this result in the following corollary.

Corollary 1. *It is not optimal to introduce quotas on the number of foreign residents.*

Let $W^*(N_f)$ denote the welfare associated with the optimal number of foreign residents, N_f. Using an envelope argument, we find that the marginal effect of an additional foreigner on welfare is given by:

$$\frac{dW^*(N_f)}{dN_f} = y_f - c_f - r_c h_f = T_f.$$

The marginal effect of increasing the number of foreigners equals the marginal value of selling h_f houses and buying $y_f - c_f$ additional consumption goods.

The difference between the value of additional consumption goods and the value of houses sold equals the entry fee T_f. If the trade fee is positive, $T_f > 0$, then letting in an additional foreigner strictly increases welfare. Instead, if the trade fee is negative, $T_f < 0$, then allowing in an additional foreigner strictly decreases welfare. Intuitively, suppose the valuation of the consumption goods brought by the marginal foreigner is higher than the valuation of the houses they buy. In that case, it is optimal to let an additional foreigner enter the home country.

Following this logic, the planner allows additional foreigners to enter the economy until the entry fee, which sets their utility equal to the outside option, is zero:

$$\frac{dW^*(N_f)}{dN_f} = 0 \Leftrightarrow T_f = 0.$$

This surprising result implies that the optimal treatment of foreigners is a laissez-faire policy. From the previous section, we know that it is optimal not to tax foreign house purchases. Here, we show that the optimal number of foreigners is obtained when there are no quotas and the entry fee is zero. In other words, foreign entry is free and undistorted. These results are summarized in the following proposition.

Proposition 4. *In the optimal allocation, the entry fee equals zero, $T_f = 0$. So, the optimal policy towards foreign residents is laissez-faire, i.e., free and undistorted entry is optimal:*

1. *There are no quotas/restrictions on foreign entry.*
2. *Taxes on foreigner's house purchases are zero, $\tau_h = 0$.*
3. *Entry fees are zero, $T_f = 0$.*

From an international trade perspective, this result states that the optimal number of trading partners (foreigners) is such that the gains from trade of the marginal partner are zero. This policy maximizes the gains from trade in the home country and, therefore, maximizes welfare. In appendix F, we further elaborate on the relation between our results and those obtained in a standard trade model.

From a public finance perspective, these results can be interpreted as the optimality of production efficiency (Diamond and Mirrlees, 1971). At an abstract level, foreigners can be interpreted as a technology that transforms houses into consumption goods. In the previous section, we assumed that N_f was fixed so the entry fee did not distort the number of entering foreigners. When the number of foreigners is endogenous, it is not optimal to distort the inflow of foreigners, so the optimal entry fee is zero.

Surprisingly, production efficiency is optimal despite the presence of externalities. This result emerges because the externality does not directly involve the number of foreign residents but only the labor supply of locals in each location. The Mirrleesian planner has enough instruments to get locals to internalize these agglomeration effects. As shown in the previous section, these instruments take the form of higher transfers for individuals with location decisions where they obtain higher than average labor income and lower transfers for individuals with location decisions where they obtain lower than average labor income. Production efficiency is not optimal when there are other externalities to which foreigners contribute directly. In Section 4, we extend our results, focusing on a more extensive set of potential impacts of foreign residents and discuss how our benchmark results change.

4 Congestion, remote work, authenticity, amenities

In this section, we extend the baseline model to include four additional issues often mentioned in policy discussions.

The first extension is to endogeneize commuting time. In the baseline model, the time spent commuting between the locations is an exogenous parameter. In practice, commuting time is likely to increase with the number of commuters. This extension introduces an additional *commuting externality*, which affects both the welfare costs of the entry of foreigners and the optimal transfers to correct externalities.

The second extension is to allow for remote work, i.e., locals can work either on-site at an office or remotely at home.[13] After the Covid pandemic, remote work has become ubiquitous, contributing significantly to the rise in the number of foreign residents. Allowing locals to work remotely is also important because it offers the possibility of working in the city center without paying commuting costs. This extension changes the welfare costs of additional foreign residents and affects optimal transfers.

The third extension is to endogeneize the amenity value that foreigners place on living in the city center. We assume that foreign residents derive utility from the *authenticity* of the city center, i.e., foreign residents derive utility from the fact that locals live in the city center. In reduced form, these effects capture a number of non-marketable attributes of the city center, such as culture and traditions. As we discuss below, this *authenticity externality* doesn't influence the welfare costs associated with foreigner influx but provides an added incentive for encouraging locals to reside in the city center.

Finally, we consider the impact of the influx of foreign residents on the amenity value that locals derive from residing in the city center. This *amenity externality* can be positive if, for instance, locals value the increase in cultural diversity. It can also be negative if, for example, foreigners reduce the authenticity of the city center, making it less appealing for locals. This extension can also capture congestion effects on the provision of public goods created by the influx of foreigners (see Guerreiro, Rebelo, and Teles, 2020). As we discuss next, this factor significantly impacts the design of the optimal entry fee.

4.1 The competitive equilibrium

In this section, we describe the environment and the competitive equilibrium.

[13]See also Monte, Porcher, and Rossi-Hansberg (2023) for a dynamic theory of remote work and city structure in which agglomeration forces can generate multiple equilibria.

4.1.1 Local households

As in the baseline model, locals choose where to live, ℓ, and where to work, j. They can also choose their "work arrangement", e. The work arrangement takes the value o and h depending on whether the individual works onsite or at home, respectively. If a local lives in ℓ, and works in j with work arrangement e, they have utility

$$\xi_{i,\ell,j,e} + u_{\ell,j,e}. \tag{20}$$

People's choices are influenced by Gumbel-distributed taste shocks, $\xi_{\ell,j,e}$, about the location of their residence, workplace, and remote versus onsite work. Their common utility is given by

$$u_{\ell,j,e} \equiv \overline{u}_\ell + c_{\ell,j,e} + v(h_{\ell,j,e}), \tag{21}$$

where c and h denote the household's traded goods and housing consumption, respectively. The function $v(\cdot)$ satisfies the same assumptions as in the baseline model.

Amenities There is a growing literature on the impact of changes in resident composition on amenities, see, e.g., Guerrieri, Hartley, and Hurst (2013), Diamond (2016), and Almagro and Dominguez-Iino (2022). To study these effects in our model, suppose that foreign residents affect the value attributed by locals to amenities in the city center:

$$\overline{u}_c = \mathcal{U}_c\left(N_f\right).$$

We assume that the elasticity of amenities to foreign residents is constant:

$$\frac{\mathcal{U}_c'\left(N_f\right)N_f}{\mathcal{U}_c\left(N_f\right)} = \phi_{\overline{u}}.$$

We make no assumptions about the sign of $\phi_{\overline{u}}$. If $\phi_{\overline{u}} > 0$, then the entry of foreign residents increases the attractiveness of the city center. If $\phi_{\overline{u}} < 0$, then the entry of foreign residents decreases the attractiveness of the city center. To simplify, we maintain the assumption that the amenity value of the periphery is an exogenous constant.

Budget constraint A local living in ℓ, working in j, with work arrangement e, faces the budget constraint:

$$c_{\ell,j,e} + r_\ell h_{\ell,j,e} = w_{j,e}\left(1 - t_{\ell,j,e}\right) + T,$$

where the variables are analogous to the baseline model. The wage $w_{j,e}$ depends both on the place of work but also on the working arrangement. As in the baseline model, if a local lives and works in the same place, they do not spend time commuting, $t_{\ell,\ell,e} = 0$ for all ℓ and e. Similarly, remote workers do not spend time commuting, so $t_{\ell,j,h} = 0$ for all ℓ and j.

Congestion In the baseline model, time spent commuting is exogenous. However, in practice, commuting time is likely to increase with the number of commuters because of traffic congestion. We model this phenomenon by assuming that

$$t_{\ell,j,o} = \mathcal{T}_{\ell,j,o}(\pi_{\ell,j,o}), \tag{22}$$

if $\ell \neq j$. We assume that the elasticity of commuting time with respect to the number of commuters is constant and positive

$$\frac{\mathcal{T}'_{\ell,j,o}(\pi_{\ell,j,o})\pi_{\ell,j,o}}{\mathcal{T}_{\ell,j,o}(\pi_{\ell,j,o})} = \psi > 0$$

Goods and housing consumption and location choices Consider a household living in location ℓ, working in location j, with work arrangement e. Their optimal goods and housing consumption satisfy

$$\begin{aligned} &v'(h_{\ell,j,e}) = r_\ell, \\ &c_{\ell,j,e} = w_{j,e}(1 - t_{\ell,j,e}) + T - r_\ell h_{\ell,j,e}. \end{aligned}$$

Their optimal location and work arrangement choices maximize

$$u_{\ell,j,e} + \xi_{\ell,j,e}.$$

So, the share of individuals living in ℓ and working in j with employment type e is

$$\pi_{\ell,j,e} = \frac{e^{\eta u_{\ell,j,e}}}{\sum_{\ell',j',e'} e^{\eta u_{\ell',j',e'}}}.$$

In this model, individuals who live and work in different places are more likely to work remotely, i.e.,

$$\frac{\pi_{j,j,o}}{\pi_{j,j,h}} = e^{\eta(w_{j,o}-w_{j,h})} > e^{\eta(w_{j,o}(1-t_{\ell,j})-w_{j,h})} = \frac{\pi_{\ell,j,o}}{\pi_{\ell,j,h}},$$

for $\ell \neq j$. So, this model suggests that an influx of foreign home buyers will incentivize remote work as locals that move to the periphery continue to work in the city center but more adopt the remote work possibility.

4.1.2 Foreign residents

The foreign resident problem is the same as in the baseline model. Their problem is to choose consumption and housing to maximize utility

$$\overline{u}_f + c_f + v(h_f),$$

subject to the budget constraint $c_f + r_c h_f = y_f$. Foreigners are willing to move if

$$\overline{u}_f + c_f + v(h_f) \geq u_f^*. \tag{23}$$

Authenticity We assume that foreign residents derive utility from the "authenticity" of the city center, which is fostered by having more locals live and work there. We model this effect by allowing the amenity value that foreigners derive to depend arbitrarily on the number of locals who live and work in the city center, i.e.,

$$\overline{u}_f = \mathcal{U}_f(\boldsymbol{\pi}),$$

where $\boldsymbol{\pi} = \{\pi_{\ell,j,e}\}_{\ell,j,e}$.

4.1.3 Firms' problem

The production function of the representative firm in location j is given by

$$Y_j = A_j \left(L_{j,o}\right) \left(l_{j,o}^{1-\alpha} k_j^{\alpha} + \zeta l_{j,h}\right),$$

where $l_{j,o}$ and $l_{j,h}$ denote the number of people working for the firm in the office and at home, respectively. The agglomeration or production externality, $A_j\left(L_{j,o}\right)$, depends on the total number of people who work in offices in location j, $L_{j,o}$. This externality benefits the productivity of all the workers. As in the baseline model, we assume that

$$A_j \left(L_{j,o}\right) = \overline{A}_j L_{j,o}^{\gamma},$$

where γ controls the strength of the production externality and $\overline{A}_j$ denotes a location specific TFP parameter. The parameter ζ determines the productivity of remote workers. The production function of the baseline model corresponds to the case of $\zeta = 0$.

A firm in location j maximizes profits which is equal to production minus the costs of hiring office workers $w_{j,o} l_{j,o}$, where $w_{j,o}$ denotes their wage, the costs of hiring remote workers $w_{j,h} l_{j,h}$, where $w_{j,h}$ denotes their wage, and the cost of renting office buildings $r_j^K k_k$, where r_j^K denotes the rental rate on office buildings in location j. The optimality conditions of this problem are given by:

$$w_{j,h} = A_j \left(L_{j,o}\right) \zeta, \tag{24}$$

$$w_{j,o} = (1-\alpha) A_j \left(L_{j,o}\right) l_{j,o}^{-\alpha} k_j^{\alpha}, \tag{25}$$

$$r_j^K = \alpha A_j \left(L_{j,o}\right) l_{j,o}^{1-\alpha} k_j^{\alpha-1}. \tag{26}$$

Market clearing and equilibrium There are two labor market clearing conditions. The first is for office workers in location j:

$$l_{j,o} = L_{j,o} = \sum_{\ell} \pi_{\ell,j,o} \left(1 - t_{\ell,j,o}\right).$$

The second is for remote workers employed by firms in location j

$$l_{j,h} = L_{j,h} = \sum_{\ell} \pi_{\ell,j,h}.$$

The market clearing conditions for office buildings in location j is

$$k_j = \overline{K}_j.$$

The goods market clearing condition is

$$\sum_{\ell,j,e} \pi_{\ell,j,e} c_{\ell,j,e} + N_f c_f = \sum_j A_j \left(L_{j,o}\right) \left(L_{j,o}^{\alpha} \overline{K}_j^{1-\alpha} + \zeta L_{j,h}\right) + N_f y_f.$$

The housing market clearing conditions for the center and the periphery are

$$\sum_{j,e} \pi_{c,j,e} h_{c,j,e} + N_f h_f = \overline{H}_c,$$

$$\sum_{j,e} \pi_{p,j,e} h_{p,j,e} = \overline{H}_p,$$

respectively. We maintain the assumption that rents on housing and office buildings are distributed equally across locals:

$$T = \sum_{\ell} r_{\ell} \overline{H}_{\ell} + \sum_j r_j^K \overline{K}_j.$$

For our results, it is useful to define the following quantities. Let Π_{ℓ}^{live} denote the share of locals that live in location ℓ, i.e., $\Pi_{\ell}^{live} \equiv \sum_{j,e} \pi_{\ell,j,e}$. We let Π^{office} denote the share of workers who are office workers $\Pi^{office} \equiv \sum_{\ell,j} \pi_{\ell,j,o}$, and $\Pi^{remote} = 1 - \Pi^{office}$ denote the share of workers who work remotely.

4.2 The welfare impact of increasing the number of foreigners

Social welfare is the average utility across the local population. Following the same steps as in the benchmark model, we can show that social welfare can be written as:

$$\mathcal{W} = \frac{\log\left(\sum_{\ell,j,e} e^{\eta u_{\ell,j,e}}\right)}{\eta} + \frac{1}{\eta} \int_0^{\infty} [-\log(y)] e^{-y} dy. \tag{27}$$

As in the benchmark model, we assume that the participation constraint of foreign residents is satisfied, and we also assume that $v(h) = h^{1-\sigma}/(1-\sigma)$. The following proposition summarizes the impact of an influx of foreign residents on social welfare, generalizing proposition 1.

Proposition 5. *The change in social welfare from a marginal increase in the number of foreign residents in the extended model can be decomposed into the sum of six terms:*

$$d\mathcal{W} = \mathcal{FS} + \mathcal{PE} - \mathcal{CE} - \mathcal{PCE} + \mathcal{AE} + \mathcal{RW},$$

where each term is constructed as follows.

1. *The foreign-residents surplus, $\mathcal{FS}$, is*

$$\mathcal{FS} = \sigma \frac{N_f}{\Pi_c^{live} + N_f} r_c h_c \left\{ d\Pi_c^{live} + dN_f \right\}.$$

2. *The production-externality effect, $\mathcal{PE}$, is*

$$\mathcal{PE} \equiv \gamma \times \Pi^{office} \times \mathbb{COV}_o \left(\frac{Y_j \left(1 - t_{\ell,j,o}\right)}{L_j}, \frac{d\pi_{\ell,j,o}}{\pi_{\ell,j,o}} \right).$$

3. *The congestion-externality effect, $\mathcal{CE}$, is*

$$\mathcal{CE} \equiv \psi \times \Pi^{office} \times \mathbb{COV}_o \left(w_{j,o} t_{\ell,j,o}, \frac{d\pi_{\ell,j,o}}{\pi_{\ell,j,o}} \right).$$

4. *The production-congestion-externalities complementarity effect, $\mathcal{PCE}$ is*

$$\mathcal{PCE} \equiv \gamma\psi \times \Pi^{office} \times \mathbb{COV}_o \left(\frac{Y_j}{L_j} t_{\ell,j,o}, \frac{d\pi_{\ell,j,o}}{\pi_{\ell,j,o}} \right).$$

5. *The amenities-externality effect, $\mathcal{AE}$, is*

$$\mathcal{AE} \equiv \phi_{\overline{u}} \Pi_c^{live} \overline{u}_c \frac{dN_f}{N_f}.$$

6. *The remote-work effect, $\mathcal{RW}$, is*

$$\mathcal{RW} \equiv \left(-\gamma \times \sum_{j} Y_j + \psi \times \sum_{\ell,j} \pi_{\ell,j,o} w_{j,o} t_{\ell,j,o} + \gamma\psi \times \sum_{\ell,j} \pi_{\ell,j,o} Y_j \frac{t_{\ell,j,o}}{L_{j,o}} \right) \times d\Pi^{remote}.$$

Generically, the covariance terms in these formulae can be written as follows. For any two variables x and y, the covariance is given by:

$$\mathbb{COV}_o(x_{\ell,j,o}, y_{\ell,j,o}) = \sum_{\ell,j} \frac{\pi_{\ell,j,o}}{\Pi^{\text{office}}} x_{\ell,j,o} y_{\ell,j,o} - \left(\sum_{\ell,j} \frac{\pi_{\ell,j,o}}{\Pi^{\text{office}}} x_{\ell,j,o} \right) \left(\sum_{\ell,j} \frac{\pi_{\ell,j,o}}{\Pi^{\text{office}}} y_{\ell,j,o} \right).$$

This operator computes the locals cross-sectional covariance of two variables x and y conditional on working from the office.

