

Online Appendix: Monetary policy without redistributive concerns

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Appendix A Appendix to Section 2

A.1 Intermediate goods firms

The monopolist of variety i chooses how much of each labor variety to demand to minimize the cost of producing. The cost minimization problem is

$$\min_{n_{i,u,t}} \int w_{u,t} n_{i,u,t} du \quad \text{subject to } y_{i,t} = Z_t \left(\int n_{i,u,t}^{\frac{\psi-1}{\psi}} du \right)^{\frac{\psi}{\psi-1}},$$

where $w_{u,t}$ is the wage of labor variety u . The solution to this problem implies that the demand for each labor variety is given by

$$n_{i,u,t} = \left(\frac{w_{u,t}}{W_t} \right)^{-\psi} \frac{y_{i,t}}{Z_t},$$

where

$$W_t = \left(\int w_{u,t}^{1-\psi} du \right)^{\frac{1}{1-\psi}}.$$

where W_t denotes the wage index of labor services.

In each period t , a random θ_p fraction of intermediate goods firms are not allowed to adjust their price. Each firm that can adjust its price chooses the price that maximizes the expected discounted stream of profits until the next time it can adjust its price. The variable $\tau_{p,t}$ denotes the ad valorem subsidy to sales revenue in period t , so that a firm receives $(1 + \tau_{p,t})p_{i,t}$ per unit sold before lump-sum taxes. The objective function of a firm that can adjust its price in period t is given by

$$\max_{p_{i,t}} \mathbb{E}_t \sum_{k=0}^{\infty} \tilde{Q}_{t,t+k} \theta_p^k \left[\left((1 + \tau_{p,t+k}) p_{i,t} - MC_{t+k} \right) y_{i,t+k} \right],$$

where $y_{i,t+k} = \left(\frac{p_{i,t}}{p_{t+k}} \right)^{-\varepsilon} Y_{t+k}$ is the demand for good i , $MC_t \equiv \frac{W_t}{Z_t}$ is the nominal marginal cost of producing good in period t , and $\tilde{Q}_{t,t+k}$ denotes the market discount factor. Following the standard approach in the NK literature, we introduce a subsidy to sales $\tau_{p,t}$, which allows us both to undo the monopoly distortion in steady state and to model cost-push shocks. We assume that the subsidy is contemporaneously financed by a lump-sum tax of the same value as the subsidy: $\tau_{p,t} p_{i,t} y_{i,t}$ to ensure budget neutrality in each period. We assume that $\tau_p = \frac{1}{\varepsilon-1}$ in steady state, so that the steady state allocation is efficient.

The first-order condition of this problem implies that the optimal price of a resetting

firm is given by:

$$p_t^* = \frac{\varepsilon}{\varepsilon - 1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \tilde{Q}_{t,t+k} \theta_p^k P_{t+k}^{1-\varepsilon} Y_{t+k} \frac{W_{t+k}}{Z_{t+k} P_{t+k}}}{\mathbb{E}_t \sum_{k=0}^{\infty} \tilde{Q}_{t,t+k} (1 + \tau_{p,t+k}) \theta_p^k P_{t+k}^{\varepsilon} Y_{t+k}}.$$

Since final-goods producers make zero profits, total profits in the economy are given by

$$\Xi_t \equiv \int [p_{i,t} - MC_{i,t}] y_{i,t} di.$$

A.2 Aggregate demand

Under the total-income rule, household h 's real income is

$$y_{h,t} = z_{h,t} \Gamma^Y (1 - L_t; \zeta_{h,t}) Y_t. \quad (29)$$

The labor-allocation function Γ determines physical hours, whereas Γ^Y determines the total income entering the household budget.

The aggregate demand of the economy is given by

$$C_t = \int c_{h,t} dh.$$

As in Angeletos et al. (2025), we can represent the aggregate consumption function by a functional $\mathcal{C}$ such that

$$C_t = \mathcal{C} \left(\{ \{ \mathbb{E}_s [\Theta_{s+h}] \}_{h=0}^{\infty} \}_{s=-\infty}^t \right),$$

where $\Theta_t \equiv \{Y_t, L_t, 1 + i_t, 1 + \delta_t, P_t / P_{t-1}, \{q_{k,t}\}, \{T_{h,t}\}\}$ collects the aggregate variables entering the household budget under the reduced-form total-income rule. In particular, profits do not enter the consumption block separately from $y_{h,t}$. Importantly, $\mathbb{E}_s [\Theta_{s+h}] = \Theta$ are constant at steady state for $s < 0$.

The dependence of aggregate demand on current fundamentals and current expectations is immediate from the dependence of current decisions on people's current expectations. The additional past expectations capture the influence of the endogenous state variables (e.g., asset distribution) on past behavior and therefore on past fundamentals and past expectations.

Note that we write the consumption function as a functional of the point expectations directly, disregarding uncertainty. Since we focus on linearized equilibrium conditions, where certainty equivalence holds, this is without loss of generality.

A.3 Unions

Each union u is a monopolist in the labor market for labor variety u . In each period, a random θ_w fraction of unions are not allowed to adjust their wage. Unions face the demand $n_{u,t}^d \equiv \int n_{i,u,t} di = \left(\frac{w_{u,t}}{W_t}\right)^{-\psi} \frac{\int y_{i,t} di}{Z_t}$ for their labor variety. The variable $\tau_{w,t}$ denotes the ad valorem subsidy to labor earnings in period t , so that the union values labor income at the subsidized wage $(1 + \tau_{w,t})w_{u,t}$ before lump-sum taxes. As in McKay and Wolf (2026), we assume that union have the following objective:

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\beta \theta_w)^k \left[(C_{t+k})^{-1} (1 + \tau_{w,t+k}) \frac{w_{u,t} n_{u,t+k}^d}{P_{t+k}} - \nu (L_{t+k})^{-1} n_{u,t+k}^d \right].$$

This assumption guarantees that the supply side of our economy remains close to the standard RANK model and leads to the same wage NK Phillips curve under our approximation.¹⁶ As in the case of the intermediate goods producer, we assume that there is a subsidy $\tau_{w,t}$ that undoes monopoly distortions and allows us to capture cost-push shocks.

Resetting unions choose $w_{u,t}$ to maximize

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\beta \theta_w)^k \left[(C_{t+k})^{-1} (1 + \tau_{w,t+k}) \frac{w_{u,t} n_{u,t+k}^d}{P_{t+k}} - \nu (L_{t+k})^{-1} n_{u,t+k}^d \right].$$

The wage subsidy undoes the monopoly distortion in the labor market and is financed by taxes on households, so that households receive pre-transfer income. We assume that $\tau_w = \frac{1}{\psi-1}$ in steady state, so that the steady state allocation is efficient.

The first-order condition of this problem implies that the optimal reset wage is given by

$$w_t^* = \frac{\psi}{\psi-1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} (\beta \theta_w)^k \nu (L_{t+k})^{-1} \left(\frac{1}{W_{t+k}}\right)^{-\psi} \frac{\int y_{i,t+k} di}{Z_{t+k}}}{\mathbb{E}_t \sum_{k=0}^{\infty} (\beta \theta_w)^k (C_{t+k})^{-1} \frac{(1+\tau_{w,t+k})}{P_{t+k}} \left(\frac{1}{W_{t+k}}\right)^{-\psi} \frac{\int y_{i,t+k} di}{Z_{t+k}}}. \quad (30)$$

A.4 Linearized equilibrium conditions

We approximate the equilibrium conditions around an efficient steady state, where all variables are constant over time. To achieve this steady state, we index exogenous variables by their deviations from steady state: $\log Z_t^\sigma = (1 - \sigma_Z) \log Z + \sigma_Z \log Z_t$, $\log(1 + \tau_{p,t}^\sigma) = (1 - \sigma_{\tau,p}) \log(1 + \tau_p) + \sigma_{\tau,p} \log(1 + \tau_{p,t})$, and $\log(1 + \tau_{w,t}^\sigma) = (1 -$

¹⁶Alternatively, we could assume that unions evaluate the benefits according to a weighted average of the marginal utilities of their members. This alternative assumption would lead to a more complex model, but it would not change the qualitative results of our analysis.

$\sigma_{\tau,w}) \log(1 + \tau_w) + \sigma_{\tau,w} \log(1 + \tau_{w,t})$, $\log(1 + \delta_t^\sigma) = \sigma_\delta \cdot \log(1 + \delta_t)$, $\log z_{h,t}^\sigma = (1 - \sigma_z) \log z_{h,0} + \sigma_z \log z_{h,t}$, and $\log(1 + i_t^\sigma) = (1 - \sigma_i) \log(1 + i) + \sigma_i \log(1 + i_t)$. We assume that $(1 + \tau_p)^{\frac{\varepsilon-1}{\varepsilon}} = (1 + \tau_w)^{\frac{\psi-1}{\psi}} = 1$, so that the steady state is efficient. We also assume that the borrowing constraint is slack for all individuals. We assume that i is such that the steady state has zero inflation, i.e., $i = \beta^{-1} - 1$. Note that our approximation preserves the cross-sectional distribution of labor income, which is crucial for capturing the efficiency implications of cyclical heterogeneity. We also assume that the borrowing constraint is slack for all individuals, i.e., for all $\mathbf{b}_{-1} \equiv (a_{h,-1}, \{\tilde{a}_{k,h,-1}\})$ we let $\Omega^\sigma(\mathbf{b}_{-1}) = \Omega^\sigma(\mathbf{b}_{-1}) + (1 - \sigma_a)\mathbf{r}$ for a small $\mathbf{r} \gg 0$. This assumption ensures that, in the limit economy, keeping the original portfolio of assets is feasible and strictly satisfies the borrowing constraint.

Let $\sigma = \{\sigma_Z, \sigma_{\tau,p}, \sigma_{\tau,w}, \sigma_\delta, \sigma_z, \sigma_a\}$. When $\sigma = 1$, we have the full original model. We approximate the economy around $\sigma = 0$ and evaluate at $\sigma = 1$. At $\sigma = 0$, the equilibrium is efficient and all wedges are zero.

We denote by dx_t a variable deviation from its steady-state value and by $\hat{x}_t = d \log x_t$ the log deviation between $\sigma = 1$ and $\sigma = 0$.

A.4.1 The price new Keynesian Phillips curve Equation (8)

The firm's optimal reset price can be written as

$$\frac{p_t^*}{P_{t-1}} = \frac{\varepsilon}{\varepsilon - 1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} \tilde{Q}_{t,t+k} \theta_p^k \left(\frac{P_{t+k}}{P_{t-1}}\right)^{1-\varepsilon} Y_{t+k} \frac{W_{t+k}}{Z_{t+k} P_{t+k}}}{\mathbb{E}_t \sum_{k=0}^{\infty} \frac{(1+\tau_{p,t+k})}{\prod_{j=0}^{k-1} (1+i_{t+j})} \theta_p^k \left(\frac{P_{t+k}}{P_{t-1}}\right)^\varepsilon Y_{t+k}}.$$

Let $MC_{t+k} \equiv \frac{W_{t+k}}{Z_{t+k} P_{t+k}}$ be the real marginal cost. Log-linearizing around the efficiency steady-state allocation, we obtain

$$\begin{aligned} \hat{p}_t^* - \hat{p}_{t-1} &= (1 - \beta\theta_p) \sum_{k=0}^{\infty} (\beta\theta_p)^k \mathbb{E}_t [(1 - \varepsilon)(\hat{p}_{t+k} - \hat{p}_{t-1}) + \hat{y}_{t+k} + \hat{m}c_{t+k}] \\ &\quad - (1 - \beta\theta_p) \sum_{k=0}^{\infty} (\beta\theta_p)^k \mathbb{E}_t [\hat{\tau}_{p,t+k} + \varepsilon(\hat{p}_{t+k} - \hat{p}_{t-1}) + \hat{y}_{t+k}] \end{aligned}$$

where $\hat{m}c_{t+k} = d \log MC_{t+k} = \hat{w}_t - \hat{z}_t$, $\hat{w}_t = d \log(W_t/P_t)$ and $\hat{z}_t \equiv d \log Z_t$, $\hat{p}_t^* = d \log p_t^*$, $\hat{p}_t = d \log P_t$, $\hat{\tau}_{p,t} = d \log(1 + \tau_{p,t})$, and $\hat{y}_t = d \log Y_t$. Rearranging terms and defining $\pi_t \equiv \hat{p}_t - \hat{p}_{t-1}$, we obtain

$$\hat{p}_t^* - \hat{p}_{t-1} = (1 - \beta\theta) \sum_{k=0}^{\infty} (\beta\theta_p)^k \mathbb{E}_t [\hat{m}c_{t+k}] + \sum_{k=0}^{\infty} (\beta\theta_p)^k \mathbb{E}_t [\pi_{t+k}] - (1 - \beta\theta_p) \sum_{k=0}^{\infty} (\beta\theta_p)^k \mathbb{E}_t [\hat{\tau}_{p,t+k}].$$

Now, we use the fact that in the Calvo model, the price level is given by

$$P_t^{1-\varepsilon} = \theta_p P_{t-1}^{1-\varepsilon} + (1 - \theta_p)(p_t^*)^{1-\varepsilon} \Rightarrow \pi_t = (1 - \theta_p)(\hat{p}_t^* - \hat{p}_{t-1}).$$

Combining the last two equations, we obtain

$$\begin{aligned} \pi_t = & (1 - \beta\theta_p)(1 - \theta_p) \sum_{k=0}^{\infty} (\beta\theta_p)^k \mathbb{E}_t [\hat{m}c_{t+k}] + (1 - \theta_p) \sum_{k=0}^{\infty} (\beta\theta_p)^k \mathbb{E}_t [\pi_{t+k}] \\ & - (1 - \beta\theta_p)(1 - \theta_p) \sum_{k=0}^{\infty} (\beta\theta_p)^k \mathbb{E}_t [\hat{\tau}_{p,t+k}], \end{aligned}$$

which we can write recursively as

$$\pi_t = \frac{(1 - \beta\theta_p)(1 - \theta_p)}{\theta_p} \hat{m}c_t + \beta \mathbb{E}_t [\pi_{t+1}] - \frac{(1 - \beta\theta_p)(1 - \theta_p)}{\theta_p} \hat{\tau}_{p,t}.$$

Defining $\lambda_p \equiv \frac{(1 - \beta\theta_p)(1 - \theta_p)}{\theta_p}$ and $u_{p,t} \equiv -\lambda_p \hat{\tau}_{p,t}$, we obtain the Price New Keynesian Phillips Curve:

$$\pi_t = \lambda_p (\hat{w}_t - \hat{z}_t) + \beta \mathbb{E}_t [\pi_{t+1}] + u_{p,t}.$$

A.4.2 The wage new Keynesian Phillips curve Equation (9)