Not surprisingly, the extensions we introduce increase the channels through which an influx of foreign residents affects welfare. Interestingly, despite the increased complexity, the welfare consequences can still be decomposed into several interpretable terms. We now describe and provide intuition for each component.

Foreign-residents surplus The foreign-resident surplus takes exactly the same form as in the baseline model. The influx of foreign residents increases the demand for housing in the city center. Rents rise, resulting in an increase in rental income received by the locals.

Production externalities The production- or agglomeration-externality effect also takes a form similar to that in the baseline model. Labor is better allocated to places where their average labor productivity is higher because the contribution to the agglomeration externality becomes more significant. Suppose the influx of foreign residents forces locals to leave high average productivity locations. In that case, they will either (1) continue working in the high average productivity place, but be forced to commute or (2) change their work place to lower average productivity locations. In both cases, the productivity gains associated with the agglomeration externality fall. So, the cross-sectional covariance is negative, and there is a welfare loss. Naturally, the magnitude of the welfare loss is mediated by the strength of the agglomeration externality captured by γ. If $\gamma = 0$, then there is no production externality and no effect on welfare from labor reallocation. If γ is high, then this production-externality effect is magnified. Since only office workers contribute to the production externality, the effect is multiplied by Π^{office}.

Congestion externalities The congestion externality occurs because commuting times are endogenous. As the number of foreign residents rises, locals change their living- and work-location decisions. If workers move to the periphery but continue working in the city center, the number of commuters increases. Because of congestion, commuting time also increases, reducing labor income. The covariance term captures the welfare losses associated with the change in commuting time. The term $w_{j,o}t_{\ell,j,o}$ captures the labor income loss from commuting. If the number of commuters increases for routes with high income losses from commuting, the covariance term will be positive leading to a welfare loss. Intuitively, if the rise in foreign residents leads to an increase in people living in the periphery but working in a highly productive city center, then the rise in commuting times will lead to income losses proportional to the income value of that commuting time.

The magnitude of the welfare loss is mediated by the strength of the congestion externality captured by ψ. If $\psi = 0$, commuting times are exogenous, so there is no congestion externality effect. If ψ is high, then commuting times are very sensitive to the number of commuters, and the effect is magnified. Since only office workers commute, the effect is multiplied by Π^{office}.

Complementarity between production and congestion externalities The production externality depends on the total number of hours worked in the city center. Since increased commuting times decrease the total labor supply, the associated production externalities are reduced. So, there is a complementarity between the congestion and the production externalities, which is mediated by the product $\gamma\psi$.

Amenities externalities As described above, the influx of foreign residents affects the value of the amenities that locals enjoy in the city center. Unlike the other externalities, this amenities effect directly impacts the influx of foreigners on the utility of the local population. The strength of this effect is determined by $\phi_{\overline{u}}$. If $\phi_{\overline{u}} > 0$, then the influx of foreign residents increases the attractiveness of the city center and so improves the welfare of the locals. If $\phi_{\overline{u}} < 0$, then the influx of foreign residents decreases the attractiveness of the city center and so harms the welfare of the locals. The higher the absolute value of $\phi_{\overline{u}}$, the stronger the effect.

Remote work The influx of foreign residents creates incentives for locals to move to the periphery and work remotely for firms in the city center. Because working arrangements are optimized, the increase in remote work does not affect welfare directly. However, it interacts both with the production externality and the commuting-congestion externality. Since remote workers do not contribute towards the production externality, welfare falls because labor productivity declines. This effect is controlled by γ. Since remote workers do not commute, there are two additional positive effects. The first is the decrease in commuting times, which leads to an improvement in labor income for those who do not work remotely. This effect is controlled by ψ. The second is analogous to the production-congestion complementarity: a decrease in commuting times increases the labor supplied by non-remote workers and, therefore, increases productivity through the agglomeration externality.

4.3 Mirrleesian optimal policy

In this section, we analyze the Mirrleesian optimal policy. We introduce no ex-ante restrictions on the set of instruments but work directly from the informational constraints. The planner can distinguish between locals and foreigners. It can observe where people live and work and whether they work remotely but cannot observe locals' idiosyncratic taste shocks $\{\xi_{i,\ell,j,e}\}$.

As in the baseline model, to compute the optimum, we can summarize the incentive constraints using the implied shares of the local population that make each choice:

$$\pi_{\ell,j,e} = \frac{e^{\eta u_{\ell,j,e}}}{\sum_{\ell',j',e'} e^{\eta u_{\ell',j',e'}}}. \tag{28}$$

We first discuss the optimal policy toward the local population and then the optimal treatment of foreign residents.

4.3.1 Optimal policy towards locals

We define transfers to individuals living in location ℓ, working in location j, with working arrangement e, as

$$T_{\ell,j,e} \equiv c_{\ell,j,e} + \hat{r}_{\ell,j,e} h_{\ell,j,e} - w_{j,e}(1 - t_{\ell,j,e}), \tag{29}$$

where $\hat{r}_{\ell,j,e}$, the effective rent paid, is given by $\hat{r}_{\ell,j,e} = v'(h_{\ell,j,e})$. Differences in prices across people may arise if the government taxes housing purchases, i.e., $\hat{r}_{\ell,j}$ denotes the after-tax price. We compute wages of remote and office workers and rents on offices using equations (24) and (25) and (26), respectively, replacing $l_{j,e}$ with $L_{j,e}$ and k_j with $\overline{K}_j$. The following proposition provides sufficient statistics to calculate the tax/transfer policies required to implement the optimal solution.

Proposition 6. *In the optimal solution, all locals living in the same location pay the same rent $\hat{r}_{\ell,j,e} = r_\ell$, i.e., the planner does not distort the locals' house purchases. The total* *transfers implemented by the planner are the sum of five terms*

$$T_{\ell,j,e} = \Xi + \Xi^{\mathcal{PE}}_{\ell,j,e} + \Xi^{\mathcal{CE}}_{\ell,j,e} + \Xi^{\mathcal{PCE}}_{\ell,j,e} + \Xi^{\mathcal{AE}}_{\ell,j,e}, \tag{30}$$

where

1. *the common transfer is*

$$\Xi \equiv \sum_j r^K_j \overline{K}_j + \sum_\ell r_\ell \overline{H}_\ell + \Theta_f,$$

2. *the production-externality correction term is*

$$\Xi^{\mathcal{PE}}_{\ell,j,o} \equiv \gamma \left\{ Y_j \frac{(1 - t_{\ell,j,o})}{L_{j,o}} - \sum_{\ell,j,o} \pi_{\ell,j,o} Y_j \frac{(1 - t_{\ell,j,o})}{L_{j,o}} \right\}, \quad \Xi^{\mathcal{PE}}_{\ell,j,h} \equiv -\gamma \sum_{\ell,j,o} \pi_{\ell,j,o} Y_j \frac{(1 - t_{\ell,j,o})}{L_{j,o}},$$

3. *the congestion-externality correction term is*

$$\Xi^{\mathcal{CE}}_{\ell,j,h} \equiv -\psi \left\{ w_{j,h} t_{\ell,j,h} - \sum_{\ell,j,h} \pi_{\ell,j,h} w_{j,h} t_{\ell,j,h} \right\},$$

4. *the production-congestion-externalities-complementarity correction term is*

$$\Xi^{\mathcal{PCE}}_{\ell,j,h} \equiv -\psi\gamma \left\{ Y_j \frac{t_{\ell,j,h}}{L_{j,o}} - \sum_{\ell,j,h} \pi_{\ell,j,h} Y_j \frac{t_{\ell,j,h}}{L_{j,h}} \right\},$$

5. *the authenticity-externality correction term is*

$$\Xi^{\mathcal{AE}}_{\ell,j,e} \equiv N_f \left\{ \frac{d\overline{u}_f}{d\pi_{\ell,j,h}} - \sum_{\ell,j,e} \pi_{\ell,j,e} \frac{d\overline{u}_f}{d\pi_{\ell,j,e}} \right\}.$$

Not surprisingly, the additional features of this extended model increase the number of possible externalities. Still, we can continue to decompose the optimal transfers into several interpretable terms. We now describe each in turn.

Common transfer As in the baseline model, the planner redistributes the income generated from office and residential rents, as well as taxes levied on foreigners, evenly among the local population.

Production-externality correction term As in the baseline model, the planner corrects the production externality by giving higher transfers than average to office workers in locations where average labor productivity is higher than the cross-sectional mean of average labor productivity. Since remote workers do not contribute towards the production externality, the planner reduces the transfer to remote workers to finance the positive transfers to office workers. The magnitude of this transfer is determined by the elasticity of productivity to total office labor supply, γ.

Congestion-externality correction term The congestion-externality correction term captures the transfers necessary for locals to internalize their impact on commuting costs. Intuitively, commuters receive a lower transfer than non-commuters (workers who live and work in the same place or remote workers). The magnitude of this transfer is determined by the elasticity of commuting costs with respect to the number of commuters ψ.

Production-congestion-externalities-complementarity correction term As discussed in the previous section, the production and congestion externalities are complementary. All else being equal, a decrease in commuting costs decreases labor supply, which in turn reduces average productivity. The term $\Xi^{\mathcal{PCE}}$ affects the transfers so that commuters also internalize their effects towards the overall total factor productivity.

Authenticity-externality correction term The presence of locals in the city center, either working or living, increases the amenity value for foreigners. The planner corrects this externality by giving higher transfers to location and work choices that lead to a higher-than-average effect on the amenity value of foreigners.

4.3.2 Optimal policy towards foreigners

We now turn to the optimal treatment of foreign residents. As in the baseline model, foreigners' house purchases are taxed if the marginal rate of substitution between houses and consumption is higher for foreigners than for locals, i.e.,

$$\tau_h \equiv \frac{v'(h_f)}{v'(h_{c,j,e})} - 1. \tag{31}$$

We define the entry fee paid by foreigners as

$$T_f \equiv y_f - c_f - (1+\tau_h) r_c h_f. \tag{32}$$

The following proposition summarizes the optimal treatment of foreigners in this model. We jointly optimize the optimal allocations of consumption, housing, and number of foreigners. This proposition generalizes the results in propositions 3 and 4.

Proposition 7. *In the optimal allocation, foreigners' house purchases are not taxed, and there is an optimal entry fee which sets foreigners' utility equal to their outside option:*

1. *No taxes on foreigners' house purchases,* $\tau_h = 0$.
2. *There is an optimal entry fee on foreigners which satisfies*

$$T_f = -\phi_{\overline{u}} \frac{\Pi_c^{live}}{N_f} \overline{u}_c.$$

Despite the additional externalities, it remains optimal for the planner not to tax house purchases by foreigners. The intuition for this result is that using an entry fee is better than distorting house purchases.

Second, in contrast to proposition 4, the optimal entry fee is not zero. The optimal entry fee is such that its proceeds offset the impact of foreigners on amenities $N_f T_f = -\phi_{\overline{u}} \Pi_c \overline{u}_c$. The intuition for this result is as follows. In this extended model, foreigners directly impose an externality on the welfare of natives. So, it is optimal for the planner to distort the entry margin using an entry fee. If $\phi_{\overline{u}} > 0$, foreigners improve the amenity value of the city center, so the entry fee is negative to incentivize the entry of foreign residents. If $\phi_{\overline{u}} < 0$, foreigners deteriorate the amenity value of the city center, so the entry fee is positive to deter entry.

5 Heterogeneous property ownership

This section extends the baseline model to allow for heterogeneous ownership of houses and office buildings. We assume that each individual i belongs to one of a finite number of groups: $g \in \{1, ..., G\}$. This formulation allows us to use the law of large numbers to compute each group's welfare.

The mass of group g is $\chi_g \geq 0$, which satisfies the adding-up condition $\sum_g \chi_g = 1$. Each member of group g is endowed with a share $s_g \geq 0$ of houses and $s_g^k \geq 0$ of office buildings. These shares are defined as a person's housing (office buildings) holdings in group g divided by the per capita housing stock (office building stock). In groups whose members own more houses than the per capita housing stock, $s_g \geq 1$. We denote every household variable by an additional subscript g to denote the group to which the household belongs.

The non-labor income of a person in group g is

$$T_g = s_g \sum_j r_j \overline{H}_j + s_g^K \sum_j r_j^K \overline{K}_j,$$

where $\sum_g \chi_g s_g = 1$ and $\sum_g \chi_g s_g^K = 1$.

We define the welfare of group g as the average utility across group members. Following the same steps as in the baseline model, we can write the welfare of group

g as:

$$\mathcal{W}_g = \frac{\log\left(\sum_{\ell,j} e^{\eta u_{g,\ell,j}}\right)}{\eta} + \frac{1}{\eta}\int_0^{\infty}[-\log(y)\, e^{-y}]dy.$$

We investigate the impact of home purchases by foreigners on individuals with different holdings of houses and office buildings.

Proposition 8. *The change in group-g welfare is*

$$d\mathcal{W}_g = \mathcal{PE} + \left(s_g^K - 1\right)\mathcal{CG}^K + s_g \mathcal{FS} + \left(s_g - 1\right)\mathcal{CG}^{H,L}, \tag{33}$$

where

$$\mathcal{CG}^{H,L} \equiv \sigma r_c \frac{\overline{H}_c - N_f h_f}{\Pi_c + N_f}\left(d\Pi_c + dN_f\right) + \sigma r_p \frac{\overline{H}_p}{\Pi_p} d\Pi_p,$$

denotes the capital gains on houses purchased by locals and

$$\mathcal{CG}^K \equiv \sum_j \alpha Y_j \left\{\gamma + (1-\alpha)\right\} \frac{dL_j}{L_j}.$$

denotes the capital gains on office buildings.

To understand the expression for the change in welfare in proposition 8, note that people in group g benefit from the foreign-resident surplus in proportion to the share of houses they own, s_g. To the extent that $s_g \neq 1$, they may also gain or lose from the fact that houses purchased by locals become more expensive, $\mathcal{CG}^{H,L}$.

People with $s_g = 0$ have to pay higher rents but do not benefit from housing capital gains. More generally, if $s_g < 1$, their capital gains are lower than the increase in housing costs. People who own more shares than average, $s_g > 1$, receive capital gains that exceed the rise in housing costs.

The change in wage income of people in group g is

$$\sum_j (\gamma - \alpha)(1-\alpha)\, Y_j \frac{dL_j}{L_j},$$

and the change in their capital income is

$$s_g^K \sum_j \alpha Y_j \left\{\gamma + (1-\alpha)\right\} \frac{dL_j}{L_j}.$$

Adding these two effects, we obtain

$$\gamma \sum_j \frac{dL_j}{L_j} Y_j + \left(s_g^K - 1\right) \sum_j \alpha Y_j \left\{\gamma + (1-\alpha)\right\} \frac{dL_j}{L_j} = \mathcal{PE} + \left(s_g^K - 1\right) \mathcal{CG}^K.$$

So, $\mathcal{PE}$ has two components: the change in wages and the changes in rents to office buildings. Implicitly, $\mathcal{PE}$ is defined as if offices are equally distributed among the population. The term $\left(s_g^K - 1\right) \mathcal{CG}^K$ corrects $\mathcal{PE}$ for the fact that the change in office rents is unequally distributed among the population. When $s_g^K = 0$, people in group g receive no capital income. So, the production externality effect must be subtracted from the change in office rents to obtain only the change in wage income.

5.1 Mirrleesian optimal policy

We compute the Mirrleesian optimal policy, assuming that the planner cannot observe the idiosyncratic preferences shocks. The planner can condition allocations on people's residential and workplace choices and their holdings of houses and office buildings.

The planner chooses the share of people in each location, $\pi_{g,\ell,j}$. These shares must satisfy

$$\pi_{g,\ell,j} = \frac{e^{\eta u_{g,\ell,j}}}{\sum_{\ell',j'} e^{\eta u_{g,\ell',j'}}}. \tag{34}$$

Similarly to Fajgelbaum and Gaubert (2020), we compute optimal policies maximizes the welfare of group 1, $\mathcal{W}_1$, subject to achieving specific welfare levels for other groups $\mathcal{W}_g \geq \overline{u}_g^p$. By varying $\{\overline{u}_g^p\}$ we can characterize the group-welfare possibilities frontier. The optimum allocations must also satisfy the resource constraints for goods,

$$\sum_g \chi_g \sum_{\ell,j} \pi_{g,\ell,j} c_{\ell,j} + N_f c_f = L_c^{\gamma+1-\alpha} \overline{K}_c^{\alpha} + L_p^{\gamma+1-\alpha} \overline{K}_p^{\alpha} + N_f y_f,$$

the housing resource constraints:

$$\sum_g \chi_g \sum_j \pi_{g,c,j} h_{g,c,j} + N_f h_f = \overline{H}_c, \quad \text{and} \sum_g \chi_g \sum_j \pi_{g,p,j} h_{g,p,j} = \overline{H}_p,$$

the incentive-compatibility constraints (34), and the foreign resident participation constraints, (7).

We now characterize the optimal policy towards locals. We define transfers to individuals living in location ℓ and working in location j as

$$T_{g,\ell,j} \equiv c_{g,\ell,j} + \hat{r}_{g,\ell,j} h_{g,\ell,j} - w_j (1 - t_{\ell,j}), \tag{35}$$

Where $\hat{r}_{g,\ell,j}$ denote the effective rent paid by individuals who live in ℓ and work in j, and is defined as $\hat{r}_{g,\ell,j} = v'(h_{\ell,j})$. Differences in prices across individuals may arise if the government taxes housing purchases, i.e., $\hat{r}_{\ell,j}$ denotes the after-tax price. We compute wages and rents on offices using equations (8) and (9), respectively, replacing $l_j = L_j$ and $k_j = \overline{K}_j$.

In the optimum, location choices $\pi_{g,\ell,j}$ are constant across groups. Analogously to the baseline model, the marginal rates of substitution across locals in the same location are equalized, i.e., all locals pay the housing rent $\hat{r}_{g,\ell,j} = r_\ell$. The transfers to individuals in group g who live in location ℓ and work in location j are

$$T_{g,\ell,j} = T_g + \gamma \left\{ \frac{Y_j}{L_j} (1 - t_{\ell,j}) - \sum_{\ell,j} \pi_{\ell,j} \frac{Y_j}{L_j} (1 - t_{\ell,j}) \right\},$$

where T_g are group-specific transfers which are a function of the parameters $\overline{u}_g^p$ and satisfy

$$\sum_g \chi_g T_g = \alpha \sum_j Y_j + \sum_\ell r_\ell \overline{H}_\ell.$$

By varying the elements of the vector that comprises the welfare of the different groups, $\{\overline{u}_g\}$, we can calculate the set of transfers for each group that satisfies this equation. It is always possible to find a distribution of welfare across groups $\{\overline{u}_\ell^p\}$ such that the second-best solution does not involve redistributing the rental income received by different groups in the competitive equilibrium.

As in the baseline model, the optimal treatment of foreigners is free and unrestricted entry, zero taxes on house purchases, $\tau_h = 0$, and zero entry fees, $T_f = 0$.

The key result in this section is that, given an initial distribution of property ownership, it is always possible to implement a transfer and tax policy such that, ex-ante, before taste shocks materialize, all groups are better off under the optimal policy. The reason for this result is as follows. In this model, the entry of foreign residents creates three effects: (1) a foreign-residents surplus, (2) a production externality, and (3) a redistribution effect coming from house and office capital gains. The first of these effects is always positive. The second is corrected by the specific transfers described above. The third effect may be positive or negative but always averages zero across groups. It follows that there is always a possible redistribution of the welfare gains, using T_g, that improves the welfare of all groups.

In the model, capital gains can be redistributed through lump-sum taxes and transfers. In practice, this redistribution can be implemented by taxing capital gains on housing and making transfers to those with property holdings below average. In a static model like ours, this tax does not distort the decisions of individuals. In a dynamic setting, capital gain taxes are also not distorting as long as investment expenses can be deducted from the tax base (see Abel, 2007).

6 Long run: the future of global cities

In this section, we study how an influx of foreign residents affects the optimal long-run city design. So far, we have assumed that the costs of converting offices into homes and vice versa are prohibitively high. We now consider the possibility that offices can be converted into homes and homes into offices. We think of this exercise as characterizing the long-run optimum.

Consider first the marginal social welfare effect of converting offices into houses in the city center.

$$d\mathcal{W} = v'\left(\frac{\overline{H}_c}{\Pi_c + N_f}\right) - \alpha\frac{L_c^{1+\gamma-\alpha}}{\overline{K}_c^{1-\alpha}}.$$

Suppose that before the influx of foreign residents, the rental rates of houses and offices are equalized in the center and the periphery: $r_c = r_c^K$ and $r_p = r_p^K$. The condition $r_c = r_c^K$ can be rewritten as:

$$v'(\overline{H}_c/\Pi_c) - \alpha\frac{L_c^{1+\gamma-\alpha}}{\overline{K}_c^{1-\alpha}} = 0.$$

Foreign home purchases reduce housing consumption in the center ($\frac{\overline{H}_c}{\Pi_c+N_f}$), increasing the utility of additional homes. At the same time, locals move away from the center, reducing labor supply L_c and the rents of office buildings. It is optimal to increase home supply in the city center, decreasing office supply: $d\mathcal{W} > 0$.

Consider now the effect on social welfare of converting houses into offices in the periphery,

$$d\mathcal{W} = \alpha\frac{L_p^{1+\gamma-\alpha}}{\overline{K}_p^{1-\alpha}} - v'\left(\frac{\overline{H}_p}{\Pi_p}\right).$$

Locals move to the periphery, reducing per-capita housing consumption $\left(\overline{H}_p/\Pi_p\right)$. At the same time, the labor supply increases in the periphery. As a result, the total marginal effect on social welfare is ambiguous: $d\mathcal{W} \lesseqgtr 0$.

Proposition 9. *In response to an influx of foreign residents, it is optimal to convert offices into houses in the city center. In contrast, the welfare effect of converting offices into houses in the periphery is ambiguous.*

7 Conclusion

Many nations and urban areas are grappling with the challenge of devising policies to ensure that the local population benefits from a potentially large influx of foreign residents.

We show that policy has a role in dealing with agglomeration, congestion, amenities, and other externalities affected by the influx of foreign residents. Implementing the optimum requires designing taxes and transfers to locals based on their residential and work-arrangement choices. These transfers incentivize workers to internalize the external effects of their living and work choices.

To the extent that foreign residents may directly affect amenities for locals, their entry should be distorted by an entry fee, analogous to the per-diem tax levied by some cities.

Once all these external effects are internalized, the marginal impact of additional foreign residents is positive. Restricting foreign property purchases or imposing taxes on those purchases is never optimal.

In the event of an unequal distribution of housing- and office-building ownership in the population, it can be optimal to implement taxes or transfers that redistribute the capital gains produced by the influx of foreign residents.

Looking toward the future, it is optimal in the long run to convert office spaces in the city center into residential units and relocate production facilities to the periphery. This urban design mirrors the one adopted in Paris. In the 19th century, Napoleon III granted Baron Hausmann broad powers to remodel Paris. The result was the monumental city we know today, with wide boulevards, impressive squares, and views of the Eiffel Tower that are not obstructed by towering skyscrapers. Office buildings, production structures, and residential complexes, where most of the local population resides, were shifted to La Defense and other peripheral areas. The ability of Paris to accommodate foreign residents impressed Ernest Hemingway, who wrote, “There are only two places in the world where we can live happy—at home and in Paris.”

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A Appendix to Section 2

A.1 Location shares

Define $x_{i,\ell,j} = u_{\ell,j} + \xi_{i,\ell,j}$ for $\ell, j = c, p$. The cross-sectional cumulative density function of $x_{i,\ell,j}$ is

$$G_{\ell,j}\left(x\right) = \mathbb{P}\left[x_{i,\ell,j} \leq x\right] = F\left(x - u_{\ell,j}\right) = e^{-e^{-\eta\left(x-u_{\ell,j}\right)}},$$

and the associated probability density function is

$$g_{\ell,j}\left(x\right) = \eta e^{-\eta\left(x-u_{\ell,j}\right)} e^{-e^{-\eta\left(x-u_{\ell,j}\right)}}$$

Then, by the law of large numbers

$$\begin{aligned}
\pi_{\ell,j} = \mathbb{P}\left[x_{i,\ell,j} = \max_{\ell',j'} x_{i,\ell',j'}\right] &= \int_{-\infty}^{\infty} g_{\ell,j}(x) \prod_{(\ell',j')\neq(\ell,j)} G_{\ell',j'}(x)dx \\
&= \int_{-\infty}^{\infty} \eta e^{-\eta\left(x-u_{\ell,j}\right)} e^{-e^{-\eta\left(x-u_{\ell,j}\right)}} \prod_{(\ell',j')\neq(\ell,j)} e^{-e^{-\eta\left(x-u_{\ell',j'}\right)}} dx \\
&= \frac{e^{\eta u_{\ell,j}}}{\sum_{\ell',j'} e^{\eta u_{\ell',j'}}} \int_{-\infty}^{\infty} \eta e^{-\eta x}\left(\sum_{\ell',j'} e^{\eta u_{\ell',j'}}\right) e^{-e^{-\eta x}\left(\sum_{\ell',j'} e^{\eta u_{\ell',j'}}\right)} dx \\
&= \frac{e^{\eta u_{\ell,j}}}{\sum_{\ell',j'} e^{\eta u_{\ell',j'}}}.
\end{aligned}$$

A.2 Social welfare

By the law of large numbers

$$\mathcal{W} \equiv \int_0^1 \max\left\{u_{\ell,j} + \xi_{i,\ell,j}\right\} di = \mathbb{E}\left[\max_{\ell,j}\left\{u_{\ell,j} + \xi_{i,\ell,j}\right\}\right].$$

Let $x^* \equiv \max_{\ell,j}\left\{u_{\ell,j} + \xi_{i,\ell,j}\right\}$. The cumulative distribution function of x^* is :

$$F^*\left(x\right) = \mathbb{P}\left[x^* \leq x\right] = \mathbb{P}\left[\xi_{\ell,j} \leq x - u_{\ell,j}, \quad \forall(\ell,j)\right] = \prod_{\ell,j} e^{-e^{-\eta\left(x-u_{\ell,j}\right)}} = e^{-e^{-\eta x}\sum_{\ell,j} e^{\eta u_{\ell,j}}}$$

and the probability density function is :

$$f^{*}(x) = \eta e^{-\eta x}\left(\sum_{\ell,j} e^{\eta u_{\ell,j}}\right) e^{-e^{-\eta x}\sum_{\ell,j} e^{\eta u_{\ell,j}}}$$

So, social welfare is given by

$$\mathcal{W} = \mathbb{E}\left[\max\left\{u_{\ell,j} + \xi_{\ell,j}\right\}\right] = \int_{-\infty}^{\infty} x\eta e^{-\eta x}\left(\sum_{\ell,j} e^{\eta u_{\ell,j}}\right) e^{-e^{-\eta x}\sum_{\ell,j} e^{\eta u_{\ell,j}}} dx$$

It is useful to do a change of variables: $y = e^{-\eta x}\sum_{\ell,j} e^{\eta u_{\ell,j}}$. Then,

$$x = -\frac{1}{\eta}\log\left(\frac{y}{\sum_{\ell,j} e^{\eta u_{\ell,j}}}\right),$$

$dx = -\frac{1}{\eta}\frac{dy}{y}$, $\lim_{x\to\infty} y = 0$ and $\lim_{x\to-\infty} y = \infty$. We can rewrite social welfare as follows:

$$\begin{aligned}
\mathcal{W} &= \int_{\infty}^{0}\left(-\log\left(\frac{y}{\sum_{\ell,j} e^{\eta u_{\ell,j}}}\right)\right) y e^{-y}\left(-\frac{1}{\eta}\frac{dy}{y}\right) \\
&= \frac{1}{\eta}\int_{0}^{\infty}\left(\log\left(\sum_{\ell,j} e^{\eta u_{\ell,j}}\right) - \log(y)\right) e^{-y} dy \\
&= \frac{\log\left(\sum_{\ell,j} e^{\eta u_{\ell,j}}\right)}{\eta} + \frac{1}{\eta}\underbrace{\int_{0}^{\infty}[-\log(y)]e^{-y} dy}_{\text{Euler-Mascheroni Constant}}
\end{aligned}$$

A.3 The welfare impact of an increase in foreign residents

Using the fact that $r_\ell = v'(h_\ell)$, we can write common utility as

$$u_{\ell,j} = \overline{u}_{\ell,j} + w_j \cdot (1 - t_{\ell,j}) + T - v'(h_\ell)\cdot h_\ell + v(h_\ell),$$

where h_ℓ denote the quantity of housing purchased by people who live in location ℓ.

Then,

$$du_{\ell,j} = dw_j \cdot (1 - t_{\ell,j}) + dT - v''(h_\ell)\cdot h_\ell \cdot dh_\ell$$

Assuming that $v(h) = h^{1-\sigma}/(1-\sigma)$ we can write

$$du_{\ell,j} = dw_j \cdot (1 - t_{\ell,j}) + dT + \sigma \cdot r_\ell \cdot dh_\ell.$$

Note that, because

$$h_c = \frac{\overline{H}_c}{\Pi_c + N_f} \Rightarrow \frac{dh_c}{h_c} = -\left(\frac{\Pi_c}{\Pi_c + N_f} \frac{d\Pi_c}{\Pi_c} + \frac{N_f}{\Pi_c + N_f} \frac{dN_f}{N_f} \right),$$
$$h_p = \frac{\overline{H}_p}{\Pi_p} \Rightarrow \frac{dh_p}{h_p} = -\frac{d\Pi_p}{\Pi_p}.$$

Since $w_j = L_j^\gamma \left(\frac{\overline{K}_j}{L_j} \right)^\alpha (1-\alpha)$ and $L_j = \sum_\ell \pi_{\ell,j}(1 - t_{\ell,j})$ then

$$\frac{dw_j}{w_j} = (\gamma - \alpha) \frac{dL_j}{L_j} = (\gamma - \alpha) \sum_\ell \frac{\pi_{\ell,j}}{L_j} \left(1 - t_{\ell,j}\right) \frac{d\pi_{\ell,j}}{\pi_{\ell,j}}.$$

Finally, the change in total rents is

$$\begin{aligned}
dT =& d\left(\sum_\ell v'\left(h_\ell\right) \overline{H}_\ell + \sum_\ell \alpha A\left(L_\ell\right) L_\ell^{1-\alpha} \overline{K}_\ell^\alpha \right) \\
=& \sum_\ell v''\left(h_\ell\right) \overline{H}_\ell dh_\ell + \sum_\ell \alpha A\left(L_\ell\right) L_\ell^{1-\alpha} \overline{K}_\ell^\alpha \left\{ \frac{A'\left(L_\ell\right)}{A\left(L_\ell\right)} L_\ell + (1-\alpha) \right\} \frac{dL_\ell}{L_\ell} \\
=& -\sum_\ell \sigma \overline{H}_\ell v'\left(h_\ell\right) \frac{dh_\ell}{h_\ell} + \sum_\ell \alpha Y_\ell \left\{ \gamma + (1-\alpha) \right\} \frac{dL_\ell}{L_\ell} \\
=& \sigma \overline{H}_c v'\left(h_c\right) \left(\frac{\Pi_c}{\Pi_c + N_f} \frac{d\Pi_c}{\Pi_c} + \frac{N_f}{\Pi_c + N_f} \frac{dN_f}{N_f} \right) + \sigma \overline{H}_p v'\left(h_p\right) \frac{d\Pi_p}{\Pi_p} \\
& + \sum_\ell \alpha Y_\ell \left\{ \gamma + (1-\alpha) \right\} \frac{dL_\ell}{L_\ell}.
\end{aligned}$$

Putting everything together, we find that

$$
\begin{aligned}
d\mathcal{W} = & (\gamma-\alpha) L_c w_c \frac{dL_c}{L_c} + (\gamma-\alpha) L_p w_p \frac{dL_p}{L_p} \\
& + \sigma v'(h_c)\left(\overline{H}_c - \Pi_c h_c\right)\left(\frac{\Pi_c}{\Pi_c+N_f}\frac{d\Pi_c}{\Pi_c} + \frac{N_f}{\Pi_c+N_f}\frac{dN_f}{N_f}\right) \\
& + \sigma v'(h_p)\left(\overline{H}_p - \Pi_p h_p\right)\frac{d\Pi_p}{\Pi_p} \\
& + \sum_j \alpha Y_j \left\{\gamma + (1-\alpha)\right\}\frac{dL_j}{L_j}.
\end{aligned}
$$

Using the fact that $\overline{H}_c = \Pi_c h_c + N_f h_f$, $\overline{H}_p = \Pi_p h_p$, and $w_j = (1-\alpha)\frac{Y_j}{L_j}$, we can write

$$
d\mathcal{W} = \gamma \sum_{\ell,j} \pi_{\ell,j}\frac{Y_j}{L_j}\left(1-t_{\ell,j}\right)\frac{d\pi_{\ell,j}}{\pi_{\ell,j}} + \sigma\frac{N_f}{\Pi_c+N_f} r_c h_f\left(d\Pi_c + dN_f\right),
$$

or, equivalently,

$$
d\mathcal{W} = \frac{\gamma}{1-\alpha}\sum_{\ell,j}\pi_{\ell,j} w_j\left(1-t_{\ell,j}\right)\frac{d\pi_{\ell,j}}{\pi_{\ell,j}} + \sigma\frac{N_f}{\Pi_c+N_f} r_c h_f\left(d\Pi_c + dN_f\right),
$$

B Appendix to Section 3

B.1 Second-best problem and incentive compatibility

Let $c(\boldsymbol{\xi})$, $h(\boldsymbol{\xi})$, $\ell(\boldsymbol{\xi})$ and $j(\boldsymbol{\xi})$ denote, respectively, the consumption, housing, living location, and working location of a person with idiosyncratic location preferences $\boldsymbol{\xi} = [\xi_{c,c}, \xi_{c,p}, \xi_{p,c}, \xi_{p,p}]$.