The union's optimal reset wage can be written as

$$\frac{w_t^*}{W_{t-1}} = \frac{\psi}{\psi - 1} \frac{\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \nu (L_{t+k})^{-1} \left(\frac{W_{t-1}}{W_{t+k}} \right)^{-\psi} N_{t+k}}{\mathbb{E}_t \sum_{k=0}^{\infty} (\beta\theta_w)^k (C_{t+k})^{-1} (1 + \tau_{w,t+k}) \frac{P_{t-1}}{P_{t+k}} \frac{W_{t-1}}{P_{t-1}} \left(\frac{W_{t-1}}{W_{t+k}} \right)^{-\psi} N_{t+k}}$$

where $N_{t+k} = \int y_{i,t+k} / Z_{t+k} di$ is aggregate labor supply. Let $MRS_{t+k}^w \equiv \frac{\nu(L_{t+k})^{-1}}{(C_{t+k})^{-1} W_{t+k} / P_{t+k}}$ denote the ratio of the marginal rate of substitution to the real wage. Log-linearizing

around the efficiency steady-state allocation, we obtain

$$\begin{aligned}\hat{w}_t^* - \hat{w}_{t-1}^{nom} &= (1 - \beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k \mathbb{E}_t \left[-\hat{l}_{t+k} + \psi(\hat{w}_{t+k}^{nom} - \hat{w}_{t-1}^{nom}) + \hat{n}_{t+k} \right] \\ &\quad - (1 - \beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k \mathbb{E}_t \left[-\hat{y}_{t+k} + \hat{\tau}_{w,t+k} - (\hat{p}_{t+k} - \hat{p}_{t-1}) + (\hat{w}_{t-1}^{nom} - \hat{p}_{t-1}) \right. \\ &\quad \left. + \psi(\hat{w}_{t+k}^{nom} - \hat{w}_{t-1}^{nom}) + \hat{n}_{t+k} \right]\end{aligned}$$

where $\hat{w}_t^* = d \log w_t^*$, $\hat{w}_t^{nom} = d \log W_t$, $\hat{l}_t = d \log L_t$, $\hat{n}_t = d \log N_t$, and $\hat{\tau}_{w,t} = d \log(1 + \tau_{w,t})$. Note that the $\psi(\hat{w}_{t+k}^{nom} - \hat{w}_{t-1}^{nom})$ and $\hat{n}_{t+k}$ terms cancel between numerator and denominator. After cancelling these terms, using $\hat{w}_{t-1}^{nom} - \hat{p}_{t-1} = \hat{w}_{t-1}$, and collecting terms using $m\hat{r}s_{t+k}^w \equiv \hat{y}_{t+k} - \hat{l}_{t+k} - \hat{w}_{t+k}$, we can write the difference as

$$\hat{w}_t^* - \hat{w}_{t-1}^{nom} = (1 - \beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k \mathbb{E}_t [m\hat{r}s_{t+k}^w + (\hat{w}_{t+k}^{nom} - \hat{w}_{t-1}^{nom})] - (1 - \beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k \mathbb{E}_t [\hat{\tau}_{w,t+k}]$$

where $m\hat{r}s_{t+k}^w \equiv d \log MRS_{t+k}^w = \hat{y}_{t+k} - \hat{l}_{t+k} - \hat{w}_{t+k}$. Defining $\pi_{w,t} \equiv \hat{w}_t^{nom} - \hat{w}_{t-1}^{nom}$ and noting that $(1 - \beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k (\hat{w}_{t+k}^{nom} - \hat{w}_{t-1}^{nom}) = \sum_{k=0}^{\infty} (\beta\theta_w)^k \pi_{w,t+k}$, we obtain

$$\hat{w}_t^* - \hat{w}_{t-1}^{nom} = (1 - \beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k \mathbb{E}_t [m\hat{r}s_{t+k}^w] + \sum_{k=0}^{\infty} (\beta\theta_w)^k \mathbb{E}_t [\pi_{w,t+k}] - (1 - \beta\theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k \mathbb{E}_t [\hat{\tau}_{w,t+k}].$$

Now, we use the fact that in the Calvo model, the wage index is given by

$$W_t^{1-\psi} = \theta_w W_{t-1}^{1-\psi} + (1 - \theta_w)(w_t^*)^{1-\psi} \Rightarrow \pi_{w,t} = (1 - \theta_w)(\hat{w}_t^* - \hat{w}_{t-1}^{nom}).$$

Combining the last two equations, we obtain

$$\begin{aligned}\pi_{w,t} &= (1 - \beta\theta_w)(1 - \theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k \mathbb{E}_t [m\hat{r}s_{t+k}^w] + (1 - \theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k \mathbb{E}_t [\pi_{w,t+k}] \\ &\quad - (1 - \beta\theta_w)(1 - \theta_w) \sum_{k=0}^{\infty} (\beta\theta_w)^k \mathbb{E}_t [\hat{\tau}_{w,t+k}],\end{aligned}$$

which we can write recursively as

$$\pi_{w,t} = \frac{(1 - \beta\theta_w)(1 - \theta_w)}{\theta_w} m\hat{r}s_t^w + \beta \mathbb{E}_t [\pi_{w,t+1}] - \frac{(1 - \beta\theta_w)(1 - \theta_w)}{\theta_w} \hat{\tau}_{w,t}.$$

Defining $\lambda_w \equiv \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w}$ and $u_{w,t} \equiv -\lambda_w \hat{\tau}_{w,t}$, we obtain

$$\pi_{w,t} = \lambda_w m\hat{r}s_t^w + \beta \mathbb{E}_t[\pi_{w,t+1}] + u_{w,t}.$$

To express $m\hat{r}s_t^w$ in terms of the output gap and the real wage gap, note that with log utility, $\hat{l}_t = -\frac{1-L}{L}(\hat{y}_t - \hat{z}_t) = -\nu^{-1}(\hat{y}_t - \hat{z}_t)$, where $\nu \equiv L/(1-L)$ is the Frisch elasticity of labor supply. This follows from $Y_t = Z_t(1-L_t)$ and $C_t = Y_t$. Furthermore, in the efficient allocation, leisure is constant at $L^* = \nu/(1+\nu)$, so that $\hat{y}_t^* = \hat{z}_t$. Therefore,

$$\begin{aligned} m\hat{r}s_t^w &= \hat{y}_t + \nu^{-1}(\hat{y}_t - \hat{z}_t) - \hat{w}_t = (1 + \nu^{-1})\hat{y}_t - \nu^{-1}\hat{y}_t^* - \hat{w}_t \\ &= \left\{1 + \nu^{-1}\right\}(\hat{y}_t - \hat{y}_t^*) - (\hat{w}_t - \hat{z}_t). \end{aligned}$$

Substituting, we obtain the Wage New Keynesian Phillips Curve:

$$\pi_{w,t} = \lambda_w \left(\left\{1 + \nu^{-1}\right\}(\hat{y}_t - \hat{y}_t^*) - (\hat{w}_t - \hat{z}_t) \right) + \beta \mathbb{E}_t[\pi_{w,t+1}] + u_{w,t}$$

A.4.3 The law of motion for the real wage Equation (35)

The real wage is defined as $\hat{w}_t \equiv d \log(W_t/P_t) = \hat{w}_t^{nom} - \hat{p}_t$. Therefore,

$$\hat{w}_t - \hat{w}_{t-1} = (\hat{w}_t^{nom} - \hat{w}_{t-1}^{nom}) - (\hat{p}_t - \hat{p}_{t-1}) = \pi_{w,t} - \pi_t,$$

which gives the law of motion for the real wage:

$$\hat{w}_t = \hat{w}_{t-1} + \pi_{w,t} - \pi_t$$

A.4.4 Aggregate demand Equation (36)

The aggregate demand of the economy is given by

$$C_t = \int c_{h,t} dh. \quad (31)$$

As in Angeletos et al. (2025), we can represent the aggregate consumption function by a functional $\mathcal{C}$ such that

$$C_t = \mathcal{C} \left(\left\{ \mathbb{E}_s[\Theta_{s+h}] \right\}_{h=0}^{\infty} \right)_{s=-\infty}^t, \quad (32)$$

where $\Theta_t \equiv \{L_t, Y_t, i_t, \delta_t, P_t/P_{t-1}, \{q_k\}, \{\{z_{h,t}\}\}\}$, for $t \geq 1$, and

$\Theta_0 \equiv \{L_0, Y_0, i_0, \delta_0, P_0/P_{-1}, \{q_k\}, \{T_h, a_{h,0}, \{z_{h,0}\}\}\}$ are all the variables that enter the consumption block of the model. Importantly, $\mathbb{E}_s[\Theta_{s+h}] = \Theta$ are constant at steady state for

$s < 0$. Linearizing around the steady state, we represent by $d\Theta_t$ the deviation of each variable in Θ_t from its steady state value, induced by the deviation of the exogenous variables from their steady state values.

The dependence of aggregate demand on current fundamentals and current expectations is immediate from the dependence of current decisions on people's current expectations. The additional past expectations capture the influence of the endogenous state variables (e.g., asset distribution) on past behavior and therefore on past fundamentals and past expectations.

Note that we write the consumption function as a functional of the point expectations directly, disregarding uncertainty. Since we focus on linearized equilibrium conditions, where certainty equivalence holds, this is without loss of generality.

Following Angeletos et al. (2025) for properties of this functional, we can write the linearized aggregate demand as:

$$\mathbb{E}_t[d\mathbf{C}_{t:\infty}] = \mathbb{E}_{t-1}[d\mathbf{C}_{t:\infty}] + \sum_{\Theta} \mathbf{M}^{\Theta} (\mathbb{E}_t[d\Theta_{t:\infty}] - \mathbb{E}_{t-1}[d\Theta_{t:\infty}]).$$

A.4.5 Log-linearized equilibrium

We now briefly characterize the log-linearized equilibrium conditions in the economy.

The log-linearized Phillips curves for price and wage inflation are standard and given, in log deviations, by

$$\pi_t = \lambda_p (\hat{w}_t - \hat{z}_t) + \beta \mathbb{E}_t[\pi_{t+1}] + u_{p,t} \quad (33)$$

$$\pi_{w,t} = \lambda_w \left(\left\{ 1 + \nu^{-1} \right\} (\hat{y}_t - \hat{y}_t^*) - (\hat{w}_t - \hat{z}_t) \right) + \beta \mathbb{E}_t[\pi_{w,t+1}] + u_{w,t}, \quad (34)$$

where $\hat{y}_t^*$ is the efficient level of output, $\hat{z}_t$ is the aggregate technology shock, and $u_{p,t}$ and $u_{w,t}$ are the cost-push shocks to prices and wages, respectively. The parameters λ_p and λ_w capture the slope of the price and wage Phillips curves, respectively, and depend on the degree of nominal rigidity in prices and wages.

As is standard, the combination of price and wage stickiness makes the real wage a state variable, with dynamics given by

$$\hat{w}_t = \hat{w}_{t-1} + \pi_{w,t} - \pi_t. \quad (35)$$

We can represent the aggregate demand ($C_t = \int c_{i,t} di$) block of the economy as a functional of the sequence of expectations of the fundamentals and policy variables. We leverage the sequence-space Jacobian approach of Auclert et al. (2021, 2024) to write the

aggregate demand block as a linear combination of the sequence of expectations of the fundamentals and policy variables, as follows. Let $d\mathbf{C}_{t:\infty} = [dC_t, dC_{t+1}, \dots]$ be the vector of changes in consumption over time. Then,

$$\mathbb{E}_t[d\mathbf{C}_{t:\infty}] = \mathbb{E}_{t-1}[d\mathbf{C}_{t:\infty}] + \sum_{\Theta} \mathbf{M}^{\Theta} (\mathbb{E}_t[d\Theta_{t:\infty}] - \mathbb{E}_{t-1}[d\Theta_{t:\infty}]), \quad (36)$$

given $\mathbb{E}_{-1}[d\mathbf{C}_{t:\infty}] = \mathbb{E}_{-1}[d\Theta_{t:\infty}] = 0$. Here, $\mathbf{M}^{\Theta}$ is the sequence-space Jacobian associated with input of the household block Θ . In equilibrium, $dC_t = dY_t$. Collectively, these equations pin down the way equilibrium variables respond to exogenous shocks.

Appendix B Characterization of the status-quo misallocation

We now provide an analytical characterization of the status-quo misallocation term.

Proposition 9 (Status-quo misallocation). *The status-quo misallocation, defined as the distance to the Pareto efficient frontier, is given by*

$$\log \mathcal{D} = U(\{C_t^*, L_t^*\}) - \log \int \bar{z}_h^{\frac{v}{1+v}} \exp\{U(\mathbf{x}_h^0)\} dh. \quad (37)$$

Here, $\log \bar{z}_h \equiv (1 - \beta) \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \log z_{h,t}$. We denote the frontier aggregate allocations with superscript $*$ to highlight the fact that they are the “potential levels” in our economy, i.e., they are the levels that would obtain in the absence of any distortions.

The notion of status-quo misallocation is also useful in characterizing the second-order approximation of aggregate surplus. This follows from the fact that it is useful to decompose aggregate surplus into the status-quo misallocation and the gap that still remains open relative to the frontier under a given policy $\log A(\mathcal{P}) = \log \mathcal{D} + \log A(\mathcal{P})/\mathcal{D}$. To see this fact in practice, we return to the RANK example above.

B.1 Proof of Proposition 9

In the technological frontier, the marginal rates of intertemporal substitution are equalized across households and the marginal rates of substitution between consumption and leisure divided by idiosyncratic productivity are equalized across households. It follows

that there exists $\omega_h \geq 0$ such that

$$c_{h,t} = \omega_h C_t, \quad \text{and} \quad l_{h,t} = \frac{\omega_h L_t}{z_{h,t}}.$$

and $\int \omega_h dh = 1$.

Furthermore, these optimality conditions, together with the production function imply that the frontier allocations satisfy the following conditions:

$$\frac{\nu C_t}{L_t} = Z_t, \quad \text{and} \quad Y_t = C_t = Z_t(1 - L_t).$$

Solving this system of equations implies (15).

Finally, as in Example 1, since preferences are homothetic, the frontier allocations can also be derived in scaled variables, which implies that

$$c_{h,t} = \omega_h \frac{C_t^*}{H}, \quad \text{and} \quad l_{h,t} = \frac{\omega_h L_t^*}{z_{h,t} H}.$$

Then, the utility of agent h is given by

$$U(\mathbf{x}_h^*/H) = \log \omega_h - \frac{\nu}{1+\nu} \log \bar{z}_h - \log H + U(\mathbf{X}^*),$$

where $\log \bar{z}_h \equiv (1 - \beta) \mathbb{E}_{-1} [\sum_{t=0}^{\infty} \beta^t \log z_{h,t}]$.

At $H = \mathcal{D}$, it must be that

$$U(\mathbf{x}_h^*/\mathcal{D}) = \log \omega_h - \frac{\nu}{1+\nu} \log \bar{z}_h - \log \mathcal{D} + U(\mathbf{X}^*) = U(\mathbf{x}_h^0)$$

for all h .

Solving these equations first for $\tilde{\omega}_h$ implies

$$\begin{aligned} \omega_h &= \frac{\bar{z}_h^{\frac{\nu}{1+\nu}} \exp\{U(\mathbf{x}_h^0)\}}{\exp\{U(\mathbf{X}^*)\}} \cdot \mathcal{D} \Rightarrow 1 = \int \omega_h dh = \frac{\int \bar{z}_h^{\frac{\nu}{1+\nu}} \exp\{U(\mathbf{x}_h^0)\} dh}{\exp\{U(\mathbf{X}^*)\}} \cdot \mathcal{D} \\ &\Leftrightarrow \log \mathcal{D} = U(\mathbf{X}^*) - \log \int \bar{z}_h^{\frac{\nu}{1+\nu}} \exp\{U(\mathbf{x}_h^0)\} dh. \end{aligned}$$

B.2 Further details on Example 1

In the RANK model, $\mathcal{X}(1/H, \mathcal{P})$ is a singleton for all H , since there is a unique equilibrium allocation for each policy, i.e., no transfers are possible since all agents are identical.

Then, note that, because preferences are homothetic, it is easy to show that the equi-

librium can be derived in scaled variables, i.e., for each variable x , we can write $x = H\tilde{x}$, where $\tilde{x}$ is the equilibrium value of the variable in the RANK model under policy $\mathcal{P}$ and $H = 1$.