The utility net of taste shocks of this person is

$$
U(\boldsymbol{\xi}) \equiv \overline{u}_{\ell(\boldsymbol{\xi}),j(\boldsymbol{\xi})} + c(\boldsymbol{\xi}) + v\left(h(\boldsymbol{\xi})\right)
$$

The incentive compatibility constraints of the direct revelation mechanism can be written as

$$
U(\boldsymbol{\xi}) + \xi_{\ell(\boldsymbol{\xi}),j(\boldsymbol{\xi})} \geq U(\boldsymbol{\xi}') + \xi_{\ell(\boldsymbol{\xi}'),j(\boldsymbol{\xi}')} \tag{36}
$$

for all $\boldsymbol{\xi}$ and $\boldsymbol{\xi}'$.

It follows from (36) that if two people have the same location choices, then they must have the same level of common utility, i.e., assuming $(\ell(\boldsymbol{\xi}), j(\boldsymbol{\xi})) = (\ell(\boldsymbol{\xi}'), j(\boldsymbol{\xi}'))$, then

$$U(\boldsymbol{\xi}) = U(\boldsymbol{\xi}'). \tag{37}$$

Let $u_{\ell,j}$ denote the level of common utility attained by individuals with location choices ℓ, j.

Incentive compatibility can now be equivalently written as

$$\{\ell(\boldsymbol{\xi}), j(\boldsymbol{\xi})\} = \arg\max\{u_{\ell,j} + \xi_{\ell,j}\}, \tag{38}$$

and $U(\boldsymbol{\xi}) = u_{\ell(\boldsymbol{\xi}),j(\boldsymbol{\xi})}$.

Using the properties of the Gumbel distribution, these equations imply that the share of individuals with location choices ℓ, j is

$$\pi_{\ell,j} = \frac{e^{\eta u_{\ell,j}}}{\sum_{\ell',j'} e^{\eta u_{\ell',j'}}}, \tag{39}$$

and the social welfare function is

$$\mathcal{W} = \frac{\log\left(\sum_{\ell,j} e^{\eta u_{\ell,j}}\right)}{\eta} + \frac{\text{Euler-Mascheroni Constant}}{\eta}. \tag{40}$$

These are the only restrictions on aggregate shares and social welfare implied by incentive compatibility. This means that if the planner chooses common utility levels $u_{\ell,j}$, location shares $\pi_{\ell,j}$, and welfare $\mathcal{W}$ which satisfy (39) and (40), then we can always find individual location choices which are consistent with incentive compatibility.

Because utility is concave in housing and all attain the same level of common utility, the optimal plan must always feature equal housing consumption for all people with the same location choices. It also follows that consumption is the same for all individuals with the same location choices.

B.2 Mirrleesian optimal policy

We write the Lagrangian for this optimization problem as follows,

$$\mathcal{L} = \frac{\log\left(\sum_{\ell,j} e^{\eta u_{\ell,j}}\right)}{\eta} + \lambda_c \left(L_c^{\gamma+1-\alpha} \overline{K}_c^{\alpha} + N_f \left(y_f - c_f\right) + L_p^{\gamma+1-\alpha} \overline{K}_p^{\alpha} - \sum_{\ell,j} \pi_{\ell,j} c_{\ell,j} \right)$$
$$+ \lambda_{h,c} \left(\overline{H}_c - \sum_j \pi_{c,j} h_{c,j} - N_f h_f \right) + \lambda_{h,p} \left(\overline{H}_p - \sum_j \pi_{p,j} h_{p,j} \right)$$
$$+ \sum_{\ell,j} \lambda_{\ell,j}^{\text{loc}} \left(\pi_{\ell,j} - \frac{e^{\eta u_{\ell,j}}}{\sum_{\ell',j'} e^{\eta u_{\ell',j'}}} \right) + N_f \lambda_f \left(\overline{u}_f + c_f + v\left(h_f\right) - u_f^* \right).$$

And let $W^*(N_f)$ denote the optimized value.

The first-order conditions for this problem can be written as

$$[c_{\ell,j}] \quad 1 - \eta \lambda_{\ell,j}^{\text{loc}} + \eta \sum_{\ell',j'} \lambda_{\ell',j'}^{\text{loc}} \pi_{\ell',j'} = \lambda_c \tag{41}$$

$$[h_{\ell,j}] \quad v'\left(h_{\ell,j}\right) \left(1 - \eta \lambda_{\ell,j}^{\text{loc}} + \eta \sum_{\ell',j'} \lambda_{\ell',j'}^{\text{loc}} \pi_{\ell',j'} \right) = \lambda_{h,\ell} \tag{42}$$

$$[c_f] \quad \lambda_f = \lambda_c \tag{43}$$

$$[h_f] \quad \lambda_f v'\left(h_f\right) = \lambda_{h,c} \tag{44}$$

$$[\pi_{\ell,j}] \quad \lambda_c \left\{ (1+\gamma-\alpha) \frac{Y_j}{L_j} \left(1 - t_{\ell,j}\right) - c_{\ell,j} \right\} - \lambda_{h,\ell} h_{\ell,j} = \lambda_{\ell,j}^{\text{loc}} \tag{45}$$

and all constraints bind with equality.

Averaging across (41) for different ℓ, j we obtain

$$\sum_{\ell,j} \pi_{\ell,j} [1 - \eta \lambda_{\ell,j}^{\text{loc}} + \eta \sum_{\ell',j'} \lambda_{\ell',j'}^{\text{loc}} \pi_{\ell',j'}] = \sum_{\ell,j} \pi_{\ell,j} \lambda_c$$
$$\Leftrightarrow 1 - \eta \sum_{\ell,j} \pi_{\ell,j} \lambda_{\ell,j}^{\text{loc}} + \eta \sum_{\ell',j'} \lambda_{\ell',j'}^{\text{loc}} \pi_{\ell',j'} = \lambda_c \Leftrightarrow 1 = \lambda_c.$$

Using this finding back in (41) we find that

$$\lambda_{\ell,j}^{\text{loc}} = \sum_{\ell',j'} \lambda_{\ell',j'}^{\text{loc}} \pi_{\ell',j'} = \lambda^{\text{loc}}$$

is constant across location choices adn where $Y_j = A(L_j)\overline{K}_j^{\alpha} L_j^{1-\alpha}$.

These first-order conditions can be simplified to

$$
\begin{aligned}
[c_{\ell,j}] &\quad 1 = \lambda_c && (46)\\
[h_{\ell,j}] &\quad v'(h_{\ell,j}) = \lambda_{h,\ell} && (47)\\
[c_f] &\quad \lambda_f = 1 && (48)\\
[h_f] &\quad \lambda_f v'\left(h_f\right) = \lambda_{h,c} && (49)\\
[\pi_{\ell,j}] &\quad \left\{(1+\gamma-\alpha)\frac{Y_j}{L_j}\left(1-t_{\ell,j}\right) - c_{\ell,j}\right\} - \lambda_{h,\ell} h_{\ell,j} = \lambda^{\text{loc}} && (50)
\end{aligned}
$$

B.2.1 Proof of proposition 2

The first order conditions (47) imply that

$$v'(h_{\ell,\ell}) = v'(h_{\ell,j}),$$

for all ℓ and j. It follows that the marginal rates of substitution are equalized across individuals who live in the same location. In other words, they all pay the same marginal price $\hat{r}_{\ell,j} = r_\ell$.

Now, averaging equation (50) across ℓ, j we find that

$$(1+\gamma-\alpha)\sum_j Y_j - \sum_{\ell,j} \pi_{\ell,j} c_{\ell,j} - \sum_{\ell,j} \pi_{\ell,j} \lambda_{h,\ell} h_{\ell,j} = \lambda^{\text{loc}}$$

Using the fact that $\sum_{\ell,j} \pi_{\ell,j} c_{\ell,j} = \sum Y_j + N_f(y_f - c_f) = \sum Y_j + N_f r_c h_f + \Theta_f$ where $\Theta_f \equiv N_f(y_c - c_f - r_c h_f)$ denotes total taxes on foreigners and $r_c = \lambda_{h,c}$, we can rewrite that equation as

$$(\gamma-\alpha)\sum_j Y_j - \sum_\ell \lambda_{h,\ell} \overline{H}_\ell - \Theta_f = \lambda^{\text{loc}}$$

Replacing λ^{loc} in equation (50), we find

$$c_j + \lambda_{\ell,j} = (1-\alpha)\frac{Y_j}{L_j}(1-t_{\ell,j}) + \gamma\frac{Y_j}{L_j}(1-t_{\ell,j}) + (\alpha-\gamma)\sum_j Y_j + \sum_\ell \lambda_{h,\ell}\overline{H}_\ell + \Theta_f$$

$$c_j + \lambda_{\ell,j}h_{\ell,j} = (1-\alpha)\frac{Y_j}{L_j}(1-t_{\ell,j}) + \gamma\left[\frac{Y_j}{L_j}(1-t_{\ell,j}) - \sum_{\ell',j'}\frac{Y_{j'}}{L_{j'}}(1-t_{\ell',j'})\right] + \sum_j \alpha Y_j + \sum_\ell \lambda_{h,\ell}\overline{H}_\ell + \Theta_f.$$

The decentralized equilibrium features the price of housing $r_\ell = \lambda_{h,\ell}$, the wage $w_j = (1-\alpha)Y_j/L_j$ and the rent of capital $r_j^K = \alpha Y_j/\overline{K}_j$. It follows that the transfer to individuals with location choices ℓ, j is

$$T_{\ell,j} = \gamma\left[\frac{Y_j}{L_j}(1-t_{\ell,j}) - \sum_{\ell',j'}\frac{Y_{j'}}{L_{j'}}(1-t_{\ell',j'})\right] + \sum_j r_j^K\overline{K}_j + \sum_\ell r_\ell\overline{H}_\ell + \Theta_f. \tag{51}$$

B.2.2 Proof of proposition 3

In the optimum,

$$v'(h_f) = v'(h_{c,j}).$$

So, it is optimal to set the marginal rate of substitution between houses and consumption equal between foreigners and locals. It follows that the optimal tax on foreigners house purchases is zero $\tau_h = 0$.

The participation constraint binds

$$\overline{u}_c + c_f + v(h_f) = u_f^*.$$

Let h_f denote the solution the housing which is such that $h_f = h_{c,j} = \overline{H}_c/(\Pi_c + N_f)$ and define the entry fee as $T_f = y_f - c_f - (1+\tau_h)r_c h_f$, then T_f solves

$$\overline{u}_c + y_f - r_c h_f - T_f + v(h_f) = u_f^*,$$

since $\tau_h = 0$.

B.2.3 Proof of proposition 4

By the envelope theorem

$$\frac{dW^*(N_f)}{dN_f} = \lambda_c(y_f - c_f) - \lambda_{h,c}h_f = y_f - c_f - r_c h_f = T_f.$$

The optimal N_f is such that

$$\frac{dW^*(N_f)}{dN_f} = 0 \Leftrightarrow T_f = 0.$$

Since the participation constraint binds

$$\overline{u}_f + c_f + v(h_f) = u_f^*,$$

then the implementation of the optimum is consistent with free entry by foreigners, i.e., no quotas are necessary. Finally, it follows from the previous section that $\tau_h = 0$.

C Appendix to Section 4

C.1 Proof of proposition 5

In the extended model, welfare is

$$\mathcal{W} = \frac{\log\left(\sum_{\ell,j,e} e^{\eta u_{\ell,j,e}}\right)}{\eta} + \frac{1}{\eta}\int_0^\infty [-\log(y)]e^{-y}dy, \tag{52}$$

where

$$u_{\ell,j,e} = \overline{u}_\ell + w_{j,e}\left(1 - t_{\ell,j,e}\right) + T - v'\left(h_\ell\right) h_\ell + v\left(h_\ell\right), \tag{53}$$

$$h_c = \frac{H_c}{\Pi_c + N_f}, \tag{54}$$

$$h_p = \frac{H_p}{\Pi_p}, \tag{55}$$

$$t_{\ell,j,e} = \mathcal{T}_{\ell,j,e}\left(\pi_{\ell,j,e}\right) \tag{56}$$

$$r_\ell = v'(h_{\ell,j}) \quad r_c = v'(h_f), \tag{57}$$

$$w_{j,o} = (1-\alpha) A_j\left(L_{j,o}\right) L_{j,o}^{-\alpha} K_j^{\alpha}, \tag{58}$$

$$w_{j,h} = A_j\left(L_{j,o}\right)\zeta, \tag{59}$$

$$r_j^K = \alpha A_j\left(L_{j,o}\right) L_{j,o}^{1-\alpha} K_j^{\alpha-1}, \tag{60}$$

$$T = \sum_\ell r_\ell \overline{H}_\ell + \sum_j r_j^K \overline{K}_j, \tag{61}$$

$$L_{j,e} = \sum_\ell \pi_{\ell,j,e}\left(1 - t_{\ell,j,e}\right). \tag{62}$$

Analogously to the baseline model

$$d\mathcal{W} = \sum_{\ell,j,e} \pi_{\ell,j,e} du_{\ell,j,e},$$

and

$$\begin{aligned} u_{\ell,j,e} &= \overline{u}_\ell + w_{j,e}\left(1 - t_{\ell,j,e}\right) + T - v'\left(h_\ell\right) h_\ell + v\left(h_\ell\right) \\ &= d\overline{u}_\ell + dw_{j,e}\left(1 - t_{\ell,j,e}\right) - w_{j,e} dt_{\ell,j,e} + dT + \sigma r_\ell h_\ell \frac{dh_\ell}{h_\ell}. \end{aligned}$$

It follows that

$$d\mathcal{W} = \sum_{\ell,j,e} \pi_{\ell,j,e} du_{\ell,j,e} = \Pi_c^{live} d\overline{u}_c + \sum_{j,e} L_{j,e} dw_{j,e} - \sum_{\ell,j} \pi_{\ell,j,o} w_{j,o} dt_{\ell,j,o} + dT + \sigma \sum_\ell \Pi_\ell r_\ell h_\ell \frac{dh_\ell}{h_\ell}$$

Change in ammenities The change in ammenities is

$$d\overline{u}_c = \mathcal{U}_c'\left(N_f\right) dN_f = \phi_{\overline{u}} \overline{u}_c \frac{dN_f}{N_f}$$

Change in wages The change in wages of remote workers is

$$dw_{j,h} = \gamma w_{j,h} \frac{dL_{j,o}}{L_{j,o}}$$

and for office workers

$$dw_{j,o} = (\gamma - \alpha) \, w_{j,o} \frac{dL_{j,o}}{L_{j,o}}$$

Change in commuting times The change in commuting times is

$$dt_{\ell,j,o} = \mathcal{T}'_{\ell,j,o} d\pi_{\ell,j,o} = \psi t_{\ell,j,o} \frac{d\pi_{\ell,j,o}}{\pi_{\ell,j,o}}$$

Change in labor supply The change in labor supply is

$$\begin{aligned}
dL_{j,o} &= \sum_{\ell} d\pi_{\ell,j,o} \left(1 - t_{\ell,j,o}\right) - \sum_{\ell} \pi_{\ell,j,o} dt_{\ell,j,o} \\
&= \sum_{\ell} \pi_{\ell,j,o} \left(1 - t_{\ell,j,o}\right) \frac{d\pi_{\ell,j,o}}{\pi_{\ell,j,o}} - \sum_{\ell} \pi_{\ell,j,o} t_{\ell,j,o} \frac{\mathcal{T}'_{\ell,j,o}}{\mathcal{T}_{\ell,j,o}} d\pi_{\ell,j,o} \\
&= \sum_{\ell} \pi_{\ell,j,o} \left(1 - t_{\ell,j,o}\right) \frac{d\pi_{\ell,j,o}}{\pi_{\ell,j,o}} - \psi \sum_{\ell} \pi_{\ell,j,o} t_{\ell,j,o} \frac{d\pi_{\ell,j,o}}{\pi_{\ell,j,o}}
\end{aligned}$$

and

$$dL_{j,h} = \sum_{\ell} \pi_{\ell,j,h} \frac{d\pi_{\ell,j,h}}{\pi_{\ell,j,h}}.$$