It follows that the utility of the representative agent is $U(\mathbf{x}(1, \mathcal{P})/H)$. Then, aggregate surplus is given by:

$$A(\mathcal{P}) \equiv \max\{H > 0 : U(\mathbf{x}(1, \mathcal{P})/H) \geq U(\mathbf{x}^0)\}.$$

Since U is strictly increase in all arguments we can write

$$\begin{aligned} U(\mathbf{x}(1, \mathcal{P})/A(\mathcal{P})) = U(\mathbf{x}^0) &\Leftrightarrow U(\mathbf{x}(1, \mathcal{P})) - \log A(\mathcal{P}) = U(\mathbf{x}^0) \\ &\Leftrightarrow \log A(\mathcal{P}) = U(\mathbf{x}(1, \mathcal{P})) - U(\mathbf{x}^0). \end{aligned}$$

Note that, using the results of Proposition 9, we can write

$$\log A(\mathcal{P}) = U(\mathbf{x}(1, \mathcal{P})) - U(\mathbf{x}^0) = \log \mathcal{D} + [U(\mathbf{x}(1, \mathcal{P})) - U(\mathbf{X}^*)].$$

Then, to second order, $\log A(\mathcal{P})$ is equal to the difference between $\log \mathcal{D}$ and the second-order approximation of the loss $\mathcal{L} \equiv U(\mathbf{X}^*) - U(\mathbf{x}(1, \mathcal{P}))$. This loss $\mathcal{L}$ is the standard object approximated in Woodford (2003) for the case with only one source of nominal rigidity, or in Erceg et al. (2000) for the case with both price and wage rigidities. So, following the standard approximation procedure, we obtain that

$$\mathcal{L} \equiv \frac{(1-\beta)\omega_C}{2} \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \left[(1+\nu^{-1})(\hat{y}_t - \hat{y}_t^*)^2 + \frac{\varepsilon}{\lambda_p} \pi_t^2 + \frac{\psi}{\lambda_w} \pi_{w,t}^2 \right].$$

Appendix C Appendix to Section 4

C.1 A constructive proof of the second-order approximation

Wedges and the sources of inefficiency. Before turning to the analysis of optimal monetary policy, it is useful to emphasize the sources of inefficiency in the economy. To identify the sources of inefficiencies, we follow Baqaee and Burstein (2025a), and consider an alternative Arrow-Debreu economy with wedges that decentralizes the same allocations. This step is also key in analyzing optimal monetary policy, since it allows us to translate the problem into the general setting developed in Baqaee et al. (2026).

We use four types of wedges to decentralize the equilibrium allocation in Arrow-Debreu: (1) a price-markup wedge to capture nominal price rigidity, (2) a wage-markup wedge to capture nominal wage rigidity, (3) a labor-wedge to capture the incidence function Γ , and (4) household-specific consumption wedges to capture incomplete financial markets.

First, the presence of sticky prices introduces heterogeneity in markups across firms due to the differences in the time since their last price adjustment. This heterogeneity leads to an inefficient allocation of production across firms. The price-markup wedges, denoted by $\mu_{i,t}^p$, are given by the gap between prices and marginal costs:

$$\log \mu_{i,t}^p \equiv \log p_{i,t} - \log \frac{W_t}{Z_t}. \quad (38)$$

Second, the presence of sticky wages introduces heterogeneity in wages across unions also due to differences in the time since their last wage adjustment. These wedges create an inefficient allocation of labor across unions. We denote the wedge in wages by $\mu_{u,t}^w$, and define them to be

$$\log \mu_{u,t}^w \equiv \log w_{u,t} - \log \tilde{W}_t, \quad (39)$$

where $\tilde{W}_t \equiv W_t^{1-\psi} \cdot \left(\int w_{u,t}^{-\psi} du \right)^{-1}$ is the average wage per effective labor unit.

Third, the incidence function Γ can also create an inefficient allocation of labor across households captured via household-specific labor wedges. Household h 's labor wedge, denoted by $\mu_{h,t}^L$, satisfies

$$\log \mu_{h,t}^L \equiv \log \left(\frac{\nu c_{h,t}}{z_{h,t} l_{h,t}} \right) - \log \frac{\tilde{W}_t}{P_t}. \quad (40)$$

Jointly, the price, wage, and individual labor wedges also imply the standard aggregate labor wedge present in NK models. The wage and labor wedges in this appendix use

the average paid wage as their reference. To distinguish them from the definitions in Section 4, denote the latter by $\mu_{u,t}^{w,\text{main}}$ and $\mu_{h,t}^{L,\text{main}}$. With $B_t \equiv \tilde{W}_t L_t / (P_t C_t)$, the exact relation is

$$\mu_{u,t}^{w,\text{main}} = B_t \mu_{u,t}^w, \quad \mu_{h,t}^{L,\text{main}} = \frac{B_t}{\nu} \mu_{h,t}^L.$$

For the wedge-induced quantity changes used below, idiosyncratic productivity is held fixed. The common change in $\log B_t$ reallocates the aggregate labor-wedge contribution between the wage and labor triangles, leaving their sum unchanged. Wedge changes are measured relative to the efficient reference, so the constant normalization by ν has no effect.

Finally, as in Baqaee and Burstein (2025b), the presence of incomplete markets and financial frictions introduces inefficiencies in households' allocation of consumption over time and across idiosyncratic states. We denote the consumption wedge of household h by $\mu_{h,t}^{IM}$, and they satisfy

$$\log \mu_{h,t}^{IM} \equiv \log \left(\frac{(c_{h,t})^{-1}}{(C_t)^{-1}} \right) - \log \left(\frac{(c_{h,0})^{-1}}{(C_0)^{-1}} \right). \quad (41)$$

In Appendix C.2, we show that the equilibrium allocation in our economy can be decentralized as an Arrow-Debreu equilibrium with the wedges defined above. In particular, we show that there exists a set of transfers such that the equilibrium allocation in our economy is supported by an AD equilibrium with these wedges. This result is key for our analysis of optimal monetary policy, since it allows us to leverage the general results in Baqaee et al. (2026) to characterize the approximately optimal policy.

Together, these four sets of wedges capture the sources of inefficiency in the economy. These wedges are endogenous and a result both of the underlying fundamentals and monetary policy. They are the key objects that the central bank needs to address to maximize aggregate surplus. For instance, pursuing a policy of inflation targeting reduces price-markup heterogeneity across firms, which is a key source of inefficiency in the product market. Pursuing a policy of wage inflation targeting reduces wage-markup heterogeneity, which is a key source of inefficiency in the labor market. Pursuing a policy that stabilizes output can affect the distribution of individual labor wedges across households, which are a key source of inefficiency in the allocation of labor across households. Finally, pursuing a policy that affects the distribution of income across households can affect the consumption wedges, which is a key source of inefficiency due to incomplete financial markets.

Deriving the approximate aggregate surplus. To derive the second-order approximation of aggregate surplus, we use the general results in Baqaee et al. (2026), who show how to characterize the approximate optimal policy in a general equilibrium model with endogenous wedges. For this, it is useful to first characterize the approximated aggregate surplus in terms of the wedges defined above, and then to rewrite the resulting expression in a more interpretable way. The next proposition characterizes the second-order approximation of aggregate surplus in terms of the Harberger (1964) triangles associated with each wedge (i.e., wedge times the distortion in the quantity divided by two).

Lemma 1 (Aggregate surplus as Harberger triangles). *To a second-order approximation, aggregate surplus can be written as*

$$\log A(\mathcal{P}) \approx \log \mathcal{D} + \triangleright^{IM} + \triangleright^p + \triangleright^w + \triangleright^L, \quad (42)$$

where

i. *The losses from incomplete markets are given by*

$$\triangleright^{IM} \equiv (1 - \beta) \mathbb{E}_{-1} \sum_t \beta^t \int_0^1 \chi_h \cdot \frac{d \log \mu_{h,t}^{IM} \cdot \partial \log x_{h,t}}{2} dh, \quad (43)$$

where $\log x_{h,t} \equiv \left[\frac{1}{1+\nu} \log c_{h,t} + \frac{\nu}{1+\nu} \log l_{h,t} \right]$ is the effective consumption-leisure composite of household h in period t and χ_h denotes the consumption share of household h .

ii. *The losses from price markups are given by*

$$\triangleright^p \equiv (1 - \beta) \omega_C \cdot \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \int_0^1 \frac{d \log \mu_{i,t}^p \cdot \partial \log y_{i,t}}{2} di, \quad (44)$$

where $\omega_C \equiv C / (wL + C)$ is the market consumption share of the aggregate time endowment.

iii. *The losses from wage markups are given by*

$$\triangleright^w \equiv (1 - \beta) \omega_C \cdot \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \int_0^1 \frac{d \log \mu_{u,t}^w \cdot \partial \log n_{u,t}^d}{2} du. \quad (45)$$

iv. *The losses from the person-specific labor wedges are given by*

$$\triangleright^L \equiv (1 - \beta)(1 - \omega_C) \cdot \mathbb{E}_{-1} \sum_t \beta^t \int_0^1 \chi_h \cdot \frac{d \log \mu_{h,t}^L \cdot \partial \log l_{h,t}}{2} dh. \quad (46)$$

where all partial derivatives, $\partial \log x_{h,t}$, $\partial \log y_{i,t}$, $\partial \log n_{u,t}^d$, and $\partial \log l_{h,t}$, denote the changes in the corresponding quantities, around the efficient allocation, caused only by the wedges in the Arrow-Debreu general equilibrium with compensating transfers. In contrast, we use $d \log \mu_{h,t}^{IM}$, $d \log \mu_{i,t}$, $d \log \mu_{u,t}^w$, and $d \log \mu_{h,t}^L$ to denote changes in wedges in the decentralized equilibrium due to all shocks.

Lemma 1 shows that, to a second-order approximation, aggregate surplus can be written as the sum of four terms, each of which captures the losses from a different source of inefficiency in the economy. Each term is given by the expected discounted sum of the product of the change in the corresponding wedge and the change in the corresponding quantity. Hence, each term can be interpreted as a general-equilibrium analogue of a deadweight-loss triangle, which captures the efficiency losses from a wedge in a partial equilibrium setting.

Lemma 1 relies on translating the results in Baqaee and Farhi (2024) and Baqaee and Burstein (2025a) to our setting. This result is the key step in characterizing the optimal policy, since it allows us to express aggregate surplus in terms of a set of interpretable wedges. In particular, by setting monetary policy, the central bank can affect the deadweight-loss triangles. In this way, the problem of the efficiency-mandate central bank is to choose policy to minimize the losses associated with the deadweight-loss triangles.

Our next set of results characterize how the different wedges respond to changes in aggregate variables, and ultimately monetary policy. In the main text, we provide some key equations that guide the intuition for the results which are derived in full detail in Appendix C.

In equilibrium, the change in $y_{i,t}$ and $n_{u,t}^d$ caused by the wedges satisfy

$$\begin{aligned}\partial \log y_{i,t} &= -\varepsilon \left(d \log \mu_{i,t}^p - \int d \log \mu_{i',t}^p di' \right) + \partial \log Y_t, \\ \partial \log n_{u,t}^d &= -\psi \left(d \log \mu_{u,t}^w - \int d \log \mu_{u',t}^w du' \right) + \partial \log Y_t,\end{aligned}$$

where $\partial \log Y_t$ is the response of aggregate output to wedges only. We can write this, in more familiar terms, as

$$\partial \log Y_t = d \log Y_t - d \log Y_t^* = (\hat{y}_t - \hat{y}_t^*),$$

which is the gap between output and its efficient level. Using these expressions, we can

rewrite the deadweight-loss triangles for price and wage markups using the fact that

$$\begin{aligned}\int_0^1 d \log \mu_{i,t}^p \cdot \partial \log y_{i,t} di &= -\varepsilon \text{Var}_i(d \log \mu_{i,t}^p) + E_i[d \log \mu_{i,t}^p] \partial \log Y_t, \\ \int_0^1 d \log \mu_{u,t}^w \cdot \partial \log n_{u,t}^d du &= -\psi \text{Var}_u(d \log \mu_{u,t}^w) + E_u[d \log \mu_{u,t}^w] \partial \log Y_t,\end{aligned}$$

where $\text{Var}_i(d \log \mu_{i,t}^p)$ and $\text{Var}_u(d \log \mu_{u,t}^w)$ denote the cross-sectional variance of price and wage markups across firms and unions, respectively, $E_i[d \log \mu_{i,t}^p]$ and $E_u[d \log \mu_{u,t}^w]$ denote the cross-sectional average of price and wage markups across firms and unions, respectively. Using these expressions, we obtain the following result.

Lemma 2 (Characterizing the deadweight-loss triangles for price and wage markups). *The Harberger triangles for price and wage markups can be written as*

$$\triangleright^p = \frac{(1-\beta)\omega_C}{2} \cdot \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \left[-\frac{\varepsilon}{\lambda_p} \pi_t^2 + E_i[d \log \mu_{i,t}^p] \partial \log Y_t \right]$$

and

$$\triangleright^w = \frac{(1-\beta)\omega_C}{2} \cdot \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \left[-\frac{\psi}{\lambda_w} \pi_{w,t}^2 + E_u[d \log \mu_{u,t}^w] \partial \log Y_t \right]$$

respectively, where π_t and $\pi_{w,t}$ are the price and wage inflation rates, respectively, and $\lambda_p \equiv (1-\beta\theta_p)(1-\theta_p)/\theta_p$ and $\lambda_w \equiv (1-\beta\theta_w)(1-\theta_w)/\theta_w$ are the price and wage Phillips curve slopes, respectively.

The proof for this result uses the well known fact that, with Calvo pricing rigidities, the variance of markups across firms (unions) is proportional to the square of accumulated inflation (wage inflation) appropriately weighed by the probability of price (wage) adjustment. Hence, the first term in the expression for $\triangleright^p$ and $\triangleright^w$ captures the familiar losses from price and wage dispersion caused by inflation, while the second term captures the losses from the average level of markups across firms and unions, which also make up the average labor wedge.

Turning to the person-specific labor wedges, we know that, in equilibrium, the change in $l_{h,t}$ is given by

$$\partial \log l_{h,t} = (1 + \gamma_h) \partial \log L_t = -(1 + \gamma_h) \frac{\omega_C}{1 - \omega_C} \partial \log Y_t.$$

where $\gamma_h \equiv \frac{\partial \log \Gamma(1-L, \zeta_{h,0})}{\partial \log(1-L)}$ is the incidence elasticity of aggregate labor fluctuations. Using this expression, we can rewrite the deadweight-loss triangle for person-specific labor

wedges as

$$\triangleright^L \equiv -\frac{(1-\beta)\omega_C}{2} \cdot \mathbb{E}_0 \sum_t \beta^t \left\{ E_{\chi_h} \left[d \log \mu_{h,t}^L \right] \partial \log Y_t + Cov_{\chi_h} \left(d \log \mu_{h,t}^L, \gamma_h \right) \partial \log Y_t \right\}$$

where $E_{\chi_h} \left[d \log \mu_{h,t}^L \right]$ and $Cov_{\chi_h} \left(d \log \mu_{h,t}^L, \gamma_h \right)$ denote the cross-sectional average and covariance of the labor wedges across households weighted by the consumption share χ_h , respectively. The first term captures the losses from the average level of labor wedges across households, which also make up the average labor wedge, while the second term captures the losses from the heterogeneity in labor wedges across households, which is a key feature of HANK models.

Lemma 3 (Characterizing the deadweight-loss triangles for person-specific labor wedges). *The deadweight-loss triangle for person-specific labor wedges can be written as*

$$\begin{aligned} \triangleright^L = & -\frac{\omega_C(1-\beta)}{2} \left[\mathbb{E}_0 \sum_t \beta^t \left[\left[(1+\nu^{-1}) \partial \log Y_t + E_i[d \log \mu_{i,t}] + E_u[d \log \mu_{u,t}] \right] \partial \log Y_t \right. \right. \\ & \left. \left. + \nu^{-1} Var_{\chi_h}(\gamma_h) (\partial \log Y_t)^2 \right] + Cov_{\chi_h}(\gamma_h, \gamma_h^Y) \cdot (1-\beta) \mathbb{E}_0 \left(\sum_t \beta^t \partial \log Y_t \right)^2 \right]. \quad (47) \end{aligned}$$

The terms in the first row capture the effect of the average labor wedge, $E_{\chi_h}[d \log \mu_{h,t}^L]$, on aggregate surplus. The heterogeneous terms have two distinct incidence statistics. The contemporaneous output-gap square uses $Var_{\chi_h}(\gamma_h)$ because γ_h determines the physical assignment of labor. The present-value term uses $Cov_{\chi_h}(\gamma_h, \gamma_h^Y)$ because it pairs that physical assignment with the lifetime-wealth effect of total-income incidence.

Denote the lifetime discounted expectation operator, which weighs dates and states by $(1-\beta)\beta^t\pi(s)$, using $E_{\beta,\pi}[\cdot]$ and let $Cov_{\beta,\pi}[\cdot]$ and $Var_{\beta,\pi}[\cdot]$ denote the associated covariance and variance operators. Then we can rewrite the deadweight-loss triangle for incomplete markets, solving out for $\partial \log x_{h,t}$, as

$$\triangleright^{IM} = -\frac{1}{2} E_{\chi_h} \left[Var_{\beta,\pi} \left(d \log \left(\mu_{h,t}^{IM} \right) \right) - (1-\omega_C) Cov_{\beta,\pi} \left(d \log \mu_{h,t}^{IM}, d \log \mu_{h,t}^L \right) \right].$$

Hence, the losses scale with the cross-sectional expectation of the discounted life-time variance and covariance of incomplete markets and labor wedges. We now use the expressions for wedges in the decentralized equilibrium to rewrite this expression in terms of the deviations of quantities from their efficient allocations.

Lemma 4 (Characterizing the deadweight-loss triangles for incomplete-market wedges). *The deadweight-loss triangles for incomplete markets can be written as*

$$\triangleright^{IM} = -\frac{1}{2}E_{\chi_h} \left[\text{Var}_{\beta, \pi} \left(d \log \left(\frac{c_{h,t}}{Y_t} \right) \right) \right] + \frac{(1-\beta)\omega_C}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \text{Cov}_{\chi_h} \left(d \log \frac{c_{h,t}}{\bar{c}_h}, \gamma_h \right) \partial \log Y_t.$$

where $d \log c_{h,t}$ represents the log-deviation of household h 's consumption from that in the efficient allocation induced by the wedges, and $d \log \bar{c}_h = (1-\beta)\mathbb{E}_{-1} \sum_t \beta^t d \log c_{h,t}$ is the average lifetime consumption of household h .

This lemma decomposes the losses from incomplete markets into two components. The first component captures the losses from incomplete markets driven by idiosyncratic income risk, which is given by the average life-time volatility of each households' share of aggregate consumption. The second component captures the losses from incomplete markets driven by aggregate shocks, which is given by the covariance between the consumption of household h relative to its average lifetime consumption, $d \log \left(\frac{c_{h,t}}{\bar{c}_h} \right)$, and the incidence elasticity of aggregate labor fluctuations, γ_h , weighted by the change in aggregate output relative to the efficient allocation, $\partial \log Y_t = (\hat{y}_t - \hat{y}_t^*)$. The intuition for the second component is as follows. With incomplete markets, households over- and under-consume consumption goods, but also leisure, relative to the efficient allocation. By introducing an output gap, monetary policy can affect the distribution of labor and leisure in the cross section provided γ_h is heterogeneous across households. The covariance term therefore captures whether aggregate stabilization pushes labor effort toward households that are currently consuming relatively too much, and away from households that are currently consuming relatively too little. The explicit covariance continues to load on γ_h , because the distorted quantity is physical labor. Total-income incidence γ_h^Y affects this term indirectly through the equilibrium consumption path.

Combining these lemmas results in Proposition 1.

C.2 The Arrow-Debreu economy with wedges

We now show formally that the baseline equilibrium allocations can be implemented as the allocation of an Arrow-Debreu economy with wedges. Fix any equilibrium of the baseline economy and denote its allocation and prices by

$$\left\{ c_{h,t}, l_{h,t}, C_t, Y_t, y_{i,t}, n_{i,u,t}, n_{u,t}^d, P_t, p_{i,t}, W_t, w_{u,t}, \tilde{W}_t \right\}_{h,i,u,t}.$$

The Arrow-Debreu economy is static in the sense that all state-contingent commodities dated at all histories are traded at date 0. Let Q_t denote the Arrow-Debreu price of one unit of the final good in period t , normalized so that

$$Q_t = \beta^t \frac{(C_t)^{-1}}{P_t}.$$

Household h chooses $\{c_{h,t}, l_{h,t}\}_{t=0}^{\infty}$ to maximize Equation (1) subject to

$$\mathbb{E}_{-1} \sum_{t=0}^{\infty} Q_t \mu_{h,t}^{IM} \left(P_t c_{h,t} + \mu_{h,t}^L \tilde{W}_t z_{h,t} l_{h,t} \right) \leq \mathcal{W}_h + T_h, \quad (48)$$

where $\mathcal{W}_h$ is the value of household h 's endowments and profit claims in the Arrow-Debreu economy, and T_h is a time-0 transfer. The transfers are chosen to satisfy Equation (48) at equality for the baseline allocation; aggregating the household budget constraints and the firms' zero-profit conditions gives $\int T_h dh = 0$.

The wedges are chosen from the baseline equilibrium prices and allocations and are exactly those defined in Equation (38)–Equation (41). The price wedge makes each intermediate producer's price satisfy

$$p_{i,t} = \mu_{i,t}^p \frac{W_t}{Z_t}.$$

The wage wedge makes each union wage satisfy

$$w_{u,t} = \mu_{u,t}^w \tilde{W}_t,$$

where $\tilde{W}_t \equiv W_t^{1-\psi} \left(\int w_{u,t}^{-\psi} du \right)^{-1}$. The household labor wedge makes the household's marginal rate of substitution satisfy

$$\frac{v(l_{h,t})^{-1}}{(c_{h,t})^{-1}} = \mu_{h,t}^L \frac{z_{h,t} \tilde{W}_t}{P_t},$$

and the incomplete-markets wedge satisfies

$$\mu_{h,t}^{IM} = \frac{(c_{h,t})^{-1} / (C_t)^{-1}}{(c_{h,0})^{-1} / (C_0)^{-1}}.$$

We verify that the baseline allocation satisfies the equilibrium conditions of this Arrow-Debreu economy. Let λ_h be the multiplier on household h 's Arrow-Debreu

budget constraint and set

$$\lambda_h = \frac{1 - \beta}{1 + \nu} \frac{(c_{h,0})^{-1}}{(C_0)^{-1}}.$$

The first-order condition for consumption in the Arrow-Debreu economy is

$$\frac{1 - \beta}{1 + \nu} \beta^t (c_{h,t})^{-1} = \lambda_h Q_t \mu_{h,t}^{IM} P_t.$$

Using the definition of Q_t and (41), the right-hand side is

$$\frac{1 - \beta}{1 + \nu} \frac{(c_{h,0})^{-1}}{(C_0)^{-1}} \beta^t (C_t)^{-1} \frac{(c_{h,t})^{-1} / (C_t)^{-1}}{(c_{h,0})^{-1} / (C_0)^{-1}} = \frac{1 - \beta}{1 + \nu} \beta^t (c_{h,t})^{-1},$$

so the consumption first-order condition is satisfied. The first-order condition for leisure is

$$\frac{1 - \beta}{1 + \nu} \beta^t \nu (l_{h,t})^{-1} = \lambda_h Q_t \mu_{h,t}^{IM} \mu_{h,t}^L \tilde{W}_t z_{h,t}.$$

By the consumption first-order condition, this is equivalent to

$$\frac{\nu (l_{h,t})^{-1}}{(c_{h,t})^{-1}} = \mu_{h,t}^L \frac{z_{h,t} \tilde{W}_t}{P_t},$$

which holds exactly by (40). Thus each household optimally chooses the baseline paths $\{c_{h,t}, l_{h,t}\}_{t=0}^{\infty}$.

On the production side, firms in the Arrow-Debreu economy take wedges as taxes or markups that separate marginal rates of transformation from relative prices. Given (38), the baseline price $p_{i,t}$ is exactly the price implied by the Arrow-Debreu price wedge. The final-good producer is competitive and has the CES technology from the baseline economy. Its cost-minimization conditions imply

$$y_{i,t} = \left(\frac{p_{i,t}}{P_t} \right)^{-\varepsilon} Y_t, \quad P_t = \left(\int p_{i,t}^{1-\varepsilon} di \right)^{\frac{1}{1-\varepsilon}},$$

which are the same demand system and price index that hold in the baseline equilibrium. Hence the baseline allocation of intermediate goods is optimal for the final-good producer.

Similarly, (39) implies that the baseline wage $w_{u,t}$ is exactly the wage implied by the Arrow-Debreu wage wedge. Intermediate firms minimize costs over labor varieties using

the same CES labor aggregator as in the baseline economy, so their labor demand satisfies

$$n_{i,u,t} = \left(\frac{w_{u,t}}{W_t} \right)^{-\psi} \frac{y_{i,t}}{Z_t}, \quad W_t = \left(\int w_{u,t}^{1-\psi} du \right)^{\frac{1}{1-\psi}}.$$

Since the wages $w_{u,t}$ and the wage index W_t are the baseline ones, these conditions implement exactly the baseline allocation of labor across unions and firms. The induced demand for union u is therefore

$$n_{u,t}^d = \int n_{i,u,t} di.$$

It remains to check the household allocation of labor. In the baseline economy, the household-level incidence of union labor demand is given by

$$n_{h,u,t}^s = \Gamma(1 - L_t; \zeta_{h,t}) \int n_{i,u,t} di,$$

with $\int z_{h,t} \Gamma(1 - L_t; \zeta_{h,t}) dh = 1$. Therefore

$$\int \int z_{h,t} n_{h,u,t}^s dh du = \int n_{u,t}^d du = 1 - L_t,$$

and the household leisure allocation in the Arrow-Debreu economy coincides with the baseline allocation. The wedge $\mu_{h,t}^L$ is exactly the wedge that makes this incidence-implied leisure allocation optimal for household h at the productivity-adjusted price $z_{h,t} \tilde{W}_t / P_t$.

Finally, goods and labor markets clear because they clear in the baseline equilibrium:

$$C_t = \int c_{h,t} dh = Y_t, \quad \int \int n_{i,u,t} di du = 1 - L_t.$$

Thus, for any baseline equilibrium allocation, the wedges defined in Equation (38)–Equation (41), together with Arrow-Debreu prices Q_t and budget-balancing transfers $\{T_h\}$, support exactly the same allocation as an equilibrium of the Arrow-Debreu economy with wedges. The four wedges therefore provide a complete decentralization of the sources of inefficiency in the baseline economy: price dispersion, wage dispersion, heterogeneous incidence of aggregate labor fluctuations, and incomplete financial markets.

C.3 Proof of Lemma 1

See Baqaee et al. (2026).

C.4 Proof of Lemma 2

Note that, in equilibrium, the introduction of wedges causes a change in the the final good's demand for intermediate good i as well as firm i 's demand for labor. The change in the final good's demand for intermediate good i is given by

$$\partial \log y_{i,t} = -\varepsilon \left(d \log \mu_{i,t}^p - \int d \log \mu_{i',t}^p di' \right) + \partial \log Y_t,$$

where $\partial \log Y_t$ captures the change in aggregate demand associated with the introduction of wedges, i.e., the output gap $\hat{y}_t - \hat{y}_t^*$.

The change in the demand for labor input u by firm i is given by

$$\partial \log n_{i,u,t} = -\psi \left(d \log \mu_{u,t}^w - \int d \log \mu_{u',t}^w du' \right) + \partial \log y_{i,t}.$$

Note that, to first order around the efficient allocation, $\int \partial \log y_{i,t} di = \partial \log Y_t$. So, we can write the change in the demand for labor input u , $n_{u,t}^d = \int n_{i,u,t} di$ as

$$\partial \log n_{u,t}^d = -\psi \left(d \log \mu_{u,t}^w - \int d \log \mu_{u',t}^w du' \right) + \partial \log Y_t.$$

We first solve the integral in the definition of the price-wedge contribution. Substituting the expression for $\partial \log y_{i,t}$ gives

$$\begin{aligned} \triangleright^p &\equiv (1 - \beta) \omega_C \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \int_0^1 \frac{d \log \mu_{i,t}^p \cdot \partial \log y_{i,t}}{2} di \\ &= (1 - \beta) \omega_C \mathbb{E}_{-1} \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \int d \log \mu_{i,t}^p \left[-\varepsilon \left(d \log \mu_{i,t}^p - \int d \log \mu_{i',t}^p di' \right) + \partial \log Y_t \right] di \\ &= (1 - \beta) \omega_C \mathbb{E}_{-1} \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left[-\varepsilon \text{Var}_i(d \log \mu_{i,t}^p) + E_i[d \log \mu_{i,t}^p] \partial \log Y_t \right]. \end{aligned}$$

The variance of price markups equals the variance of prices, since W_t/Z_t is common across firms in period t . Hence, the standard Calvo price-setting recursion implies

$$\begin{aligned} \text{Var}_i(d \log \mu_{i,t}^p) &= \text{Var}_i(d \log p_{i,t}) \\ &= \theta_p \text{Var}_i(d \log p_{i,t-1}) + \frac{\theta_p}{1 - \theta_p} \pi_t^2. \end{aligned}$$

Using $\text{Var}_i(d \log p_{i,-1}) = 0$, solve the recursion backward and substitute into the preced-

ing expression to obtain

$$\triangleright^p = \frac{(1-\beta)\omega_C}{2} \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \left[-\varepsilon \frac{\theta_p}{(1-\theta_p)(1-\beta\theta_p)} \pi_t^2 + E_i[d \log \mu_{i,t}^p] \partial \log Y_t \right].$$

Using $\lambda_p \equiv (1-\beta\theta_p)(1-\theta_p)/\theta_p$, this becomes

$$\triangleright^p = \frac{(1-\beta)\omega_C}{2} \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \left[-\frac{\varepsilon}{\lambda_p} \pi_t^2 + E_i[d \log \mu_{i,t}^p] \partial \log Y_t \right].$$

We next solve the integral in the definition of the wage-wedge contribution. Substituting the expression for $\partial \log n_{u,t}^d$ gives

$$\begin{aligned} \triangleright^w &\equiv (1-\beta)\omega_C \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \int_0^1 \frac{d \log \mu_{u,t}^w \cdot \partial \log n_{u,t}^d}{2} du \\ &= (1-\beta)\omega_C \mathbb{E}_{-1} \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \int d \log \mu_{u,t}^w \left[-\psi \left(d \log \mu_{u,t}^w - \int d \log \mu_{u',t}^w du' \right) + \partial \log Y_t \right] du \\ &= (1-\beta)\omega_C \mathbb{E}_{-1} \frac{1}{2} \sum_{t=0}^{\infty} \beta^t \left[-\psi \text{Var}_u(d \log \mu_{u,t}^w) + E_u[d \log \mu_{u,t}^w] \partial \log Y_t \right]. \end{aligned}$$

The variance of wage markups equals the variance of wages, since $\tilde{W}_t$ is common across unions in period t . Hence, the standard Calvo wage-setting recursion implies

$$\begin{aligned} \text{Var}_u(d \log \mu_{u,t}^w) &= \text{Var}_u(d \log w_{u,t}) \\ &= \theta_w \text{Var}_u(d \log w_{u,t-1}) + \frac{\theta_w}{1-\theta_w} \pi_{w,t}^2. \end{aligned}$$

Using $\text{Var}_u(d \log w_{u,-1}) = 0$, solve the recursion backward and substitute into the preceding expression to obtain

$$\triangleright^w = \frac{(1-\beta)\omega_C}{2} \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \left[-\psi \frac{\theta_w}{(1-\theta_w)(1-\beta\theta_w)} \pi_{w,t}^2 + E_u[d \log \mu_{u,t}^w] \partial \log Y_t \right].$$

Using $\lambda_w \equiv (1-\beta\theta_w)(1-\theta_w)/\theta_w$, this becomes

$$\triangleright^w = \frac{(1-\beta)\omega_C}{2} \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \left[-\frac{\psi}{\lambda_w} \pi_{w,t}^2 + E_u[d \log \mu_{u,t}^w] \partial \log Y_t \right],$$

which is the desired wage-wedge expression.