Change in total rents from houses and office buildings Since

$$T = \sum_{\ell} v' \left(\frac{H_\ell}{\Pi_\ell + N_{f,\ell}} \right) H_\ell + \alpha \sum_j Y_{j,o},$$

where $Y_{j,o} \equiv A_j(L_{j,o}) L_{j,o}^{1-\alpha} \overline{K}_j^{\alpha}$. Then

$$\begin{aligned}
dT &= - \sum_{\ell} v'' \left(\frac{H_\ell}{\Pi_\ell + N_{f,\ell}} \right) H_\ell \frac{H_\ell}{\Pi_\ell + N_{f,\ell}} \frac{d\Pi_\ell + dN_{f,\ell}}{\Pi_\ell + N_{f,\ell}} + \alpha \sum_j dY_j \\
&= \sum_{\ell} \sigma r_\ell H_\ell \frac{d\Pi_\ell + dN_{f,\ell}}{\Pi_\ell + N_{f,\ell}} + \alpha \sum_j dY_{j,o},
\end{aligned}$$

Change in housing consumption The change in consumption of houses is

$$\frac{dh_c}{h_c} = -\frac{d\Pi_c + dN_f}{\Pi_c + N_f}$$

$$\frac{dh_p}{h_p} = -\frac{d\Pi_p}{\Pi_p}$$

Change in output The change in output is

$$\begin{aligned} dY_j =& A_j'\left(L_{j,o}\right)\left(L_{j,o}^{1-\alpha}K_j^{\alpha} + \zeta L_{j,h}\right)dL_{j,o} + A_j\left(L_{j,o}\right)\left(\left(1-\alpha\right)L_{j,o}^{-\alpha}K_j^{\alpha}dL_{j,o} + \zeta dL_{j,h}\right) \\ dY_j =& \gamma A_j\left(L_{j,o}\right)\left(L_{j,o}^{1-\alpha}K_j^{\alpha} + \zeta L_{j,h}\right)\frac{dL_{j,o}}{L_{j,o}} + w_{j,o}dL_{j,o} + w_{j,h}dL_{j,h} \\ dY_j =& \gamma Y_j \frac{dL_{j,o}}{L_{j,o}} + w_{j,o}dL_{j,o} + w_{j,h}dL_{j,h}, \end{aligned}$$

and since since $Y_j = A_j\left(L_{j,o}\right)\left\{L_{j,o}^{1-\alpha}K_j^{\alpha} + \zeta L_{j,h}\right\} = \frac{w_{j,o}L_{j,o}}{1-\alpha} + w_{j,h}L_{j,h}$

$$dY_j = \gamma\left\{\frac{w_{j,o}L_{j,o}}{1-\alpha} + w_{j,h}L_{j,h}\right\}\frac{dL_{j,o}}{L_{j,o}} + w_{j,o}dL_{j,o} + w_{j,h}dL_{j,h}.$$

Considering only office output $Y_{j,o} = A_j\left(L_{j,o}\right)L_{j,o}^{1-\alpha}K_j^{\alpha}$ we obtain

$$dY_{j,o} = \left\{\gamma + 1 - \alpha\right\}Y_{j,o}\frac{dL_{j,o}}{L_{j,o}} = \left\{\frac{\gamma + 1 - \alpha}{1-\alpha}\right\}w_{j,o}L_{j,o}\frac{dL_{j,o}}{L_{j,o}} = \left\{\frac{\gamma}{1-\alpha} + 1\right\}w_{j,o}L_{j,o}\frac{dL_{j,o}}{L_{j,o}}$$

The change in welfare Welfare is given by

$$d\mathcal{W} = \Pi_c^{live}d\overline{u}_c + \sum_j L_{j,o}dw_{j,o} + \sum_j L_{j,h}dw_{j,h} - \sum_{\ell,j}\pi_{\ell,j,o}w_{j,o}dt_{\ell,j,o} + dT + \sigma\sum_{\ell}\Pi_{\ell}r_{\ell}h_{\ell}\frac{dh_{\ell}}{h_{\ell}}.$$

Replacing $dt_{\ell,j,o}$ and dT obtains

$$\begin{aligned} d\mathcal{W} =& \Pi_c^{live}d\overline{u}_c + \sum_j L_{j,o}dw_{j,o} + \sum_j L_{j,h}dw_{j,h} - \psi\sum_{\ell,j}\pi_{\ell,j,o}w_{j,o}t_{\ell,j,o}\frac{d\pi_{\ell,j,o}}{\pi_{\ell,j,o}} \\ &+ \sum_{\ell}\sigma r_{\ell}H_{\ell}\frac{d\Pi_{\ell} + dN_{f,\ell}}{\Pi_{\ell} + N_{f,\ell}} + \alpha\sum_j dY_{j,o} + \sigma\sum_{\ell}\Pi_{\ell}r_{\ell}h_{\ell}\frac{dh_{\ell}}{h_{\ell}}, \end{aligned}$$

where $N_{f,c} = N_f$ and $N_{f,p} = 0$ for brevity of notation. Replace dh_ℓ / h_ℓ and combine housing terms

$$\begin{aligned} d\mathcal{W} =& \Pi_c^{live} d\overline{u}_c + \sum_j L_{j,o} dw_{j,o} + \sum_j L_{j,h} dw_{j,h} - \psi \sum_{\ell,j} \pi_{\ell,j,o} w_{j,o} t_{\ell,j,o} \frac{d\pi_{\ell,j,o}}{\pi_{\ell,j,o}} \\ & + \sum_\ell \sigma r_\ell \left[H_\ell - \Pi_\ell h_\ell \right] \frac{d\Pi_\ell + dN_{f,\ell}}{\Pi_\ell + N_{f,\ell}} + \alpha \sum_j dY_{j,o}. \end{aligned}$$

Note that $H_\ell - \Pi_\ell h_\ell = N_{f,\ell} h_\ell$ and replace wages and office output

$$\begin{aligned} d\mathcal{W} =& \Pi_c^{live} d\overline{u}_c + (\gamma - \alpha) \sum_j L_{j,o} w_{j,o} \frac{dL_{j,o}}{L_{j,o}} + \gamma \sum_j L_{j,h} w_{j,h} \frac{dL_{j,o}}{L_{j,o}} - \psi \sum_{\ell,j} \pi_{\ell,j,o} w_{j,o} t_{\ell,j,o} \frac{d\pi_{\ell,j,o}}{\pi_{\ell,j,o}} \\ & + \sum_\ell \sigma r_\ell N_{f,\ell} h_\ell \frac{d\Pi_\ell + dN_{f,\ell}}{\Pi_\ell + N_{f,\ell}} + \sum_j \left\{ \frac{\alpha\gamma}{1-\alpha} + \alpha \right\} w_{j,o} L_{j,o} \frac{dL_{j,o}}{L_{j,o}}. \end{aligned}$$

Combine the second and last terms

$$\begin{aligned} d\mathcal{W} =& \Pi_c^{live} d\overline{u}_c + \left(\gamma - \alpha + \frac{\alpha\gamma}{1-\alpha} + \alpha \right) \sum_j L_{j,o} w_{j,o} \frac{dL_{j,o}}{L_{j,o}} + \gamma \sum_j L_{j,h} w_{j,h} \frac{dL_{j,o}}{L_{j,o}} \\ & - \psi \sum_{\ell,j} \pi_{\ell,j,o} w_{j,o} t_{\ell,j,o} \frac{d\pi_{\ell,j,o}}{\pi_{\ell,j,o}} + \sum_\ell \sigma r_\ell N_{f,\ell} h_\ell \frac{d\Pi_\ell + dN_{f,\ell}}{\Pi_\ell + N_{f,\ell}} \\ \Leftrightarrow d\mathcal{W} =& \Pi_c^{live} d\overline{u}_c + \frac{\gamma}{1-\alpha} \sum_j L_{j,o} w_{j,o} \frac{dL_{j,o}}{L_{j,o}} + \gamma \sum_j L_{j,h} w_{j,h} \frac{dL_{j,o}}{L_{j,o}} - \psi \sum_{\ell,j} \pi_{\ell,j,o} w_{j,o} t_{\ell,j,o} \frac{d\pi_{\ell,j,o}}{\pi_{\ell,j,o}} \\ & + \sum_\ell \sigma r_\ell N_{f,\ell} h_\ell \frac{d\Pi_\ell + dN_{f,\ell}}{\Pi_\ell + N_{f,\ell}}. \end{aligned}$$

Since $Y_{j,o} = L_{j,o} w_{j,o} / (1 - \alpha)$ and $Y_{j,h} = L_{j,h} w_{j,h}$ then

$$\begin{aligned} d\mathcal{W} =& \Pi_c^{live} d\overline{u}_c + \gamma \sum_j Y_{j,o} \frac{dL_{j,o}}{L_{j,o}} + \gamma \sum_j Y_{j,h} \frac{dL_{j,o}}{L_{j,o}} - \psi \sum_{\ell,j} \pi_{\ell,j,o} w_{j,o} t_{\ell,j,o} \frac{d\pi_{\ell,j,o}}{\pi_{\ell,j,o}} \\ & + \sum_\ell \sigma r_\ell N_{f,\ell} h_\ell \frac{d\Pi_\ell + dN_{f,\ell}}{\Pi_\ell + N_{f,\ell}} \\ \Leftrightarrow d\mathcal{W} =& \Pi_c^{live} d\overline{u}_c + \gamma \sum_j Y_j \frac{dL_{j,o}}{L_{j,o}} - \psi \sum_{\ell,j} \pi_{\ell,j,o} w_{j,o} t_{\ell,j,o} \frac{d\pi_{\ell,j,o}}{\pi_{\ell,j,o}} + \sum_\ell \sigma r_\ell N_{f,\ell} h_\ell \frac{d\Pi_\ell + dN_{f,\ell}}{\Pi_\ell + N_{f,\ell}} \end{aligned}$$

Finally, replace changes in office labor supply in each location

$$\begin{aligned}
d\mathcal{W} =&\Pi_c^{live} d\overline{u}_c + \gamma \sum_j Y_j \left\{ \sum_\ell \pi_{\ell,j,o} \frac{\left(1-t_{\ell,j,o}\right)}{L_{j,o}} \frac{d\pi_{\ell,j,o}}{\pi_{\ell,j,o}} - \psi \sum_\ell \pi_{\ell,j,o} \frac{t_{\ell,j,o}}{L_{j,o}} \frac{d\pi_{\ell,j,o}}{\pi_{\ell,j,o}} \right\} \\
&- \psi \sum_{\ell,j} \pi_{\ell,j,o} w_{j,o} t_{\ell,j,o} \frac{d\pi_{\ell,j,o}}{\pi_{\ell,j,o}} + \sum_\ell \sigma r_\ell N_{f,\ell} h_\ell \frac{d\Pi_\ell + dN_{f,\ell}}{\Pi_\ell + N_{f,\ell}} \\
\Leftrightarrow \mathcal{W} =&\Pi_c^{live} d\overline{u}_c + \gamma \sum_{\ell,j} \pi_{\ell,j,o} Y_j \frac{\left(1-t_{\ell,j,o}\right)}{L_{j,o}} \frac{d\pi_{\ell,j,o}}{\pi_{\ell,j,o}} - \gamma\psi \sum_{\ell,j} Y_j \frac{t_{\ell,j,o}}{L_{j,o}} \frac{d\pi_{\ell,j,o}}{\pi_{\ell,j,o}} \\
&- \psi \sum_{\ell,j} \pi_{\ell,j,o} w_{j,o} t_{\ell,j,o} \frac{d\pi_{\ell,j,o}}{\pi_{\ell,j,o}} + \sum_\ell \sigma \frac{N_{f,\ell}}{\Pi_\ell + N_{f,\ell}} r_\ell h_\ell \left\{ d\Pi_\ell + dN_{f,\ell} \right\}
\end{aligned}$$

and replacing the amenity effect

$$\begin{aligned}
d\mathcal{W} =&\phi_{\overline{u}} \Pi_c \overline{u}_c \frac{dN_f}{N_f} + \gamma \sum_{\ell,j} \pi_{\ell,j,o} \frac{Y_j\left(1-t_{\ell,j,o}\right)}{L_{j,o}} \frac{d\pi_{\ell,j,o}}{\pi_{\ell,j,o}} - \gamma\psi \sum_{\ell,j} \pi_{\ell,j,o} \frac{Y_j}{L_{j,o}} t_{\ell,j,o} \frac{d\pi_{\ell,j,o}}{\pi_{\ell,j,o}} \\
&- \psi \sum_{\ell,j} \pi_{\ell,j,o} w_{j,o} t_{\ell,j,o} \frac{d\pi_{\ell,j,o}}{\pi_{\ell,j,o}} + \sigma \frac{N_f}{\Pi_\ell + N_f} r_c h_c \left\{ d\Pi_c + dN_{f,c} \right\}.
\end{aligned}$$

Finally, note that

$$\begin{aligned}
\sum_{\ell,j} \pi_{\ell,j,o} \frac{Y_j\left(1-t_{\ell,j,o}\right)}{L_j} \frac{d\pi_{\ell,j,o}}{\pi_{\ell,j,o}} &= \Pi^{\text{office}} \mathbb{COV}\left(\frac{Y_j\left(1-t_{\ell,j,o}\right)}{L_j}, \frac{d\pi_{\ell,j,o}}{\pi_{\ell,j,o}} \right) + \sum_j Y_j \times d\Pi^{\text{office}} \\
\sum_{\ell,j} \pi_{\ell,j,o} w_{j,o} t_{\ell,j,o} \frac{d\pi_{\ell,j,o}}{\pi_{\ell,j,o}} &= \Pi^{\text{office}} \mathbb{COV}\left(w_{j,o} t_{\ell,j,o}, \frac{d\pi_{\ell,j,o}}{\pi_{\ell,j,o}} \right) + \left(\sum_{\ell,j} \pi_{\ell,j,o} w_{j,o} t_{\ell,j,o} \right) d\Pi^{\text{office}} \\
\sum_{\ell,j} \pi_{\ell,j,o} \frac{Y_j}{L_j} t_{\ell,j,o} \frac{d\pi_{\ell,j,o}}{\pi_{\ell,j,o}} &= \Pi^{\text{office}} \mathbb{COV}\left(\frac{Y_j}{L_j} t_{\ell,j,o}, \frac{d\pi_{\ell,j,o}}{\pi_{\ell,j,o}} \right) + \left(\sum_{\ell,j} \pi_{\ell,j,o} \frac{Y_j}{L_j} t_{\ell,j,o} \right) d\Pi^{\text{office}},
\end{aligned}$$

and $d\Pi^{\text{office}} = -d\Pi^{\text{remote}}$. Replacing these definitions and rearranging terms obtains the intended formula.