C.5 Proof of Lemma 3

Start from the Harberger triangle for the household-specific labor wedge, written in the normalized form used in the lemma,

$$\triangleright^L \equiv (1 - \beta)(1 - \omega_C) \mathbb{E}_0 \sum_t \beta^t \int_0^1 \chi_h \frac{d \log \mu_{h,t}^L \cdot \partial \log l_{h,t}}{2} dh.$$

The incidence rule implies that, to a first order around the efficient allocation,

$$\partial \log l_{h,t} = (1 + \gamma_h) \partial \log L_t = -(1 + \gamma_h) \frac{\omega_C}{1 - \omega_C} \partial \log Y_t.$$

Substituting this expression and collecting cross-sectional terms gives

$$\triangleright^L = -\frac{(1 - \beta)\omega_C}{2} \mathbb{E}_0 \sum_t \beta^t \left\{ E_{\chi_h} [d \log \mu_{h,t}^L] \partial \log Y_t + \text{Cov}_{\chi_h} (d \log \mu_{h,t}^L, \gamma_h) \partial \log Y_t \right\}. \quad (49)$$

Thus, the proof reduces to characterizing the average labor wedge and the component of the labor wedge that covaries with the incidence elasticity γ_h .

We first compute the average labor wedge. Using log utility, the definition of the labor wedge implies

$$d \log \mu_{h,t}^L = -(d \log z_{h,t} + d \log l_{h,t} - d \log c_{h,t}) - (d \log \tilde{W}_t - d \log P_t).$$

The price and wage indices imply, to a first order,

$$d \log P_t = E_i [d \log \mu_{i,t}^p] + d \log W_t - d \log Z_t, \quad d \log W_t = E_u [d \log \mu_{u,t}^w] + d \log \tilde{W}_t.$$

Therefore,

$$\begin{aligned} d \log \tilde{W}_t - d \log P_t &= d \log W_t - E_u [d \log \mu_{u,t}^w] - \left(E_i [d \log \mu_{i,t}^p] + d \log W_t - d \log Z_t \right) \\ &= d \log Z_t - E_i [d \log \mu_{i,t}^p] - E_u [d \log \mu_{u,t}^w]. \end{aligned}$$

At the efficient reference, $z_{h,t} l_{h,t} = \chi_h L_t$, so $E_{\chi_h} [d \log z_{h,t} + d \log l_{h,t}] = d \log L_t$. Taking the consumption-share weighted average across households gives

$$\begin{aligned} E_{\chi_h} [d \log \mu_{h,t}^L] &= -(E_{\chi_h} [d \log z_{h,t} + d \log l_{h,t}] - E_{\chi_h} [d \log c_{h,t}]) - (d \log \tilde{W}_t - d \log P_t) \\ &= -(d \log L_t - d \log C_t) - d \log Z_t + E_i [d \log \mu_{i,t}^p] + E_u [d \log \mu_{u,t}^w]. \end{aligned}$$

Around the efficient allocation, $d \log C_t = d \log Z_t + \partial \log Y_t$ and $d \log L_t = -\nu^{-1} \partial \log Y_t$. Hence,

$$E_{\chi_h}[d \log \mu_{h,t}^L] = (1 + \nu^{-1}) \partial \log Y_t + E_i[d \log \mu_{i,t}^p] + E_u[d \log \mu_{u,t}^w]. \quad (50)$$

This is the average labor-wedge component. It combines the usual aggregate labor wedge, $(1 + \nu^{-1}) \partial \log Y_t$, with the average price and wage markup wedges.

We next compute the dispersion component. Since the aggregate price term $d \log \tilde{W}_t - d \log P_t$ is common across households, it drops out of the covariance with γ_h . Therefore,

$$Cov_{\chi_h}(d \log \mu_{h,t}^L, \gamma_h) = Cov_{\chi_h}(-(d \log z_{h,t} + d \log l_{h,t} - d \log c_{h,t}), \gamma_h).$$

Recall that we have assumed that $\Gamma(\cdot)$ is such that labor supply is optimal in the absence of aggregate shocks. This implies that, to a first order around the efficient allocation, the labor wedge does not respond to changes in idiosyncratic characteristics. Collecting terms common across households in k_t^L , which drops out of the covariance, it follows that

$$d \log \mu_{h,t}^L = - \left[(1 + \gamma_h) \partial \log L_t - \mathbb{E}_{-1} \sum_{s=0}^{\infty} \sum_{v_s \in \hat{\Theta}_s} \frac{\partial \log c_{h,t}}{\partial v_s} dv_s \right] + k_t^L,$$

where $\hat{\Theta}_t \equiv \{Y_t, L_t, i_t, \delta_t, P_t/P_{t-1}, \{q_{k,t}\}\}$.

The physical leisure response is governed by γ_h , whereas the heterogeneous consumption response is governed by total-income incidence γ_h^Y . In an output-gap direction,

$$\partial \log \frac{y_{h,s}}{Y_s} = \gamma_h^Y \partial \log(1 - L_s) = -\nu \gamma_h^Y \partial \log L_s.$$

Since aggregate risks are spanned, log utility annuitizes this relative-income innovation across the household's consumption path. Thus the heterogeneous part of the consumption derivative is

$$\left. \frac{\partial \log c_{h,t}}{\partial \log L_s} \right|_{\text{het}} = -\nu(1 - \beta) \beta^s \gamma_h^Y.$$

All remaining aggregate consumption responses are common across households and drop out of the covariance with γ_h . Therefore,

$$\begin{aligned} Cov_{\chi_h}(d \log \mu_{h,t}^L, \gamma_h) &= -Var_{\chi_h}(\gamma_h) \partial \log L_t - \nu Cov_{\chi_h}(\gamma_h, \gamma_h^Y) \mathbb{E}_{-1} \sum_{s=0}^{\infty} (1 - \beta) \beta^s d \log L_s \\ &= \nu^{-1} Var_{\chi_h}(\gamma_h) \partial \log Y_t + Cov_{\chi_h}(\gamma_h, \gamma_h^Y) \mathbb{E}_{-1} \sum_{s=0}^{\infty} (1 - \beta) \beta^s \partial \log Y_s. \end{aligned}$$

So, substituting above we get

$$\begin{aligned}\triangleright^L &= -\frac{(1-\beta)\omega_C}{2}\mathbb{E}_0\sum_t\beta^t\left\{\left[(1+\nu^{-1})\partial\log Y_t + E_i[d\log\mu_{i,t}^p] + E_u[d\log\mu_{u,t}^w]\right]\partial\log Y_t\right. \\ &\quad \left.+ \nu^{-1}Var_{\chi_h}(\gamma_h) \cdot (\partial\log Y_t)^2 + Cov_{\chi_h}(\gamma_h, \gamma_h^Y)\mathbb{E}_{-1}\sum_{s=0}^{\infty}(1-\beta)\beta^s\partial\log Y_s\partial\log Y_t\right\}. \\ \Leftrightarrow \triangleright^L &= -\frac{(1-\beta)\omega_C}{2}\mathbb{E}_0\sum_t\beta^t\left\{\left[(1+\nu^{-1})\partial\log Y_t + E_i[d\log\mu_{i,t}^p] + E_u[d\log\mu_{u,t}^w]\right]\partial\log Y_t\right. \\ &\quad \left.+ \nu^{-1}Var_{\chi_h}(\gamma_h) \cdot (\partial\log Y_t)^2 + (1-\beta)Cov_{\chi_h}(\gamma_h, \gamma_h^Y)\mathbb{E}_{-1}\left(\sum_{s=0}^{\infty}\beta^s\partial\log Y_s\right)^2\right\}.\end{aligned}$$

C.6 Proof of Lemma 4

The Harberger triangle for incomplete markets is given by

$$\triangleright^{IM} \equiv (1-\beta)\mathbb{E}_{-1}\sum_t\beta^t E_{\chi_h}\left[\frac{d\log\mu_{h,t}^{IM} \cdot \partial\log x_{h,t}}{2}\right],$$

Note that, using household optimality, we find that

$$\beta^t\left(\frac{x_{h,t}}{x_{h,0}}\right)^{-1} = Q_t\frac{\mu_{h,t}^{IM}p_{h,t}^x}{\mu_{h,0}^{IM}p_{h,0}^x},$$

where $p_{h,t}^x \equiv P_t^{\frac{1}{1+\nu}}(\mu_{h,t}^L\tilde{W}_tz_{h,t})^{\frac{\nu}{1+\nu}}$ is the price of the composite good $x_{h,t}$ in the Arrow-Debreu economy. Therefore,

$$\frac{\mu_{h,t}^{IM}p_{h,t}^xx_{h,t}}{\mu_{h,0}^{IM}p_{h,0}^xx_{h,0}} = K_t,$$

for some variable K_t that is common across households. The wedge-induced partial changes hold idiosyncratic productivity fixed; changes in common aggregate prices are absorbed in K_t . Taking logs and approximating around the efficient allocation gives us

$$\begin{aligned}
\partial \log x_{h,t} - \partial \log x_{h,0} &= -(d \log \mu_{h,t}^{IM} - d \log \mu_{h,0}^{IM}) - (\partial \log p_{h,t}^x - \partial \log p_{h,0}^x) + \partial \log K_t \\
\partial \log x_{h,t} - \partial \log x_{h,0} &= -(d \log \mu_{h,t}^{IM} - d \log \mu_{h,0}^{IM}) - (1 - \omega_C)(d \log \mu_{h,t}^L - d \log \mu_{h,0}^L) + \partial \log K_t \\
(\partial \log x_{h,t} - \partial \log x_{h,0}) - (\partial \log x_{0,t} - \partial \log x_{0,0}) &= -(d \log \mu_{h,t}^{IM} - d \log \mu_{h,0}^{IM} - (d \log \mu_{0,t}^{IM} - d \log \mu_{0,0}^{IM})) \\
&\quad - (1 - \omega_C)(d \log \mu_{h,t}^L - d \log \mu_{h,0}^L - (d \log \mu_{0,t}^L - d \log \mu_{0,0}^L)).
\end{aligned}$$

By Corollary 1 in Baqaee et al. (2026) we know that

$$\mathbb{E}_{-1} \sum_{t=0}^{\infty} (1 - \beta) \beta^t \partial \log x_{h,t} = \mathbb{E}_{-1} \sum_{t=0}^{\infty} (1 - \beta) \beta^t \partial \log x_{0,t} = 0.$$

So, we can write

$$\begin{aligned}
\partial \log x_{h,t} - \partial \log x_{0,t} &= -[(d \log \mu_{h,t}^{IM} - d \log \bar{\mu}_h^{IM}) - (d \log \mu_{0,t}^{IM} - d \log \bar{\mu}_0^{IM})] \\
&\quad - (1 - \omega_C)[(d \log \mu_{h,t}^L - d \log \bar{\mu}_h^L) - (d \log \mu_{0,t}^L - d \log \bar{\mu}_0^L)],
\end{aligned}$$

where $d \log \bar{\mu}_h = (1 - \beta) \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t d \log \mu_{h,t}$ is the average lifetime wedge for household h .

By the definition of the consumption–leisure composite,

$$\omega_C \partial \log c_{h,t} + (1 - \omega_C) \partial \log l_{h,t} = \partial \log x_{h,t},$$

and, since by market clearing

$$E_{\chi_h} [\omega_C \partial \log c_{h,t} + (1 - \omega_C) \partial \log l_{h,t}] = 0 \Leftrightarrow E_{\chi_h} [\partial \log x_{h,t}] = 0.$$

Using these findings in the expression above, we can rewrite

$$\begin{aligned}
\partial \log x_{h,t} &= -[(d \log \mu_{h,t}^{IM} - d \log \bar{\mu}_h^{IM}) - E_{\chi_{h'}} [(d \log \mu_{h',t}^{IM} - d \log \bar{\mu}_{h'}^{IM})]] \\
&\quad - (1 - \omega_C)[(d \log \mu_{h,t}^L - d \log \bar{\mu}_h^L) - E_{\chi_{h'}} [(d \log \mu_{h',t}^L - d \log \bar{\mu}_{h'}^L)]],
\end{aligned}$$

and note that $E_{\chi_h} [d \log \mu_{h,t}^{IM}] = -E_{\chi_h} [\partial \log c_{h,t} - \partial \log C_t - (\partial \log c_{h,0} - \partial \log C_0)] = 0$ and similarly $E_{\chi_h} [d \log \bar{\mu}_{h,t}^{IM}] = 0$.

Replacing $\partial \log x_{h,t}$ in the definition of the Harberger triangle for incomplete markets gives

$$\begin{aligned}
\triangleright^{IM} &= -\frac{1-\beta}{2}\mathbb{E}_{-1}\sum_t\beta^t E_{\chi_h}\left[d\log\mu_{h,t}^{IM}[(d\log\mu_{h,t}^{IM}-d\log\bar{\mu}_h^{IM})]\right] \\
&\quad -\frac{1-\beta}{2}(1-\omega_C)\mathbb{E}_{-1}\sum_t\beta^t E_{\chi_h}\left[d\log\mu_{h,t}^{IM}[(d\log\mu_{h,t}^L-d\log\bar{\mu}_h^L)-E_{\chi_{h'}}[(d\log\mu_{h',t}^L-d\log\bar{\mu}_{h'}^L)]]\right] \\
\\
&\Leftrightarrow \triangleright^{IM} = -\frac{1-\beta}{2}E_{\chi_h}\left[\mathbb{E}_{-1}\sum_t\beta^t d\log\mu_{h,t}^{IM}[(d\log\mu_{h,t}^{IM}-d\log\bar{\mu}_h^{IM})]\right] \\
&\quad -\frac{1-\beta}{2}(1-\omega_C)\mathbb{E}_{-1}\sum_t\beta^t Cov_{\chi_h}\left[d\log\mu_{h,t}^{IM},(d\log\mu_{h,t}^L-d\log\bar{\mu}_h^L)\right] \\
\\
&\Leftrightarrow \triangleright^{IM} = -\frac{1}{2}E_{\chi_h}\left[Var_{\beta,\pi}\left(d\log\mu_{h,t}^{IM}\right)\right] \\
&\quad -\frac{1-\beta}{2}(1-\omega_C)\mathbb{E}_{-1}\sum_t\beta^t Cov_{\chi_h}\left[d\log\mu_{h,t}^{IM},(d\log\mu_{h,t}^L-d\log\bar{\mu}_h^L)\right].
\end{aligned}$$

The heterogeneous lifetime-wealth component of $d\log\mu_{h,t}^L$ is proportional to γ_h^Y but is constant over the household's consumption dates under aggregate spanning. It therefore cancels from $d\log\mu_{h,t}^L-d\log\bar{\mu}_h^L$. The remaining time-varying component is generated by physical labor incidence and continues to load on γ_h . Now, using the fact that

$$\begin{aligned}
d\log\mu_{h,t}^{IM} &= -\left[d\log\left(\frac{c_{h,t}}{C_t}\right)-d\log\left(\frac{c_{h,0}}{C_0}\right)\right] \\
&\quad \left[d\log\mu_{h,t}^L-d\log\bar{\mu}_h^L\right]-E_{\chi_{h'}}\left[d\log\mu_{h',t}^L-d\log\bar{\mu}_{h'}^L\right] \\
&= \gamma_h\frac{\omega_C}{1-\omega_C}\left(\partial\log Y_t-(1-\beta)\mathbb{E}_{-1}\sum_{s=0}^{\infty}\beta^s\partial\log Y_s\right),
\end{aligned}$$

and $C_t = Y_t$, we can write

$$\begin{aligned}
\triangleright^{IM} &= -\frac{1}{2}E_{\chi_h}\left[Var_{\beta,\pi}\left(d\log\left(\frac{c_{h,t}}{Y_t}\right)\right)\right] \\
&\quad +\frac{1-\beta}{2}\omega_C\mathbb{E}_{-1}\sum_t\beta^t Cov_{\chi_h}\left[d\log\left(\frac{c_{h,t}}{c_{h,0}}\right),\gamma_h\right]\left(\partial\log Y_t-(1-\beta)\mathbb{E}_{-1}\sum_{s=0}^{\infty}\beta^s\partial\log Y_s\right)
\end{aligned}$$

$$\Leftrightarrow \triangleright^{IM} = -\frac{1}{2}E_{\chi_h} \left[\text{Var}_{\beta, \pi} \left(d \log \left(\frac{c_{h,t}}{Y_t} \right) \right) \right] + \frac{1-\beta}{2}\omega_C \left[\mathbb{E}_{-1} \sum_t \beta^t \text{Cov}_{\chi_h} \left[d \log \left(\frac{c_{h,t}}{c_{h,0}} \right), \gamma_h \right] \partial \log Y_t \right. \\ \left. - \text{Cov}_{\chi_h} \left[d \log \left(\frac{\bar{c}_h}{c_{h,0}} \right), \gamma_h \right] \mathbb{E}_{-1} \sum_{s=0}^{\infty} \beta^s \partial \log Y_s \right]$$

where $d \log (\bar{c}_h) \equiv (1 - \beta) \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t d \log c_{h,t}$ is the average lifetime consumption of household h . So,

$$\triangleright^{IM} = -\frac{1}{2}E_{\chi_h} \left[\text{Var}_{\beta, \pi} \left(d \log \left(\frac{c_{h,t}}{Y_t} \right) \right) \right] + \frac{1-\beta}{2}\omega_C \mathbb{E}_{-1} \sum_t \beta^t \text{Cov}_{\chi_h} \left[d \log \left(\frac{c_{h,t}}{\bar{c}_h} \right), \gamma_h \right] \partial \log Y_t$$

C.7 Proof of Proposition 1

We have found that

$$\begin{aligned} \triangleright^p &= \frac{(1-\beta)\omega_C}{2} \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \left[-\frac{\varepsilon}{\lambda_p} \pi_t^2 + E_i[d \log \mu_{i,t}^p] \partial \log Y_t \right] \\ \triangleright^w &= \frac{(1-\beta)\omega_C}{2} \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \left[-\frac{\psi}{\lambda_w} \pi_{w,t}^2 + E_u[d \log \mu_{u,t}^w] \partial \log Y_t \right], \\ \triangleright^L &= -\frac{(1-\beta)\omega_C}{2} \mathbb{E}_0 \sum_t \beta^t \left\{ \left[(1 + \nu^{-1}) \partial \log Y_t + E_i[d \log \mu_{i,t}^p] + E_u[d \log \mu_{u,t}^w] \right] \partial \log Y_t \right. \\ &\quad \left. + \nu^{-1} \text{Var}_{\chi_h}(\gamma_h) \cdot (\partial \log Y_t)^2 + (1-\beta) \text{Cov}_{\chi_h}(\gamma_h, \gamma_h^Y) \mathbb{E}_{-1} \left(\sum_{s=0}^{\infty} \beta^s \partial \log Y_s \right)^2 \right\}, \\ \triangleright^{IM} &= -\frac{1}{2}E_{\chi_h} \left[\text{Var}_{\beta, \pi} \left(d \log \left(\frac{c_{h,t}}{Y_t} \right) \right) \right] + \frac{1-\beta}{2}\omega_C \mathbb{E}_{-1} \sum_t \beta^t \text{Cov}_{\chi_h} \left[d \log \left(\frac{c_{h,t}}{\bar{c}_h} \right), \gamma_h \right] \partial \log Y_t. \end{aligned}$$

Putting these together, we get

$$\begin{aligned}
\triangleright^p + \triangleright^w + \triangleright^L + \triangleright^{IM} &= \frac{(1-\beta)\omega_C}{2} \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \left[-\frac{\varepsilon}{\lambda_p} \pi_t^2 + E_i[d \log \mu_{i,t}^p] \partial \log Y_t \right] \\
&\quad \frac{(1-\beta)\omega_C}{2} \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \left[-\frac{\psi}{\lambda_w} \pi_{w,t}^2 + E_u[d \log \mu_{u,t}^w] \partial \log Y_t \right] \\
&\quad - \frac{(1-\beta)\omega_C}{2} \mathbb{E}_0 \sum_t \beta^t \left\{ \left[(1+\nu^{-1}) \partial \log Y_t + E_i[d \log \mu_{i,t}^p] + E_u[d \log \mu_{u,t}^w] \right] \partial \log Y_t \right. \\
&\quad \left. + \nu^{-1} \text{Var}_{\chi_h}(\gamma_h) \cdot (\partial \log Y_t)^2 + (1-\beta) \text{Cov}_{\chi_h}(\gamma_h, \gamma_h^Y) \mathbb{E}_{-1} \left(\sum_{s=0}^{\infty} \beta^s \partial \log Y_s \right)^2 \right\} \\
&\quad - \frac{1}{2} E_{\chi_h} \left[\text{Var}_{\beta, \pi} \left(d \log \left(\frac{c_{h,t}}{Y_t} \right) \right) \right] \\
&\quad + \frac{1-\beta}{2} \omega_C \mathbb{E}_{-1} \sum_t \beta^t \text{Cov}_{\chi_h} \left[d \log \left(\frac{c_{h,t}}{\bar{c}_h} \right), \gamma_h \right] \partial \log Y_t
\end{aligned}$$

$$\begin{aligned}
\Leftrightarrow \log A(\mathcal{P}) &= \log \mathcal{D} - \frac{(1-\beta)\omega_C}{2} \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \left[(1+\nu^{-1}) (\partial \log Y_t)^2 + \frac{\varepsilon}{\lambda_p} \pi_t^2 + \frac{\psi}{\lambda_w} \pi_{w,t}^2 \right] \\
&\quad - \frac{(1-\beta)\omega_C}{2} \mathbb{E}_0 \sum_t \beta^t \left\{ \nu^{-1} \text{Var}_{\chi_h}(\gamma_h) \cdot (\partial \log Y_t)^2 \right. \\
&\quad \left. + (1-\beta) \text{Cov}_{\chi_h}(\gamma_h, \gamma_h^Y) \mathbb{E}_{-1} \left(\sum_{s=0}^{\infty} \beta^s \partial \log Y_s \right)^2 \right\} \\
&\quad - \frac{1}{2} E_{\chi_h} \left[\text{Var}_{\beta, \pi} \left(d \log \left(\frac{c_{h,t}}{Y_t} \right) \right) \right] + \frac{1-\beta}{2} \omega_C \mathbb{E}_{-1} \sum_t \beta^t \text{Cov}_{\chi_h} \left[d \log \left(\frac{c_{h,t}}{\bar{c}_h} \right), \gamma_h \right] \partial \log Y_t
\end{aligned}$$

$$\begin{aligned}
\Leftrightarrow \log A(\mathcal{P}) &= \log \mathcal{D} - \frac{1}{2} \left\{ \mathbb{E}_{-1} \left[\sum_{t=0}^{\infty} \omega_C (1-\beta) \beta^t \left[(1+\nu^{-1}) (\partial \log Y_t)^2 + \frac{\varepsilon}{\lambda_p} \pi_t^2 + \frac{\psi}{\lambda_w} \pi_{w,t}^2 \right] \right] \right. \\
&\quad + \mathbb{E}_0 \left[\sum_t \omega_C (1-\beta) \beta^t \nu^{-1} \text{Var}_{\chi_h}(\gamma_h) \cdot (\partial \log Y_t)^2 \right] \\
&\quad + \omega_C \text{Cov}_{\chi_h}(\gamma_h, \gamma_h^Y) \mathbb{E}_{-1} \left(\sum_{s=0}^{\infty} (1-\beta) \beta^s \partial \log Y_s \right)^2 \\
&\quad \left. + E_{\chi_h} \left[\text{Var}_{\beta, \pi} \left(d \log \left(\frac{c_{h,t}}{Y_t} \right) \right) \right] + \mathbb{E}_{-1} \sum_t \omega_C (1-\beta) \beta^t \text{Cov}_{\chi_h} \left[d \log \left(\frac{c_{h,t}}{\bar{c}_h} \right), \gamma_h \right] \partial \log Y_t \right\}
\end{aligned}$$

Finally, replacing the change in output induced by the wedges

$$\partial \log Y_t = d \log(Z_t L_t) - d \log \left(\frac{Z_t}{1+\nu} \right) = d \log Y_t - d \log Y_t^* = \hat{y}_t - \hat{y}_t^*,$$

we obtain the desired result.

Appendix D Appendix to Section 5

D.1 Proof of Proposition 2

See Baqaee et al. (2026).

The approximate optimal policy can be computed from choosing $\{\hat{y}_t, \pi_t, \pi_{w,t}, \hat{w}_t\}$ to maximize the second order approximation of aggregate surplus, subject to the constraints

$$\begin{aligned}\pi_t &= \lambda_p (\hat{w}_t - \hat{z}_t) + \beta \mathbb{E}_t[\pi_{t+1}] + u_{p,t} \\ \pi_{w,t} &= \lambda_w \left(\left\{ 1 + \nu^{-1} \right\} (\hat{y}_t - \hat{y}_t^*) - (\hat{w}_t - \hat{z}_t) \right) + \beta \mathbb{E}_t[\pi_{w,t+1}] + u_{w,t} \\ \hat{w}_t &= \hat{w}_{t-1} + \pi_{w,t} - \pi_t.\end{aligned}$$

D.2 Full accommodation of demand shocks

Note that δ_t does not show up in the reduced form program for optimal monetary policy. It follows that the quantities $\{\hat{y}_t, \pi_t, \pi_{w,t}, \hat{w}_t\}$ that maximize the second order approximation of aggregate surplus are the same as those in the case without demand shocks. In particular, the optimal policy does not respond to demand shocks. So, the central bank chooses i_t to fully offset the demand shock, i.e., $d \log(1 + i_t) = -d \log(1 + \delta_t)$.

D.3 Proof of Proposition 3

Let $\tilde{y}_t \equiv \hat{y}_t - \hat{y}_t^*$. In this model $\gamma_h = 0$ for every h . Hence the second-order objective in Proposition 1 reduces to

$$\begin{aligned}\log A(\mathcal{P}) &= \log \mathcal{D} - \frac{(1 - \beta)\omega_C}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[(1 + \nu^{-1}) \tilde{y}_t^2 + \frac{\varepsilon}{\lambda_p} \pi_t^2 + \frac{\psi}{\lambda_w} \pi_{w,t}^2 \right] \\ &\quad - \frac{1}{2} E_{\chi_h} \left[\text{Var}_{\beta, \pi} \left(d \log \frac{c_{h,t}}{Y_t} \right) \right],\end{aligned}\tag{51}$$

up to terms that are independent of policy. Thus the only point to prove is that the last term has zero derivative with respect to aggregate conditions.

Consider first the household block with idiosyncratic risk and initial shut down, $\sigma_z = 0$, but with arbitrary aggregate shocks and arbitrary monetary policy. Since $\gamma_h = 0$, aggregate fluctuations do not change household h 's physical labor share to a first order. Under the total-income rule, real household income is

$$y_{h,t} = z_{h,0} \Gamma^Y (1 - L_t; \zeta_{h,0}) Y_t.$$

The elasticity γ_h^Y may generate heterogeneous exposure of lifetime wealth to aggregate fluctuations. The financial constraint is slack at the efficient allocation because $\Omega(0, \{0\}_k) > 0$. Therefore, for a sufficiently small perturbation around that allocation, the household's Euler equations with respect to assets that span aggregate risk hold with equality. These claims insure the γ_h^Y exposure: it can change the household-specific lifetime consumption share α_h , but not that share's profile across aggregate dates and histories.

Let Q_t denote the date-0 Arrow price of one unit of nominal wealth in the aggregate history at date t . The first-order condition for any aggregate Arrow claim is

$$\frac{(1-\beta)\beta^t}{1+\nu} \frac{1}{P_t c_{h,t}} = \Lambda_h Q_t, \quad (52)$$

where Λ_h is the household-specific multiplier on lifetime wealth. This first order condition implies that

$$c_{h,t} = \alpha_h Y_t,$$

for some $\alpha_h \geq 0$ with $\int \alpha_h dh = 1$. It follows that aggregate shocks and monetary policy can affect the common factor C_t , but they do not affect the time profile of household consumption relative to aggregate consumption. Equivalently, for any aggregate perturbation, the incomplete market wedges are zero around the complete markets allocation.

D.4 Proof of Proposition 4

We compare policies at the same fundamentals and initial household states, using the efficient steady state at $\sigma = 0$ as the point of approximation. Let X_t^0 denote the aggregate allocation under the status quo, and let Δ denote the difference between the first-order responses under the candidate policy and the status quo. The expectation $\mathbb{E}_{-1}$ holds $z_{h,0}$ and inherited portfolios fixed and averages over aggregate shocks from date 0 and future idiosyncratic shocks. By iterated expectations and (1), the first-order difference in ex-ante welfare is

$$\mathbb{E}_{-1} \Delta U_h = \frac{1-\beta}{1+\nu} \sum_{t=0}^{\infty} \beta^t \{ \mathbb{E}_{-1} \Delta \log c_{h,t} + \nu \mathbb{E}_{-1} \Delta \log l_{h,t} \}. \quad (53)$$

Here $\Delta \log c_{h,t}$ and $\Delta \log l_{h,t}$ are differences in log deviations, evaluated at the common deterministic reference. Since aggregate surplus has no first-order term at the efficient allocation, $d \log A = 0$, these are also the first-order welfare differences entering the compensating-transfer problem.

Set $dT_h = 0$ for all h . The linearized household and equilibrium conditions have de-

terministic coefficients at the reference. Consequently, the policy-induced differences in individual allocations are linear functions of $\{X_t - X_t^0\}_{t \geq 0}$. Direct responses to idiosyncratic shocks are common across policies and cancel; their interactions with aggregate policy are second order in the joint approximation. Taking expectations therefore gives the same household allocation differences as imposing the mean aggregate reform path. This argument includes the revaluation of inherited assets and does not require their initial positions to be zero.

By Proposition 3, the optimal policy is the RANK policy. With centered shocks and no initial aggregate distortion, this policy has no deterministic component. The status quo is centered by assumption, so

$$\mathbb{E}_{-1}(X_t - X_t^0) = 0, \quad t \geq 0.$$

It follows that $\mathbb{E}_{-1}\Delta \log c_{h,t} = \mathbb{E}_{-1}\Delta \log l_{h,t} = 0$ for every h, t . Equation (53) then implies $\mathbb{E}_{-1}\Delta U_h = 0$ for every household. Thus zero transfers satisfy all first-order household indifference conditions and budget balance. Any compensation required beyond this approximation is second order. Thus, with unrestricted access to lump-sum transfers, the optimum admits zero transfers in the first-order compensation system.