C.2 Mirrleesian optimal policy

The Mirrleesian problem is

$$\max \frac{\log\left(\sum_{\ell,j,e} e^{\eta u_{\ell,j,e}}\right)}{\eta}$$

$$u_{\ell,j,e} = \overline{u}_\ell\left(N_f\right) + c_{\ell,j,e} + v\left(h_{\ell,j,e}\right)$$

$$L_{j,e} = \sum_\ell \pi_{\ell,j,e}\left(1 - \mathcal{T}_{\ell,j}\left(\pi_{\ell,j,e}\right)\right)$$

$$\sum_{\ell,j,e} \pi_{\ell,j,e} c_{\ell,j,e} + N_f c_f = \sum_j A_j\left(L_{j,o}\right)\left(L_{j,o}^{\alpha}\overline{K}_j^{1-\alpha} + \zeta L_{j,h}\right) + N_f y_f$$

$$\sum_{j,e} \pi_{c,j,e} h_{c,j,e} + N_f h_f = \overline{H}_c$$

$$\sum_{j,e} \pi_{p,j,e} h_{p,j,e} = \overline{H}_p$$

$$\mathcal{U}_f(\pi_{c,c}, \pi_{c,p}, \pi_{p,c}) + c_f + v(h_f) \geq u_f^*$$

$$\pi_{\ell,j,e} = \frac{e^{\eta u_{\ell,j,e}}}{\sum_{\ell',j',e'} e^{\eta u_{\ell',j',e'}}}$$

The Lagrangian is

$$\begin{aligned}\mathcal{L} = {} & \frac{\log\left(\sum_{\ell,j,e} e^{\eta u_{\ell,j,e}}\right)}{\eta} + \lambda_c\left(\sum_j A_j\left(L_{j,o}\right)\left(L_{j,o}^{\alpha}\overline{K}_j^{1-\alpha} + \zeta L_{j,h}\right) + N_f y_f - \sum_{\ell,j,e}\pi_{\ell,j,e}c_{\ell,j,e} - N_f c_f\right) \\ & + \lambda_{h,c}\left(\overline{H}_c - \sum_{j,e}\pi_{c,j,e}h_{c,j,e} - N_f h_f\right) + \lambda_{h,p}\left(\overline{H}_p - \sum_{j,e}\pi_{p,j,e}h_{p,j,e}\right) \\ & + \sum_{\ell,j,e}\lambda_{\ell,j,e}^{\mathrm{IC}}\left(\pi_{\ell,j,e} - \frac{e^{\eta u_{\ell,j,e}}}{\sum_{\ell',j',e'} e^{\eta u_{\ell',j',e'}}}\right) + N_f\lambda_f\left(\mathcal{U}_f(\boldsymbol{\pi}) + c_f + v(h_f) - u_f^*\right)\end{aligned}$$

We use the labor and utility as definitions implicitly there. The first-order conditions for consumption and housing are

$$\left[c_{\ell,j,e}\right] \quad \pi_{\ell,j,e} - \lambda_{\ell,j,e}^{\mathrm{IC}}\pi_{\ell,j,e} + \sum_{\ell',j',e'}\lambda_{\ell',j',e'}^{\mathrm{IC}}\pi_{\ell',j',e'}\pi_{\ell,j,e} = \pi_{\ell,j,e}\lambda_c$$

$$\left[h_{\ell,j,e}\right] \quad v'\left(h_{\ell,j,e}\right)\left\{\pi_{\ell,j,e}-\lambda^{\text{IC}}_{\ell,j,e}\pi_{\ell,j,e}+\sum_{\ell',j',e'}\lambda^{\text{IC}}_{\ell',j',e'}\pi_{\ell',j',e'}\pi_{\ell,j,e}\right\}=\pi_{\ell,j,e}\lambda_{h,\ell}$$

Summing the first one across ℓ, j, e we get

$$1-\sum_{\ell',j',e'}\lambda^{\text{IC}}_{\ell',j',e'}\pi_{\ell',j',e'}+\sum_{\ell',j',e'}\lambda^{\text{IC}}_{\ell',j',e'}\pi_{\ell',j',e'}=\lambda_c \Leftrightarrow \lambda_c=1$$

then

$$\lambda^{\text{IC}}_{\ell,j,e}=\sum_{\ell',j',e'}\lambda^{\text{IC}}_{\ell',j',e'}\pi_{\ell',j',e'}$$

for all ℓ, j, e which implies that $\lambda^{\text{IC}}_{\ell,j,e}=\lambda^{\text{IC}}$ is constant across ℓ, j, e.

The first order conditions with respect to $\pi_{\ell,j,e}$ are :

$$\begin{aligned}\left[\pi_{\ell,j,o}\right] \quad & \lambda_c\left(A'_j\left(L_{j,o}\right)\left(L^{\alpha}_{j,o}\overline{K}^{1-\alpha}_j+\zeta L_{j,h}\right)\left(1-t_{\ell,j,o}\right)+\alpha A_j\left(L_{j,o}\right)L^{\alpha-1}_{j,o}\overline{K}^{1-\alpha}_j\left(1-t_{\ell,j,o}\right)-c_{\ell,j,o}\right)\\ & -\lambda_c\left(A'_j\left(L_{j,o}\right)\left(L^{\alpha}_{j,o}\overline{K}^{1-\alpha}_j+\zeta L_{j,h}\right)+\alpha A_j\left(L_{j,o}\right)L^{\alpha-1}_{j,o}\overline{K}^{1-\alpha}_j\left(1-t_{\ell,j,o}\right)\right)\pi_{\ell,j,o}\mathcal{T}'_{\ell,j,o}-\lambda_{h,\ell}h_{\ell,j,o}\\ & +N_f\lambda_f\frac{d\overline{u}_f}{d\pi_{\ell,j,o}}+\lambda^{\text{IC}}_{\ell,j,o}=0\end{aligned}$$

$$\left[\pi_{\ell,j,h}\right] \quad \lambda_c\left(A_j\left(L_{j,o}\right)\zeta-c_{\ell,j,h}\right)-\lambda_{h,\ell}h_{\ell,j,h}+N_f\lambda_f\frac{d\overline{u}_f}{d\pi_{\ell,j,h}}+\lambda^{\text{IC}}_{\ell,j,h}=0$$

The first order conditions with respect to c_f, h_f and N_f are

$$\left[c_f\right] \quad -N_f\lambda_c+N_f\lambda_f=0 \Leftrightarrow \lambda_c=\lambda_f$$

$$\left[h_f\right] \quad -N_f\lambda_{h,c}+N_f\lambda_f v'\left(h_f\right)=0 \Leftrightarrow v'\left(h_f\right)=\lambda_{h,c}/\lambda_f$$

$$\left[N_f\right] \quad \Pi^{live}_c\mathcal{U}'_c(N_f)+\lambda_c\left(y_f-c_f\right)-\lambda_{h,c}h_f=0.$$

C.2.1 Proof of proposition 6

Using the fact that $\lambda^{\text{IC}}_{\ell,j,e}$ is constant for all ℓ, j, e we find that

$$v'(h_{\ell,j,e})=\lambda_{h,\ell}$$

for all ℓ, j, e. Letting $r_\ell = \lambda_{h,\ell}$, this shows that all locals who live in location ℓ pay the same rent.

Replacing $\lambda_f = \lambda_c = 1$ and $\lambda^{\mathrm{IC}}_{\ell,j,o} = \lambda^{\mathrm{IC}}$ in the first order conditions with respect to $\pi_{\ell,j,o}$ we obtain:

$$[\pi_{\ell,j,o}] \quad \left(\gamma Y_j \frac{(1-t_{\ell,j,o})}{L_j} + w_{j,o}\left(1-t_{\ell,j,o}\right) - c_{\ell,j,o}\right) - \psi\gamma Y_j \frac{t_{\ell,j,o}}{L_{j,o}} + \psi w_{j,o} t_{\ell,j,o}$$
$$-r_\ell h_{\ell,j,o} + N_f \frac{d\overline{u}_f}{d\pi_{\ell,j,o}} + \lambda^{\mathrm{IC}} = 0$$

Define the transfer to these locals as $T_{\ell,j,o} \equiv c_{\ell,j,o} + r_\ell h_{\ell,j,o} - w_{j,o}\left(1-t_{\ell,j,o}\right)$ then

$$T_{\ell,j,o} = \gamma Y_j \frac{(1-t_{\ell,j,o})}{L_j} - \psi\gamma Y_j \frac{t_{\ell,j,o}}{L_{j,o}} + \psi w_{j,o} t_{\ell,j,o} + N_f \frac{d\overline{u}_f}{d\pi_{\ell,j,o}} + \lambda^{\mathrm{IC}}.$$

Analogously, the first order condition with respect to $\pi_{\ell,j,h}$ can be written as

$$w_{j,h} - c_{\ell,j,h} - r_\ell h_{\ell,j,h} + N_f \frac{d\overline{u}_f}{d\pi_{\ell,j,h}} + \lambda^{\mathrm{IC}} = 0.$$

Define the transfers to ℓ, j, h as $T_{\ell,j,h} \equiv c_{\ell,j,h} + r_\ell h_{\ell,j,h} - w_{j,h}$ then

$$T_{\ell,j,h} = N_f \frac{d\overline{u}_f}{d\pi_{\ell,j,h}} + \lambda^{\mathrm{IC}}$$

Summing the transfers across $\{\ell, j, e\}$ obtains:

$$\sum_{\ell,j,e} \pi_{\ell,j,e} T_{\ell,j,e} = \sum_{\ell,j,e} \pi_{\ell,j,e} c_{\ell,j,e} + \sum_{\ell,j,e} \pi_{\ell,j,e} r_\ell h_{\ell,j,e} - \sum_{\ell,j,e} \pi_{\ell,j,e} w_{j,e}\left(1-t_{\ell,j,e}\right) \tag{63}$$

From the resource constraint, we also know that:

$$\sum_{\ell,j,e} \pi_{\ell,j,e} c_{\ell,j,e} = \sum r^K_j \overline{K}_j + \sum_{\ell,j,e} \pi_{\ell,j,e} w_{j,h}\left(1-t_{\ell,j,e}\right) + N_f\left(y_f - c_f\right)$$

Since $\Theta_f \equiv N_f(y_f - c_f - r_c h_f)$, where $r_c = \lambda_{h,c}$, we can also write this resource constraint as

$$\sum_{\ell,j,e} \pi_{\ell,j,e} c_{\ell,j,e} = \sum r^K_j \overline{K}_j + \sum_{\ell,j,e} \pi_{\ell,j,e} w_{j,h}\left(1-t_{\ell,j,e}\right) + N_f r_c h_f + \Theta_f.$$

Replacing the sum of consumptions in equation (63), we find

$$\sum_{\ell,j,e} \pi_{\ell,j,e} T_{\ell,j,e} = \sum r_j^K \overline{K}_j + \sum_{\ell} r_\ell \overline{H}_\ell + \Theta_f \tag{64}$$

Averaging the first-order conditions, we find

$$\begin{aligned}\sum_{\ell,j,e} \pi_{\ell,j,e} T_{\ell,j,e} = \sum_{\ell,j,o} \pi_{\ell,j,o} \gamma Y_j \frac{(1 - t_{\ell,j,o})}{L_j} - \sum_{\ell,j,o} \pi_{\ell,j,o} \psi \gamma Y_j \frac{t_{\ell,j,o}}{L_{j,o}} - \sum_{\ell,j,o} \pi_{\ell,j,o} \psi w_{j,o} t_{\ell,j,o} \\ + \sum_{\ell,j,h} \pi_{\ell,j,h} N_f \frac{d\overline{u}_f}{d\pi_{\ell,j,h}} + \lambda^{\mathrm{IC}}\end{aligned}$$

and finally, replacing the transfers with (64), we can solve for the Lagrange multiplier λ^{IC}:

$$\begin{aligned}\lambda^{\mathrm{IC}} = \left\{ \sum r_j^K \overline{K}_j + \sum_{\ell} r_\ell \overline{H}_\ell + \Theta_f \right\} - \sum_{\ell,j,o} \pi_{\ell,j,o} \gamma Y_j \frac{(1 - t_{\ell,j,o})}{L_j} \\ + \sum_{\ell,j,o} \pi_{\ell,j,o} \psi \gamma Y_j \frac{t_{\ell,j,o}}{L_{j,o}} + \sum_{\ell,j,o} \pi_{\ell,j,o} \psi w_{j,o} t_{\ell,j,o} - \sum_{\ell,j,h} \pi_{\ell,j,h} N_f \frac{d\overline{u}_f}{d\pi_{\ell,j,h}}\end{aligned}$$

Replacing the lagrange multiplier λ^{IC} in the solution to the transfers above, we obtain

$$\begin{aligned}T_{\ell,j,o} = \sum r_j^K \overline{K}_j + \sum_{\ell} r_\ell \overline{H}_\ell + \Theta_f + \gamma \left\{ Y_j \frac{(1 - t_{\ell,j,o})}{L_{j,o}} - \sum_{\ell,j,o} \pi_{\ell,j,o} Y_j \frac{(1 - t_{\ell,j,o})}{L_{j,o}} \right\} + \\ - \psi \left\{ w_{j,o} t_{\ell,j,o} - \sum_{\ell,j,o} \pi_{\ell,j,o} \psi w_{j,o} t_{\ell,j,o} \right\} - \psi \gamma \left\{ Y_j \frac{t_{\ell,j,o}}{L_{j,o}} - \sum_{\ell,j,o} \pi_{\ell,j,o} Y_j \frac{t_{\ell,j,o}}{L_{j,o}} \right\} \\ + N_f \left\{ \frac{d\overline{u}_f}{d\pi_{\ell,j,o}} - \sum_{\ell,j,h} \pi_{\ell,j,h} \frac{d\overline{u}_f}{d\pi_{\ell,j,e}} \right\}\end{aligned}$$

and

$$T_{\ell,j,h} = \sum r_j^K \overline{K}_j + \sum_{\ell} r_{\ell} \overline{H}_{\ell} + \Theta_f + \gamma \left\{ - \sum_{\ell,j,o} \pi_{\ell,j,o} Y_j \frac{(1 - t_{\ell,j,o})}{L_{j,o}} \right\} +$$

$$- \psi \left\{ - \sum_{\ell,j,o} \pi_{\ell,j,o} \psi w_{j,o} t_{\ell,j,o} \right\} - \psi\gamma \left\{ - \sum_{\ell,j,o} \pi_{\ell,j,o} Y_j \frac{t_{\ell,j,o}}{L_{j,o}} \right\}$$

$$+ N_f \left\{ \frac{d\overline{u}_f}{d\pi_{\ell,j,h}} - \sum_{\ell,j,e} \pi_{\ell,j,e} \frac{d\overline{u}_f}{d\pi_{\ell,j,e}} \right\}.$$

C.2.2 Proof of proposition 7

The first order conditions with respect to c_f and h_f are

$$[c_f] \quad - N_f \lambda_c + N_f \lambda_f = 0 \Leftrightarrow \lambda_c = \lambda_f$$

$$[h_f] \quad - N_f \lambda_{h,c} + N_f \lambda_f v'(h_f) = 0 \Leftrightarrow v'(h_f)$$

Since $\lambda_c = 1$ then $\lambda_f = 1$, which implies that

$$v'(h_f) = \lambda_{h,c} = r_c.$$

So, foreigners' house purchases are not taxed, $\tau_h = 0$.

The first order condition with respect to N_f is :

$$[N_f] \quad \Pi_c^{live} \overline{u}'_{\ell}(N_f) + \lambda_c (y_f - c_f) - \lambda_{h,c} h_f = 0 \Leftrightarrow \Pi_c^{live} \overline{u}_c \phi_{\overline{u}} N_f + \lambda_c (y_f - c_f) - \lambda_{h,c} h_f = 0$$

So, foreigners face a lump-sum tax,

$$T_f = -\phi_{\overline{u}} \frac{\Pi_c^{live}}{N_f} \overline{u}_c$$

and $\Theta_f = N_f T_f$.

D Appendix to Section 5

D.1 The welfare impact of an increase in foreign residents

Note that, in equilibrium, all locals in a location ℓ consume the same number of houses $v'(h_{g,\ell,j}) = v'(h_\ell)$ where h_ℓ satisfies $v'(h_\ell) = r_\ell$. It follows that common utility is given by

$$u_{g,\ell,j} = \overline{u}_\ell + w_j(1 - t_{\ell,j}) + T_g - r_\ell h_\ell + v(h_\ell)$$

and living- and work-place shares are constant across groups g:

$$\pi_{g,\ell,j} = \frac{e^{\eta u_{g,\ell,j}}}{\sum_{\ell',j'} e^{\eta u_{g,\ell',j'}}} = \frac{e^{\eta(\overline{u}_\ell + w_j(1-t_{\ell,j}) - r_\ell h_\ell + v(h_\ell))}}{\sum_{\ell',j'} e^{\eta(\overline{u}_{\ell'} + w_{j'}(1-t_{\ell',j'}) - r_{\ell'} h_{\ell'} + v(h_{\ell'}))}} = \pi_{\ell,j}.$$

The change in group-g welfare is given by:

$$d\mathcal{W}_g = \sum_{\ell,j} \pi_{\ell,j} du_{g,\ell,j},$$

and

$$du_{g,\ell,j} = dw_j(1 - t_{\ell,j}) + dT_g - v''\left(h_\ell\right) h_\ell dh_\ell.$$

Replacing $dw_j = (\gamma - \alpha)(1 - t_{\ell,j})\frac{dL_j}{L_j}$ and $dT_g = v''\left(h_\ell\right) h_\ell = -\sigma v'(h_\ell) = -\sigma r_\ell dh_\ell$ we get

$$du_{g,\ell,j} = (\gamma - \alpha)\left(1 - t_{\ell,j}\right)\frac{dL_j}{L_j} + dT_g + \sigma r_\ell h_\ell \frac{dh_\ell}{h_\ell},$$

and note that $dT_g = s_g d\left(\sum_j r_j \overline{H}_j\right) + s_g^K d\left(\sum_j r_j^K \overline{K}_j\right)$. Next, we use the following facts

$$\frac{dL_j}{L_j} = \sum_{\ell} \frac{\pi_{\ell,j}}{L_j}\left(1 - t_{\ell,j}\right)\frac{d\pi_{\ell,j}}{\pi_{\ell,j}},$$
$$d\left(\sum_j r_j \overline{H}_j\right) = \sigma \overline{H}_c v'\left(h_c\right)\left(\frac{\Pi_c}{\Pi_c + N_f}\frac{d\Pi_c}{\Pi_c} + \frac{N_f}{\pi_c + N_f}\frac{dN_f}{N_f}\right) + \sigma \overline{H}_p v'\left(h_p\right)\frac{d\Pi_p}{\Pi_p},$$
$$d\left(\sum_j r_j^K \overline{K}_j\right) = \sum_{\ell} \alpha Y_\ell \left\{\gamma + (1-\alpha)\right\}\frac{dL_\ell}{L_\ell},$$
$$\frac{dh_c}{h_c} = -\left(\frac{\Pi_c}{\Pi_c + N_f}\frac{d\Pi_c}{\Pi_c} + \frac{N_f}{\pi_c + N_f}\frac{dN_f}{N_f}\right),$$
$$h_p = \frac{\overline{H}_p}{\Pi_p} \Rightarrow \frac{dh_p}{h_p} = -\frac{d\Pi_p}{\Pi_p},$$

and replace all in $d\mathcal{W}_g$:

$$\begin{aligned} d\mathcal{W}_g = &\sum_{\ell,j} \pi_{\ell,j}\left(\gamma - \alpha\right) w_j (1 - t_{\ell,j})\frac{dL_j}{L_j} \\ &+ s_g \left\{\sigma \overline{H}_c v'\left(h_c\right)\left(\frac{\Pi_c}{\Pi_c + N_f}\frac{d\Pi_c}{\Pi_c} + \frac{N_f}{\pi_c + N_f}\frac{dN_f}{N_f}\right) + \sigma \overline{H}_p v'\left(h_p\right)\frac{d\Pi_p}{\Pi_p}\right\} \\ &+ s_g^K \sum_j \alpha Y_j \left\{\gamma + (1-\alpha)\right\}\frac{dL_j}{L_j} - \sigma r_c \Pi_c h_c \left(\frac{\Pi_c}{\Pi_c + N_f}\frac{d\Pi_c}{\Pi_c} + \frac{N_f}{\pi_c + N_f}\frac{dN_f}{N_f}\right) - \sigma r_p \Pi_p h_p \frac{d\Pi_p}{\Pi_p}. \end{aligned}$$