Restricting the transfer set. The relevant feasible-set argument is simple. Let $\mathcal{F}$ denote the unrestricted set of compensated policy allocations and let $\mathcal{F}_{\mathcal{R}} \subseteq \mathcal{F}$ impose a restriction $\{T_h\} \in \mathcal{R}$. If an unrestricted maximizer $(\mathcal{P}^*, \mathbf{x}^*, \{T_h^*\})$ has $T_h^* = 0$ for every household and $\mathbf{0} \in \mathcal{R}$, that same maximizer belongs to $\mathcal{F}_{\mathcal{R}}$. Restricting the feasible set cannot improve on the unrestricted optimum, and retaining that optimum prevents the value from falling. The policy is therefore optimal in both problems.

Applied to the first-order allocation and compensation system above, this observation shows that any admissible restriction containing zero transfers retains the unrestricted first-order solution. An exact version of this argument requires exact zero compensation, rather than just $dT_h = 0$. In particular, the calculation above does not by itself rule out second-order household-specific compensation. To extend the set-inclusion conclusion to the original no-transfer surplus problem, one must also establish that the unrestricted optimum remains feasible through the order at which surplus is evaluated. This is the remaining higher-order feasibility condition behind the proposed equivalence.

D.5 Proof of Proposition 5

In the sticky-wage model, prices are flexible and there are no price cost-push shocks. Hence, the price Phillips curve implies $\hat{w}_t = \hat{z}_t$, and the wage Phillips curve reduces to

$$\pi_{w,t} = \lambda_w(1 + \nu^{-1})\tilde{y}_t + \beta\mathbb{E}_t[\pi_{w,t+1}] + u_{w,t}, \quad \tilde{y}_t \equiv \hat{y}_t - \hat{y}_t^*. \quad (54)$$

Terms in the objective that are independent of policy can be dropped. After multiplying by the positive constant $(1 - \beta)\omega_C$, the relevant maximization problem is

$$\begin{aligned} \max_{\{\tilde{y}_t, \pi_{w,t}\}} & -\frac{1}{2}\mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \left[\frac{\psi}{\lambda_w} \pi_{w,t}^2 + \left(1 + \nu^{-1} + \nu^{-1}V_\gamma\right) \tilde{y}_t^2 - C_t \tilde{y}_t \right] \\ & - \frac{1}{2}(1 - \beta)C_{\gamma Y} \mathbb{E}_{-1} \left(\sum_{t=0}^{\infty} \beta^t \tilde{y}_t \right)^2, \end{aligned}$$

where

$$V_\gamma \equiv \text{Var}_{\chi_h}(\gamma_h), \quad C_t \equiv \text{Cov}_{\chi_h} \left(d \log \frac{c_{h,t}}{\bar{c}_h}, \gamma_h \right), \quad C_{\gamma Y} \equiv \text{Cov}_{\chi_h}(\gamma_h, \gamma_h^Y).$$

Since $\bar{c}_h$ is time invariant for a given household,

$$C_t - C_{t-1} = \text{Cov}_{\chi_h}(d \log c_{h,t}, \gamma_h) - \text{Cov}_{\chi_h}(d \log c_{h,t-1}, \gamma_h).$$

Let ξ_t be the multiplier on the wage Phillips curve. The Lagrangian first-order conditions are

$$\left(1 + \nu^{-1} + \nu^{-1}V_\gamma\right) \tilde{y}_t + (1 - \beta)C_{\gamma Y} \mathbb{E}_{-1} \sum_{s=0}^{\infty} \beta^s \tilde{y}_s = \frac{1}{2}C_t - \lambda_w(1 + \nu^{-1})\xi_t, \quad (55)$$

$$\frac{\psi}{\lambda_w} \pi_{w,t} - \xi_t + \xi_{t-1} = 0, \quad (56)$$

with $\xi_{-1} = 0$. The present-value term in (55) is common across dates, so it cancels when subtracting the condition at $t - 1$ from the condition at t . Using (56), for $t \geq 1$ we obtain

$$\psi(1 + \nu^{-1})\pi_{w,t} = \frac{1}{2}(C_t - C_{t-1}) - \left(1 + \nu^{-1} + \nu^{-1}V_\gamma\right) (\tilde{y}_t - \tilde{y}_{t-1}). \quad (57)$$

Rearranging,

$$\pi_{w,t} = -\frac{1 + \nu^{-1} + \nu^{-1}V_\gamma}{\psi(1 + \nu^{-1})}(\tilde{y}_t - \tilde{y}_{t-1}) + \frac{1}{2\psi(1 + \nu^{-1})}(C_t - C_{t-1}). \quad (58)$$

Now define

$$\hat{\psi} \equiv \frac{\psi}{1 + \frac{\nu^{-1}}{1+\nu^{-1}} V_\gamma}.$$

Then

$$\hat{\psi}^{-1} = \frac{1 + \nu^{-1} + \nu^{-1} V_\gamma}{\psi(1 + \nu^{-1})}.$$

Moreover,

$$\frac{\hat{\psi}^{-1} - \psi^{-1}}{2\nu^{-1}} = \frac{V_\gamma}{2\psi(1 + \nu^{-1})} \equiv \zeta_{IM}.$$

Therefore

$$\frac{1}{2\psi(1 + \nu^{-1})} (C_t - C_{t-1}) = \zeta_{IM} \cdot \frac{Cov_{\chi_h}(d \log c_{h,t}, \gamma_h) - Cov_{\chi_h}(d \log c_{h,t-1}, \gamma_h)}{V_\gamma},$$

which gives (24). For $t = 0$, use $\zeta_{-1} = 0$. The present-value term has no date-(-1) condition against which to cancel, so the initial condition is

$$\pi_{w,0} = -\hat{\psi}^{-1} \tilde{y}_0 + \zeta_{IM} \cdot \frac{Cov_{\chi_h}(d \log(c_{h,0}/\bar{c}_h), \gamma_h)}{V_\gamma} - \frac{(1 - \beta)C_{\gamma Y}}{\psi(1 + \nu^{-1})} \mathbb{E}_0 \sum_{s=0}^{\infty} \beta^s \tilde{y}_s.$$

D.6 Proof of Corollary 6

The targeting rule in Proposition 5 can be written as

$$\pi_{w,t} = -\hat{\psi}^{-1} (\tilde{y}_t - \tilde{y}_{t-1}) + \zeta_{IM} \cdot \frac{Cov_{\chi_h}(d \log c_{h,t}, \gamma_h) - Cov_{\chi_h}(d \log c_{h,t-1}, \gamma_h)}{Var_{\chi_h}(\gamma_h)}.$$

Along a bounded equilibrium path converging back to the stationary allocation, the cross-sectional distribution induced by a date-0 perturbation becomes independent of the initial household state. Hence $Cov_{\chi_h}(d \log c_{h,t}, \gamma_h)$ converges to a constant, equal to zero under the normalization of log deviations around the efficient steady state. It follows that

$$Cov_{\chi_h}(d \log c_{h,t}, \gamma_h) - Cov_{\chi_h}(d \log c_{h,t-1}, \gamma_h) \rightarrow 0.$$

Taking limits in the targeting rule yields

$$\pi_{w,t} = -\hat{\psi}^{-1} (\tilde{y}_t - \tilde{y}_{t-1}),$$

or equivalently $\tilde{y}_t - \tilde{y}_{t-1} = -\hat{\psi} \pi_{w,t}$.

D.7 Proof of Proposition 6

Let $\tilde{y}_t \equiv \hat{y}_t - \hat{y}_t^*$, $C_t \equiv \text{Cov}_{\chi_h} \left(d \log \frac{c_{h,t}}{\bar{c}_h}, \gamma_h \right)$, and $V_\gamma \equiv \text{Var}_{\chi_h}(\gamma_h)$. Also define $C_{\gamma Y} \equiv \text{Cov}_{\chi_h}(\gamma_h, \gamma_h^Y)$. Use the dual-mandate weights in Proposition 6,

$$\omega_y = 1 + \nu^{-1}(1 + V_\gamma), \quad \omega_p = \frac{\varepsilon}{\lambda_p}, \quad \omega_w = \frac{\psi}{\lambda_w}.$$

Dropping terms independent of policy and multiplying by positive constants, the policy-relevant part of the efficiency mandate objective in the sticky-price and sticky-wage model can be written as

$$- \frac{1}{2} \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \left[\omega_y \tilde{y}_t^2 + \omega_p \pi_t^2 + \omega_w \pi_{w,t}^2 - C_t \tilde{y}_t \right] - \frac{1}{2} (1 - \beta) C_{\gamma Y} \mathbb{E}_{-1} \left(\sum_{t=0}^{\infty} \beta^t \tilde{y}_t \right)^2. \quad (59)$$

The dual-mandate problem in Proposition 6 has the same linearized private-sector constraints,

$$\begin{aligned} \pi_t &= \lambda_p (\hat{w}_t - \hat{z}_t) + \beta \mathbb{E}_t [\pi_{t+1}] + u_{p,t}, \\ \pi_{w,t} &= \lambda_w \left((1 + \nu^{-1}) \tilde{y}_t - (\hat{w}_t - \hat{z}_t) \right) + \beta \mathbb{E}_t [\pi_{w,t+1}] + u_{w,t}, \\ \hat{w}_t &= \hat{w}_{t-1} + \pi_{w,t} - \pi_t, \end{aligned}$$

and the same quadratic weights ω_y , ω_p , and ω_w on $\tilde{y}_t$, π_t , and $\pi_{w,t}$, but it omits the deterministic linear term $C_t \tilde{y}_t$ and the intertemporal incidence term.

Let $\xi_{w,t}$ denote the multiplier on the wage Phillips curve. The first-order condition of the efficiency mandate problem with respect to $\tilde{y}_t$ is

$$\omega_y \tilde{y}_t + (1 - \beta) C_{\gamma Y} \mathbb{E}_{-1} \sum_{s=0}^{\infty} \beta^s \tilde{y}_s - \frac{1}{2} C_t = -\lambda_w (1 + \nu^{-1}) \xi_{w,t}. \quad (60)$$

The present-value term is constant across dates. Therefore, after differencing the date- t and date- $(t-1)$ conditions, it drops out. For $t \geq 1$,

$$\lambda_w (1 + \nu^{-1}) (\xi_{w,t} - \xi_{w,t-1}) = -\omega_y (\tilde{y}_t - \tilde{y}_{t-1}) + \frac{1}{2} (C_t - C_{t-1}). \quad (61)$$

In the dual-mandate problem, the same differenced condition holds with $C_t - C_{t-1} = 0$. All other first-order conditions and all private-sector constraints have the same time-invariant coefficients in the two problems. Thus the only non-common object in the sys-

tem is an additive deterministic forcing term proportional to $C_t - C_{t-1}$, together with a possible date-0 difference in initial multipliers induced by the present-value term.

We now use the fact that the incomplete-market covariance is a deterministic transition term. Since $\bar{c}_h$ is the discounted lifetime average of household h 's consumption response, the discounted average of C_t is zero. Moreover, along the unique bounded solution, the cross-sectional distribution induced by a date-0 perturbation converges back to the stationary distribution, so C_t converges to a finite limit and therefore $C_t - C_{t-1} \rightarrow 0$.

Because the problem is linear-quadratic with linear constraints, the full system of constraints and first-order conditions can be written as a linear rational-expectations system in a vector containing allocations and commitment multipliers. Under the maintained uniqueness and boundedness condition in the proposition, this system has a stable representation

$$V_{t+1} = AV_t + B\varepsilon_{t+1} + b_t, \quad (62)$$

for the efficiency mandate problem, where ε_{t+1} collects the one-step-ahead innovations in the exogenous shocks. The deterministic vector b_t is proportional to $C_t - C_{t-1}$ and hence satisfies $b_t \rightarrow 0$. The corresponding dual-mandate problem has the same stable matrices A and B , but no deterministic forcing term:

$$V_{t+1}^{DM} = AV_t^{DM} + B\varepsilon_{t+1}. \quad (63)$$

Any difference in the date-0 multipliers is absorbed in the initial condition.

Define $\Delta_t \equiv V_t - V_t^{DM}$. Subtracting (63) from (62) gives

$$\Delta_{t+1} = A\Delta_t + b_t.$$

The stochastic innovations enter the two systems through the same matrix B , so they cancel exactly; Δ_t is deterministic. Since the solution is stable, the spectral radius of A is strictly less than one. Let $\|\cdot\|$ denote any fixed vector norm and its induced matrix norm. Hence there are constants $M < \infty$ and $\alpha \in (0, 1)$ such that $\|A^k\| \leq M\alpha^k$ for all $k \geq 0$. Iterating,

$$\Delta_t = A^t \Delta_0 + \sum_{j=0}^{t-1} A^{t-1-j} b_j.$$

The first term converges to zero. For the second, fix any $\epsilon > 0$ and choose T such that

$\|b_j\| \leq \epsilon$ for all $j \geq T$. Then

$$\begin{aligned} \left\| \sum_{j=0}^{t-1} A^{t-1-j} b_j \right\| &\leq \sum_{j=0}^{t-1} \|A^{t-1-j} b_j\| \\ &\leq \sum_{j=0}^{t-1} \|A^{t-1-j}\| \|b_j\| \\ &\leq \sum_{j=0}^{T-1} \|A^{t-1-j}\| \|b_j\| + \epsilon \sum_{k=0}^{\infty} \|A^k\|, \end{aligned}$$

where the first step uses the triangle inequality and the second step uses compatibility of the matrix norm with the vector norm. The finite sum converges to zero as $t \rightarrow \infty$, and the second term is bounded by $\epsilon M/(1 - \alpha)$. Since ϵ is arbitrary, the whole convolution converges to zero. Thus $\Delta_t \rightarrow 0$.

The allocation vector $X_t = \{\hat{y}_t, \pi_t, \pi_{w,t}, \hat{w}_t\}$ is a linear projection of the state vector V_t and contemporaneous shocks. Since the shock component is common across the two problems and Δ_t is deterministic with $\Delta_t \rightarrow 0$, there exists a deterministic sequence D_t such that

$$X_t = S_t + D_t, \quad D_t \rightarrow 0.$$

This proves the proposition.

D.8 Proof of Proposition 7

Use the policy comparison and welfare differential in the proof of Proposition 4. Heterogeneous incidence changes the coefficients of the linearized household problem, but these coefficients are still deterministic at the efficient reference. Conditional on the inherited household distribution, first-order expected allocations therefore depend on the aggregate reform only through

$$\left\{ \mathbb{E}_{-1}(X_t - X_t^0) \right\}_{t \geq 0}$$

and the non-state-contingent transfer vector $\{dT_h\}_h$. In particular, centered changes in labor income, leisure, or inherited-asset values contribute zero to (53).

The first-order household indifference conditions $\mathbb{E}_{-1} \Delta U_h = 0$, together with the linearized equilibrium conditions and $\int dT_h dh = 0$, form a linear system. Under the maintained regular local approximation, its compensating-transfer solution defines a linear

functional $\mathcal{T}_h$ of the mean aggregate reform path:

$$dT_h = \mathcal{T}_h \left[\left\{ \mathbb{E}_{-1}(X_t - X_t^0) \right\}_{t \geq 0} \right]. \quad (64)$$

The functional includes the effects of compensation on household decisions and equilibrium prices. For a zero mean reform path, zero transfers satisfy this system, so $\mathcal{T}_h[0] = 0$.