Grouping terms obtains

$$\begin{aligned} d\mathcal{W}_g = &\gamma \sum_{\ell} Y_j \frac{dL_j}{L_j} + \left(s_g - 1\right)\left\{\sigma \overline{H}_c r_c \left(\frac{\Pi_c}{\Pi_c + N_f}\frac{d\Pi_c}{\Pi_c} + \frac{N_f}{\pi_c + N_f}\frac{dN_f}{N_f}\right) + \sigma \overline{H}_p r_p \frac{d\Pi_p}{\Pi_p}\right\} \\ &+ \left(s_g^K - 1\right)\sum_j \alpha Y_j \left\{\gamma + (1-\alpha)\right\}\frac{dL_j}{L_j} + \sigma \frac{N_f}{\Pi_c + N_f} r_c h_f \left(d\Pi_c + dN_f\right) \end{aligned}$$

Defining

$$\mathcal{PE} \equiv \gamma \sum_{\ell} Y_j \frac{dL_j}{L_j}$$

$$\mathcal{FS} \equiv \sigma \frac{N_f}{\Pi_c + N_f} r_c h_f \left(d\Pi_c + dN_f \right)$$

$$\mathcal{CG} \equiv \sigma \overline{H}_c r_c \left(\frac{\Pi_c}{\Pi_c + N_f} \frac{d\Pi_c}{\Pi_c} + \frac{N_f}{\pi_c + N_f} \frac{dN_f}{N_f} \right) + \sigma \overline{H}_p r_p \frac{d\Pi_p}{\Pi_p}$$

$$\mathcal{CG}^K \equiv \sum_j \alpha Y_j \left\{ \gamma + (1-\alpha) \right\} \frac{dL_j}{L_j}$$

delivers the result in the main text.

D.2 Mirrleesian optimal policy

The second-best problem is

$$\max \chi_1 \frac{\log \left(\sum_{\ell,j} e^{\eta u_{1,\ell,j}} \right)}{\eta}$$

$$\chi_g \frac{\log \left(\sum_{\ell,j} e^{\eta u_{g,\ell,j}} \right)}{\eta} \geq \chi_g \overline{u}_g^p$$

$$\sum_g \chi_g \sum_{\ell,j} \pi_{g,\ell,j} c_{g,\ell,j} + N_f c_f \leq L_c^{\gamma+1-\alpha} \overline{K}_c^\alpha + N_f y_f + L_p^{\gamma+1-\alpha} \overline{K}_p^\alpha$$

$$\sum_g \sum_j \chi_g \pi_{g,c,j} h_{c,j} + N_f h_f \leq \overline{H}_c$$

$$\sum_g \sum_j \chi_g \pi_{g,p,j} h_{p,j} \leq \overline{H}_p$$

$$\chi_g \pi_{g,\ell,j} = \chi_g \frac{e^{\eta u_{g,\ell,j}}}{\sum_{\ell',j'} e^{\eta u_{g,\ell',j'}}}$$

$$c_f + v\left(h_f \right) \geq \overline{u}_f$$

The first order conditions are (let $\lambda_1^u \equiv 1$)

$$\left[c_{g,\ell,j} \right] \quad \lambda_g^u + \eta \left(\sum_{\ell',j'} \lambda_{g,\ell',j'}^{\text{loc}} \pi_{g,\ell',j'} - \lambda_{g,\ell,j}^{\text{loc}} \right) = \lambda_c$$

$$[h_{g,\ell,j}] \quad v'\left(h_{g,\ell,j}\right)\left\{\lambda_g^u+\eta\left(\sum_{\ell',j'}\lambda_{g,\ell',j'}^{\text{loc}}\pi_{g,\ell',j'}-\lambda_{g,\ell,j}^{\text{loc}}\right)\right\}=\lambda_{\ell,h}$$

$$[c_f] \quad \lambda_f=\lambda_c$$

$$[h_f] \quad \lambda_f v'\left(h_f\right)=\lambda_{h,c}$$

Combining the first order equations for $c_{g,\ell,j}$ we obtain

$$\lambda_g^u=\lambda_c=\lambda_1^u=1$$

and

$$\sum_{\ell',j'}\lambda_{g,\ell',j'}^{\text{loc}}\pi_{g,\ell',j'}=\lambda_{g,\ell,j}^{\text{loc}}=\lambda_g^{\text{loc}}$$

and

$$v'\left(h_{g,\ell,j}\right)=\lambda_{\ell,h}$$

And the first order condition with respect to $\pi_{g,\ell,j}$ is

$$\lambda_c\left\{(1+\gamma-\alpha)\frac{Y_j}{L_j}\chi_g\left(1-t_{\ell,j}\right)-\chi_g c_{g,\ell,j}\right\}-\lambda_{h,\ell}\chi_g h_{g,\ell,j}=-\chi_g\lambda_g^{\text{loc}}$$

So

$$(1+\gamma-\alpha)\frac{Y_j}{L_j}\left(1-t_{\ell,j}\right)-c_{g,\ell,j}-\lambda_{h,\ell}h_{g,\ell,j}=-\lambda_g^{\text{loc}}$$

so sum across ℓ, j and obtain

$$(1+\gamma-\alpha)\sum_j Y_j-\sum_{\ell,j}\pi_{g,\ell,j}c_{g,\ell,j}-\sum_{\ell,j}\pi_{g,\ell,j}\lambda_{h,\ell}h_{g,\ell,j}=-\lambda_g^{\text{loc}}$$

and

$$\sum_{\ell,j}\pi_{g,\ell,j}c_{g,\ell,j}+\sum_{\ell,j}\pi_{g,\ell,j}\lambda_{h,\ell}h_{g,\ell,j}=(1+\gamma-\alpha)\sum_j Y_j+\lambda_g^{\text{loc}}$$

Note that

$$\sum_{\ell,j}\pi_{g,\ell,j}c_{g,\ell,j}+\sum_{\ell,j}\pi_{g,\ell,j}\lambda_{h,\ell}h_{g,\ell,j}=(1-\alpha)\sum_j Y_j+\underbrace{\sum\pi_{g,\ell,j}T_{g,\ell,j}}_{\equiv T_g}$$

so then

$$(1+\gamma-\alpha)\sum_j Y_j + \lambda_g^{\text{loc}} = (1-\alpha)\sum_j Y_j + T_g$$
$$\lambda_g^{\text{loc}} = -\gamma\sum_j Y_j + T_g$$

also

$$c_{g,\ell,j} + \lambda_{h,\ell} h_{g,\ell,j} = (1+\gamma-\alpha)\frac{Y_j}{L_j}\left(1-t_{\ell,j}\right) + \lambda_g^{\text{loc}}$$
$$c_{g,\ell,j} + \lambda_{h,\ell} h_{g,\ell,j} = (1-\alpha)\frac{Y_j}{L_j}\left(1-t_{\ell,j}\right) + \gamma\left\{\frac{Y_j}{L_j}\left(1-t_{\ell,j}\right) - \sum_j Y_j\right\} + T_g$$

Note that $h_{g,\ell,j}$ is constant across g and j since

$$v'\left(h_{g,\ell,j}\right) = \lambda_{\ell,h}.$$

Then

$$u_{g,\ell,j} = \overline{u}_{\ell,j} + (1-\alpha)\frac{Y_j}{L_j}\left(1-t_{\ell,j}\right) + \gamma\left\{\frac{Y_j}{L_j}\left(1-t_{\ell,j}\right) - \sum_j Y_j\right\} + T_g - \lambda_{h,\ell} h_\ell + v\left(h_\ell\right)$$

also for $g \geq 2$

$$\frac{\log\left(\sum_{\ell,j} e^{\eta\left\{\overline{u}_{\ell,j}+(1-\alpha)\frac{Y_j}{L_j}\left(1-t_{\ell,j}\right)+\gamma\left\{\frac{Y_j}{L_j}\left(1-t_{\ell,j}\right)-\sum_j Y_j\right\}+T_g-\lambda_{h,\ell}h_\ell+v(h_\ell)\right\}}\right)}{\eta} = \overline{u}_g^p$$

$$\frac{\log\left(e^{\eta T_g}\sum_{\ell,j} e^{\eta\left\{\overline{u}_{\ell,j}+(1-\alpha)\frac{Y_j}{L_j}\left(1-t_{\ell,j}\right)+\gamma\left\{\frac{Y_j}{L_j}\left(1-t_{\ell,j}\right)-\sum_j Y_j\right\}-\lambda_{h,\ell}h_\ell+v(h_\ell)\right\}}\right)}{\eta} = \overline{u}_g^p$$

$$T_g = \overline{u}_g^p - \frac{\log\left(\sum_{\ell,j} e^{\eta\left\{\overline{u}_{\ell,j}+(1-\alpha)\frac{Y_j}{L_j}\left(1-t_{\ell,j}\right)+\gamma\left\{\frac{Y_j}{L_j}\left(1-t_{\ell,j}\right)-\sum_j Y_j\right\}-\lambda_{h,\ell}h_\ell+v(h_\ell)\right\}}\right)}{\eta}$$

Note that since

$$\sum_{\ell,j}\pi_{g,\ell,j} c_{g,\ell,j} + \sum_{\ell,j}\pi_{g,\ell,j}\lambda_{h,\ell} h_{g,\ell,j} = (1-\alpha)\sum_j Y_j + T_g$$

$$\sum_g \chi_g \sum_{\ell,j} \pi_{g,\ell,j} c_{g,\ell,j} + \sum_g \chi_g \sum_{\ell,j} \pi_{g,\ell,j} \lambda_{h,\ell} h_{g,\ell,j} = (1-\alpha) \sum_j Y_j + \sum_g \chi_g T_g$$

and

$$\sum_j Y_j + N_f \left(y_f - c_f\right) + \sum_g \chi_g \sum_{\ell,j} \pi_{g,\ell,j} \lambda_{h,\ell} h_{g,\ell,j} = (1-\alpha) \sum_j Y_j + \sum_g \chi_g T_g$$

$$\sum_j Y_j + \sum_\ell \lambda_{h,\ell} \overline{H}_\ell = (1-\alpha) \sum_j Y_j + \sum_g \chi_g T_g$$

$$\sum_g \chi_g T_g = \alpha \sum_j Y_j + \sum_\ell \lambda_{h,\ell} \overline{H}_\ell$$

The optimal treatment of tourists is the same as in the baseline model. The foreigners participation constraint binds. Combining the first order conditions to c_f and h_f we obtain $v'(h_f) = \lambda_{h,c} = r_c$ and the first order condition with respect to N_f is given by:

$$\left[N_f\right] \quad \lambda_c \left(y_f - c_f\right) - \lambda_{h,c} h_f = 0 \Leftrightarrow y_f = c_f + v'\left(h_f\right) h_f \Rightarrow T_f = 0.$$

E What if foreigners had the same spatial distribution as locals?

The baseline model assumes that foreigners only live in the city center. However, for the purposes of our results, all that is needed is that foreigners disproportionally seek to live in the city center relative to locals. This appendix shows that if $\sigma = 1$ and foreigners choose the same geographical spread as the incumbent population, then the production externality term is zero.

Suppose that foreigners enter both locations and let $N_{f,\ell}$ denote the number of foreigners that live in location ℓ. Due to the quasi-linearity of preferences, each individual in location ℓ consumes

$$h_\ell = \frac{\overline{H}_\ell}{\Pi_\ell + N_{f,\ell}} \tag{65}$$

houses, where $\Pi_\ell = \sum_j \pi_{\ell,j}$. Locals in location ℓ who work in j obtain common utility

$$u_{\ell,j} = \overline{u}_\ell + w_j(1 - t_{\ell,j}) + T - r_\ell h_\ell + v(h_\ell), \tag{66}$$

where $r_\ell = v'(h_\ell)$. Their location choices satisfy

$$\pi_{\ell,j} = \frac{e^{\eta u_{\ell,j}}}{\sum_{\ell',j'} e^{\eta u_{\ell',j'}}}. \tag{67}$$

Suppose there is an influx of foreigners, which is located in proportion to the incumbent distribution. We denote with superscript the new equilibrium. Suppose that $N'_f > N_f$ and

$$N'_{f,\ell} = N_{f,\ell} + \frac{\Pi_\ell + N_{f,\ell}}{1 + N_f}(N'_f - N_f). \tag{68}$$

We prove via guess and verification that equilibrium location choices are unchanged. In each place, housing consumption falls proportionally

$$h'_\ell = \frac{\overline{H}_\ell}{\Pi_\ell + N'_{f,\ell}} = h_\ell \frac{1 + N_f}{1 + N'_f} < h_\ell.$$

Since $\pi_{\ell,j}$ are unchanged then $w'_j = w_j$. When $\sigma = 1$

$$\begin{aligned}
u'_{\ell,j} =& \overline{u}_\ell + w_j(1 - t_{\ell,j}) + T' - \underbrace{v'(h'_\ell)h'_\ell}_{=1} + v(h'_\ell) \\
=& \overline{u}_\ell + w_j(1 - t_{\ell,j}) + T - \underbrace{v'(h_\ell)h_\ell}_{=1} + v(h_\ell) + \log\left(\frac{1 + N_f}{1 + N'_f}\right) + (T' - T) \\
=& u_{\ell,j} + \log\left(\frac{1 + N_f}{1 + N'_f}\right) + (T' - T).
\end{aligned}$$

Finally, using this expression we see that

$$\pi'_{\ell,j} = \frac{e^{\eta u_{\ell,j} + \eta\left[\log\left(\frac{1+N_f}{1+N'_f}\right) + (T'-T)\right]}}{\sum_{\ell',j'} e^{\eta u_{\ell',j'} + \eta\left[\log\left(\frac{1+N_f}{1+N'_f}\right) + (T'-T)\right]}} = \pi_{\ell,j},$$

confirming the guess.

F Relation to the optimal tariff literature

We can interpret the sales of houses to foreigners as exports paid for in units of the tradable consumption good. So, there is a connection between our results and those in the trade literature (see, e.g., Dixit, 1985, Caliendo and Parro, 2022, and references therein). In this appendix, we discuss this relation using a simple trade model.

Consider a world with a home country and $n \in \mathbb{R}$ identical foreign countries. Countries are endowed with two consumption goods, 1 and 2. The home country has y_1 units of good 1 and y_2 units of good 2. Each foreign country has y_1^* and y_2^* units of goods 1 and 2, respectively (throughout, we use stars to denote foreign-country variables). The representative agent of the home country has utility $u(c_1, c_2)$, and the representative agent of each foreign country has utility $u^*(c_1^*, c_2^*)$.

Abstracting from location choices and goods production, this model is analogous to our main model if we interpret one good as houses and the other as consumption.

F.1 Why is the optimal tax on houses bought by foreigners zero?

To compute the optimal tariff, we assume that the home country can unilaterally impose a proportional tax τ on imports (or, equivalently, a subsidy to exports). The resulting tax revenue, T, is rebated back to the households of the home country. The budget constraints of home and foreign consumers are given by

$$c_1 - y_1 + (1+\tau)p(c_2 - y_2) - T = 0, \tag{69}$$

$$c_1^* - y_1^* + p(c_2^* - y_2^*) = 0, \tag{70}$$

where p denotes the relative price of good 2 in units of good 1. Two first-order conditions describe the equilibrium in this economy,

$$\frac{u_2}{u_1} = (1+\tau)p, \tag{71}$$

$$\frac{u_2^*}{u_1^*} = p, \tag{72}$$

the budget constraints (69) and (70), the resource constraints,

$$c_1 + nc_1^* = y_1 + ny_1^*, \tag{73}$$

$$c_2 + nc_2^* = y_2 + ny_2^*, \tag{74}$$

and the government budget constraint,

$$T = \tau p(c_2 - y_2). \tag{75}$$

We compute the optimal tariff using the primal approach developed by Lucas and Stokey (1983). This approach involves choosing $\{c_1, c_2, c_1^*, c_2^*\}$ to maximize the utility in the home country subject to the resource constraints (73) and (74), the implementability condition

$$u_1^*(c_1^* - y_1^*) + u_2^*(c_2^* - y_2^*) = 0, \tag{76}$$

and a participation constraint for the foreign countries:[14]

$$u^*(c_1^*, c_2^*) \geq \overline{u}^*. \tag{77}$$

This constraint reflects the existence of un-modelled alternatives to trading with the home country, which guarantee a level of utility $\overline{u}^*$.

Theorem 1. *Let φ and λ_p denote the Lagrange multipliers associated with (76) and (77), respectively. The optimal tariff is given by*

$$\tau = \varphi \frac{\left(\frac{u_{22}^*}{u_2^*} - \frac{u_{21}^*}{u_1^*}\right)(c_2^* - y_2^*) - \left(\frac{u_{11}^*}{u_1^*} - \frac{u_{12}^*}{u_2^*}\right)(c_1^* - y_1^*)}{\lambda_p + \varphi\left[1 + \frac{u_{11}^*}{u_1^*}(c_1^* - y_1^*) + \frac{u_{21}^*}{u_1^*}(c_2^* - y_2^*)\right]} \neq 0. \tag{78}$$

[14]These are necessary and sufficient conditions to solve for the equilibrium allocations. They are necessary because the equilibrium conditions imply them. Sufficiency can be proved as follows. Take a set of allocations $\{c_1, c_2, c_1^*, c_2^*\}$ that satisfies these conditions. These allocations can be equilibrium allocations for an appropriate choice of prices and policies. We can always find a tariff, τ, and a relative price, p, that satisfy the marginal rates of substitution (71) and (72), respectively. We can always find T that satisfies the domestic budget constraint (69). Using these values for p, τ, and T, the foreign budget constraint (70) is satisfied since the implementability condition (76) is also satisfied. The government budget constraint is satisfied by Walras' law. Finally, the resource constraints are also satisfied since they are imposed. It follows that we can always construct an equilibrium that implements the allocations $\{c_1, c_2, c_1^*, c_2^*\}$.

Suppose that $u^*(c_1^*, c_2^*) = [(c_1^*)^{1-\sigma} + (c_2^*)^{1-\sigma}]/(1-\sigma)$, then the optimal tariff takes the form

$$\tau = \sigma\varphi \frac{\left(\frac{c_1^* - y_1^*}{c_1^*}\right) - \left(\frac{c_2^* - y_2^*}{c_2^*}\right)}{\lambda_p + \varphi\left[1 - \sigma\left(\frac{c_1^* - y_1^*}{c_1^*}\right)\right]}.$$

Suppose $\varphi > 0$. If foreigners export good 2, then $c_1^* > y_1^*$ and $c_2^* < y_2^*$. It follows that the optimal tariff is positive ($\tau > 0$). If foreigners export good 1, then $c_1^* < y_1^*$ and $c_2^* > y_2^*$. It follows that the optimal tariff is negative ($\tau < 0$).

This is the classical result that a country has an incentive to unilaterally tax imports or subsidize exports to manipulate terms of trade and obtain monopolistic rents. The home country exports houses and imports traded goods in our baseline model. So, why do we find that taxing the houses foreigners purchase is not optimal?

In deriving the optimal tariff, we have assumed that it is impossible to levy a lump-sum tax on foreigners. This possibility is not precluded in our main model since the home country can impose an entry fee on foreign residents. Suppose that in our trade model, the home country can charge foreign countries a fee T^* for the right to trade. The foreigners' budget constraint is

$$c_1^* - y_1^* + p(c_2^* - y_2^*) + T^* = 0. \tag{79}$$

The domestic budget constraint takes the same form (69),

$$c_1 - y_1 + (1+\tau)p(c_2 - y_2) - T = 0$$

where the rebates to domestic households are now given by

$$T = \tau p(c_2 - y_2) + nT^*.$$

We do not need to impose the implementability condition (76), since this condition can always be satisfied by choosing an appropriate trade fee, T^*. So, the new planning problem is to maximize the welfare of the home country subject to (73), (74), and (77).

Proposition 10. *Suppose that the home country can impose a right-to-trade fee, T^*. Then, the optimal tariff is zero*

$$\tau = 0. \tag{80}$$

The right-to-trade fee is set so that foreign countries are indifferent between trading and not trading:

$$u^*(c_1^*, c_2^*) = \overline{u}^*. \tag{81}$$

When a lump-sum instrument is available, it is always better to use it to extract the gains from trade from foreign countries than to impose a distortionary tax on trade. The reason is as follows. A zero tariff maximizes the gains from trade. These gains are then taxed away by the home country using the lump-sum instrument. This scheme resembles the optimal use of a two-part tariff by a monopolist. It is optimal for the monopolist to set the price equal to the marginal cost and use a fixed fee to extract all the consumer surplus.

In our model, we impose no exogenous restrictions on the available instruments. Instead, the set of feasible instruments is determined by the primitive informational constraints faced by the planner or government. Since the planner can observe the country of origin, it can design a tax system with a lump-sum tax on foreigners. The result above implies that it is not optimal to tax houses.

In our model in the main text, for any fixed number of foreign countries N_f, it is optimal for the home country to choose a non-zero entry fee $T_f \neq 0$ to extract the gains of foreign countries relative to their outside option.

F.2 Why is a zero entry fee optimal in our model?

The third part of proposition 3 states that the optimal entry fee is zero in our main model. This result reflects the fact that the planner can choose the optimal number of foreigners, N_f.

To discuss the optimal entry fee using the trade model presented in this section, we allow the home country to choose the number of trading partners, n. Let λ_1 and λ_2 denote the Lagrange multipliers on resource constraints for good 1 and 2, respectively. The first-order condition for n is[15]

$$\lambda_1(y_1^* - c_1^*) + \lambda_2(y_2^* - c_2^*) = 0. \tag{82}$$

This equation equates marginal benefits with marginal costs. The marginal benefit of an additional trading partner is the value of the goods they bring to the table $\lambda_1 y_1^* + \lambda_2 y_2^*$. The marginal cost is the value of goods they consume $\lambda_1 c_1^* + \lambda_2 c_2^*$.

Combining (82) with the implementability condition (76), we find that

$$\frac{\lambda_1(y_1^* - c_1^*)}{u_1^*(y_1^* - c_1^*)} = \frac{\lambda_2(y_2^* - c_2^*)}{u_2^*(y_2^* - c_2^*)} \Leftrightarrow \frac{u_2}{u_1} = \frac{\lambda_2}{\lambda_1} = \frac{u_2}{u_1}. \tag{83}$$

If the home country cannot levy a lump-sum tax, T^*, then the optimal number of trading partners is $\tau = 0$.

If the home country can choose $T^* \neq 0$, then we already know that $\tau = 0$ and $p = u_2^*/u_1^* = \lambda_2/\lambda_1$. It then follows from (82) that

$$(y_1^* - c_1^*) + \frac{u_2^*}{u_1^*}(y_2^* - c_2^*) = 0 \Leftrightarrow (y_1^* - c_1^*) + p(y_2^* - c_2^*) = 0 \Leftrightarrow T^* = 0. \tag{84}$$

So, even if the home country can levy a lump-sum tax, the optimal number of trading partners is $T^* = 0$.

These results are summarized in the following proposition, which echoes the results in proposition 3.

Proposition 11. *Suppose the home country can choose the number of trading partners, n. Then, the optimal number of trading partners is such that:*

1. *If the home country cannot impose a right-to-trade fee, then the optimal tariff is zero, $\tau = 0$.*

[15] We assume throughout that the solution is interior.

2. *If the home country can impose a right-to-trade fee, then the optimal fee is zero,* $T_f = 0$.

It follows that the optimal number of trading partners is the same as in a laissez-faire solution. To explain why, we start with too few trading partners. As we increase n, each trading partner receives a smaller portion of the home country's exports. The relative price of the exported good rises, and the home country benefits more from exports.[16] To satisfy the participation constraint, the home country must reduce the rights-to-trade fee. The benefit from increasing the value of exports is strictly higher than the reduction in fee revenue.

For analogous reasons, in our model, optimizing the number of foreigners N_f requires setting the entry fee, T_f, to zero.

F.3 Numerical example

We illustrate the results described in propositions 10 and 11 with a numerical example. We assume that the utility function takes the form $u(c_1, c_2) = (c_1^{1-\sigma} + c_2^{1-\sigma})/(1-\sigma)$ and $u^*(c_1^*, c_2^*) = [(c_1^*)^{1-\sigma} + (c_2^*)^{1-\sigma}]/(1-\sigma)$ and set $\sigma = 0.25$. We also set $y_1 = 1$, $y_2 = 0.3$, $y_1^* = 0.3$ and $y_2^* = 1$. We set the foreigner's outside option to $\overline{u}^* = 1.7371$.[17]

Figure 2 displays the optimal tariff as a function of the number of trading partners, n when the rights-to-trade fee is restricted to zero. We also display the optimum under the additional assumption that trading partners are free-disposable, i.e., the home country can trade with fewer than the n countries. The dotted red line represents the results under this additional assumption. The panel in position (1,1) displays the welfare in the home country, the panel in (1,2) the optimal tariff, the panel in (2,1) the right-to-trade fee (which in this case is restricted to zero), and finally panel (2,2) the transfer of the tariff revenue to the domestic household, T.

[16] The home country also exports more in total, so it consumes a lower amount of the exported good and more of the imported good.

[17] In this numerical example, as the outside converges to the utility under autarky, $u^*(y_1^*, y_2^*)$, the optimal number of trading partners converges to infinity.

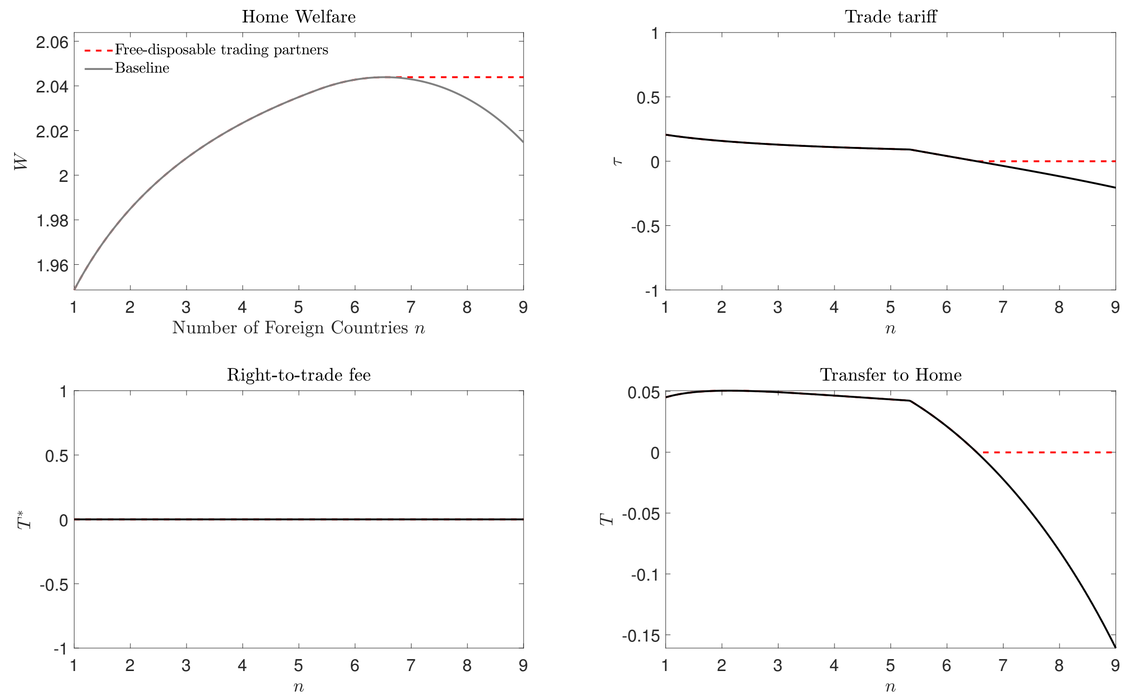

Figure 1: Optimal tariff

When the right-to-trade fee is restricted to zero, it is optimal to impose a tariff, i.e., a tax on imports. As the number of trading partners increases, the optimal tariff falls. Home welfare rises for small n and reaches a maximum when $n = n^* = 6.53$. As shown in proposition 11, the optimal tariff when the country can choose the optimal number of trade partners is zero. Past this optimal number of trade partners, home welfare falls because the home country has to subsidize imports. This subsidy transfers resources to foreign countries and helps satisfy their outside option.

So, when $n \geq n^*$ and trading partners are freely disposable, it is optimal to implement a laissez-faire policy in which tariffs are zero and foreign countries freely choose whether to trade with the home country.

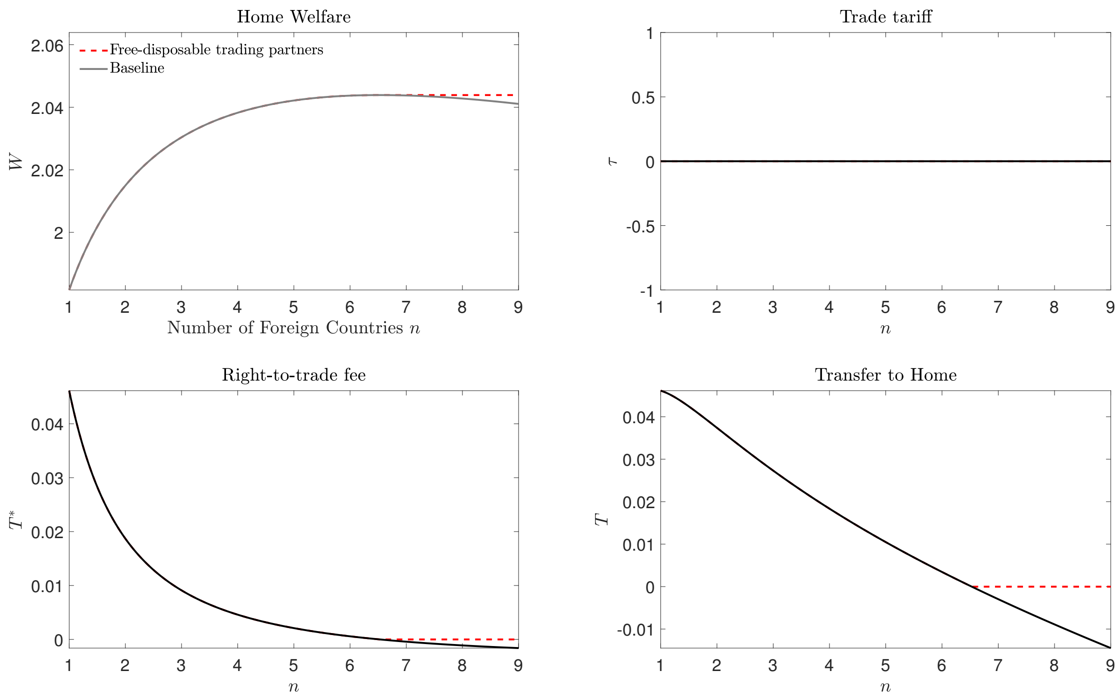

Figure 2: Optimal right-to-trade fee

Figure 2 displays the results for the case of the optimal tariff and rights-to-trade fee as a function of the number of trading partners, n. As in Figure 1, we also display the optimum under the additional assumption that there is free-disposal of trading partners, i.e., the home country can trade with fewer than the n countries. The dotted red line represents these results. The panel in position (1,1) displays the welfare in the home country, the panel in (1,2) the optimal tariff, the panel in (2,1) the trade fee (which in this case is restricted to be zero), and finally panel (2,2) the transfer of the tariff revenue to the domestic household, T.

When the home country can impose a right-to-trade fee, setting the tariff to zero is always optimal, echoing the results in proposition 10. As the number of trading partners increases, the optimal right-to-trade fee falls. Home welfare rises for small n and reaches a maximum when $n = n^* = 6.53$. If $n < n^*$, it is optimal to impose a positive rights-to-trade fee. As n increases, the optimal rights-to-trade fee falls and reaches zero when $n = n^*$, as shown in proposition 11. If $n > n^*$, the optimal right-to-trade fee becomes negative. This subsidy transfers resources to foreign countries and helps satisfy their outside option.

For $n \geq n^*$ and free-disposability of trading partners, it is optimal to implement a laissez-faire policy in which tariffs are zero and foreign countries freely choose whether to trade with the home country.