Under the stated decomposition, $X_t = S_t + D_t$, with $\mathbb{E}_{-1}S_t = \mathbb{E}_{-1}X_t^0 = 0$ and D_t known before shocks. Hence

$$\mathbb{E}_{-1}(X_t - X_t^0) = D_t.$$

Substituting into Equation (64) shows that compensatory transfers depend only on the deterministic component. Setting $D_t = 0$ proves Proposition 7. More generally, if the status quo has a deterministic component D_t^0 , the relevant argument is $\{D_t - D_t^0\}_{t \geq 0}$. If a component is deterministic only conditional on a realized date-0 shock, its pre-shock expectation enters instead. Thus the result concerns ex-ante compensation and does not impose indifference after each shock realization.

Appendix E Appendix to section 6

E.1 Calibrating the incidence function

In the economy with sticky wages only that we use for the quantitative analysis, profits are zero and we impose $\Gamma^Y = \Gamma$. Since $y_{h,t}$ denotes real total income,

$$y_{h,t} = z_{h,t} \Gamma^Y (1 - L_t; \zeta_{h,t}) Y_t = z_{h,t} \Gamma (1 - L_t; \zeta_{h,t}) Y_t,$$

and $\gamma_h^Y = \gamma_h$. Thus, to first order around $\sigma = 0$,

$$\begin{aligned} \log y_{h,t} &= \log z_{h,t} + \gamma_h^Y d \log(1 - L_t) + d \log Y_t \\ &= \log z_{h,t} + (1 + \gamma_h) d \log(Y_t) - \gamma_h d \log(Z_t), \end{aligned}$$

where the second equality follows from the fact that $d \log Y_t = d \log Z_t + d \log(1 - L_t)$. So, income growth is given by

$$\Delta \log y_{h,t} = (1 + \gamma_h) \Delta d \log(Y_t) - \gamma_h \Delta d \log(Z_t).$$

To map the elasticity to the evidence in Guvenen et al. (2017), we assume that γ_h is only

a function of the household's current productivity, i.e., with a slight abuse of notation, $\gamma_h = \gamma_g$ where g denotes the productivity group. This procedure involves choosing γ_g to match the cross-sectional distribution of the worker beta regressions

$$\Delta \log y_{h,t} = b_{0,g} + b_{1,g} \Delta \log(Y_t) + \varepsilon_{h,t},$$

where g is the productivity group to which household h belongs.

$$\begin{aligned} \text{plim } \hat{b}_{1,g} &= 1 + \gamma_g - \gamma_g \frac{\text{Cov}(\Delta \log Z_t, \Delta \log Y_t)}{\text{Var}(\Delta \log Z_t)} \frac{\text{Var}(\Delta \log Z_t)}{\text{Var}(\Delta \log Y_t)} \\ &= 1 + \gamma_g - \gamma_g \frac{\text{Cov}(\Delta \log Z_t, \Delta \log Y_t)}{\text{SD}(\Delta \log Z_t) \text{SD}(\Delta \log Y_t)} \frac{\text{SD}(\Delta \log Z_t)}{\text{SD}(\Delta \log Y_t)} \end{aligned}$$

We use the evidence in King and Rebelo (1999) to calibrate the covariance and variance terms:

$$\frac{\text{SD}(\Delta \log Z_t)}{\text{SD}(\Delta \log Y_t)} = 0.54, \quad \frac{\text{Cov}(\Delta \log Z_t, \Delta \log Y_t)}{\text{SD}(\Delta \log Z_t) \text{SD}(\Delta \log Y_t)} = 0.78.$$

It follows that we can map

$$b_g = 1 + (1 - 0.42) \gamma_g \Leftrightarrow \gamma_g = \frac{b_g - 1}{1 - 0.42}.$$

Our parameterization uses the GDP betas from Guvenen et al. (2017) for males aged 36 to 45. We map these coefficients to b_g in our model by interpolating the betas across income groups. We normalize the coefficients such that $\sum \pi_g z_g b_g = 1$. Finally, we then apply the above mapping to obtain γ_g . The final result can be found in Figure 1.

E.2 Targeting criterion

This appendix reports the quantitative implications of the targeting criterion in Proposition 5 under the calibration in Section 6. Table 3 reports its coefficients. The expenditure-weighted variance of incidence is $\text{Var}_{\chi_h}(\gamma_h) = 0.6443$, implying $\hat{\psi} = 2.92$. Heterogeneous incidence therefore raises the weight on current output-gap stabilization relative to inflation stabilization by 36.82% compared with RANK.

The second component of the targeting criterion captures the interaction between incomplete markets and heterogeneous labor incidence. Figure 6 reports the household moments underlying this term. The left panel plots expected log consumption conditional on initial productivity, averaging over the corresponding initial asset distribution.

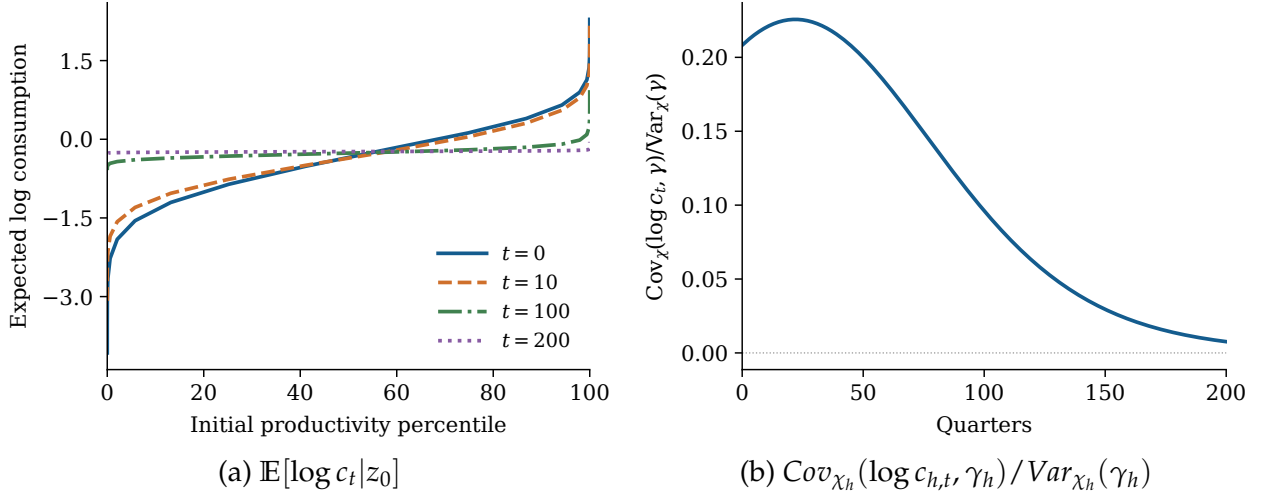
Table 3: Targeting criterion coefficients

Coefficient	Description	Value
ψ	Elasticity of substitution across labor varieties	4.00
ν	Frisch elasticity	0.75
$Var_{\chi_h}(\gamma_h)$	Variance of incidence elasticity	0.6443
$\hat{\psi}$	Targeting parameter	2.92
ζ_{IM}	Incomplete-markets coefficient	0.0345

Notes: Coefficients in Proposition 5, computed using consumption-share weights over the stationary joint distribution of productivity and assets.

These profiles become flatter as the household's future state becomes less dependent on its initial productivity. The right panel plots the covariance of log consumption with the initial incidence elasticity, normalized by $Var_{\chi_h}(\gamma_h)$.

Figure 6: Incomplete market terms in targeting criterion



Notes: The left panel averages initial assets conditional on productivity. The right panel uses fixed initial consumption-share weights and incidence labels. Horizons are quarters.

The covariance initially rises before declining toward zero. For dates $t \geq 1$, the targeting criterion depends on its change between consecutive quarters. At date zero, it depends on the covariance after subtracting each household's discounted lifetime mean log consumption. This centered initial covariance is positive in our calibration, creating an incentive for the temporary expansion shown in Figure 2.

Appendix F Variations on the definition of aggregate surplus

In this section, we discuss two modifications to the definition of aggregate surplus. What happens if we: (1) define aggregate surplus using a factor-biased technology shock, (2) alter the timing of the lump-sum compensations. We show that our characterization of the approximately optimal policy is invariant to these changes.

F.1 Biased productivity variation

Our baseline definition measures aggregate surplus by asking how much the entire time endowment can be contracted while keeping every household indifferent to the status-quo allocation. One could instead measure the same efficiency gain in biased productivity units. For example, the productivity shifter could apply only to market production time, leaving the productivity of home-production/leisure unchanged. More generally, let $\tilde{A}(\mathcal{P})$ denote the largest contraction in the productivity of a subset of factor endowments, rather than a uniform contraction of all factor endowments, such that the compensated allocation remains feasible and every household is kept indifferent to the status-quo allocation $\mathbf{x}^0$.

This alternative changes the units in which aggregate surplus is reported, but not the local policy problem. The formal argument is given in Appendix B of Baqaee et al. (2026). Around the efficient allocation, the Hessian of the measure of aggregate surplus in biased productivity units with respect to wedges is proportional to the Hessian of the baseline TFP-equivalent measure. If the biased shifter applies to a factor, or set of factors, with income share $\lambda_{F^*} > 0$, then, to a second-order approximation,

$$\log \tilde{A}(\mathcal{P}) \approx \frac{1}{\lambda_{F^*}} \log A(\mathcal{P}).$$

Thus the biased measure rescales the deadweight-loss-triangle objective by a positive constant.

It follows that all first-order policy prescriptions in the paper are unchanged. The quadratic approximation in Proposition 1 would be multiplied by a positive factor, and the status-quo misallocation term would be expressed in different productivity units. But the policy-relevant first-order conditions are identical. Hence the orthogonality results for the canonical HANK cases, the targeting criterion with cyclical income risk, and the long-run equivalence to a modified dual-mandate policy do not depend on whether aggregate

surplus is measured in uniform time-endowment units or in biased factor-productivity units.

F.2 Timing of transfers

In the baseline definition of aggregate surplus, one-off compensating transfers are paid out at date 0. This is a normalization for the local policy problem rather than a substantive restriction on the date at which compensation is delivered.¹⁷ This subsection explains why.

Consider an alternative definition in which household h receives a non-state-contingent transfer schedule $\{T_{h,t}\}_{t \geq 0}$ rather than a single date-0 transfer. The transfer schedule may vary across households and across calendar dates, but it is fixed independently of the household's future idiosyncratic shocks and satisfies budget balance at each date,

$$\int T_{h,t} dh = 0.$$

Let Q_t denote the date-0 price of one unit of the final good delivered at date t , evaluated at the allocation around which we approximate. The present value of household h 's transfer schedule is

$$\mathcal{T}_h \equiv \sum_{t=0}^{\infty} Q_t T_{h,t}.$$

If portfolio constraints are slack, the household's intertemporal problem depends on the schedule $\{T_{h,t}\}$ only through this present value. Hence the schedule is locally equivalent to a date-0 transfer $\mathcal{T}_h$.

The required slackness holds at our point of approximation. We approximate around the efficient allocation with zero household asset positions. By assumption, $\Omega(0, \{0\}_k) > 0$, so the portfolio constraints are slack at that allocation. Since $\Omega(\cdot)$ is continuously differentiable, there is an open neighborhood of zero asset positions in which the constraints remain slack. Therefore, for any sufficiently small compensating transfer schedule, households can use the available financial assets to shift resources across dates without changing which portfolio constraints bind. Delaying or accelerating a non-state-contingent payment only changes the household's financing position inside this neighborhood.

This also shows that the timing of such transfers does not create incomplete-market wedges to first order. Holding fixed aggregate prices and quantities at the efficient allocation, a non-state-contingent transfer changes household h 's lifetime wealth and therefore

¹⁷The substantive restriction is that compensating transfers are not contingent on households' future idiosyncratic histories. We explore this in more detail in Section F.3.

its multiplier on the intertemporal budget constraint. Since the constraint set is locally interior, the first-order conditions imply

$$d \log c_{h,t} - d \log C_t = \eta_h \quad \text{for all } t,$$

for some household-specific constant η_h that depends on the present value $\mathcal{T}_h$, but not on the timing of the schedule. With log utility, the incomplete-market wedge in (41) satisfies

$$\begin{aligned} d \log \mu_{h,t}^{IM} &= -[d \log c_{h,t} - d \log C_t] + [d \log c_{h,0} - d \log C_0] \\ &= -\eta_h + \eta_h = 0. \end{aligned}$$

Thus a non-state-contingent compensating transfer can redistribute the present value of resources across households, but it does not by itself provide insurance across idiosyncratic states or alter intertemporal wedges at the margin. In particular, replacing date-0 transfers with any locally small non-state-contingent payout schedule leaves unchanged the first-order incomplete-market wedges that enter the second-order approximation in Proposition 1. The quadratic policy problem and all the first-order policy prescriptions are therefore the same.

State-contingent transfers are different. If transfers can depend on the household's realized idiosyncratic history, the policymaker is no longer only choosing the timing of compensation. It is expanding the set of insurance instruments available to households. With sufficiently rich person-time-state-contingent transfers, the policymaker can equate the ratio of marginal utilities across dates and states, thereby setting the eliminating incomplete-market wedges. This is the case studied in Appendix F.3. Hence our date-0 formulation should be read as a convenient representative of the class of non-state-contingent compensation schemes, not as an assumption that the calendar timing of compensation is economically important for the local analysis.

F.3 Optimal monetary policy with state-contingent transfers

In this appendix, we study the problem of the efficiency-maximizing central bank under the assumption that it can implement arbitrary transfers in each period. This setting, while unrealistic, is instructive and allows us to characterize the efficiency-maximizing policy in a simple way.

Without any restriction on transfers, the efficiency-maximizing allocations must be such that the incomplete market wedges are always equal to zero. In other words, the Euler equation is always satisfied with equality for all households and states, since person-

time-state contingent transfers are used to substitute incomplete markets. So, in this case

$$c_{h,t} = \alpha_h C_t$$

for some $\alpha_h \geq 0$ such that $\int \alpha_h dh = 1$.

Suppose further that labor-income incidence function is such that there is no dispersion of labor wedges across households, i.e., $\log \mu_{h,t}^L = \log \mu_t^L$ for all h , and so

$$\frac{\nu c_{h,t}}{z_{h,t} l_{h,t}} = \frac{\nu C_t}{L_t}, \quad \text{for all } h.$$

This would be the case, for instance, in a model without nominal wage rigidity in which households would choose labor supply optimally. In this case, it follows that

$$z_{h,t} l_{h,t} = \alpha_h L_t.$$

Using the fact that, in the optimum, $\tilde{U}_h(\mathbf{x}_h)$ is equalized across h , and that $\int \alpha_h dh = 1$, it follows that

$$\alpha_h = \frac{\bar{z}_h^{\frac{\nu}{1+\nu}} \exp\{U(\mathbf{x}_h^0)\}}{\int \bar{z}_{h'}^{\frac{\nu}{1+\nu}} \exp\{U(\mathbf{x}_{h'}^0)\} dh'}.$$

The productivity factor $\bar{z}_h^{\nu/(1+\nu)}$, defined in Appendix B, is invariant to monetary policy. The utility of the compensated agent is

$$\min_h \tilde{U}_h(\mathbf{x}_h) = \frac{\exp\{U(\{C_t, L_t\})\}}{\int \bar{z}_{h'}^{\frac{\nu}{1+\nu}} \exp\{U(\mathbf{x}_{h'}^0)\} dh'}. \quad (65)$$

It follows that the equilibrium is as if the aggregates were determined by a representative agent with the original preferences $U(\{C_t, L_t\})$, i.e., the equilibrium aggregate quantities coincide with those in the RANK model under policy $\mathcal{P}$.

Proposition 10. *Consider the case with state-contingent transfers. Suppose furthermore that labor-income incidence is such that there is no dispersion of labor wedges across households. Then, the aggregate surplus of policy $\mathcal{P}$ is given by*

$$A(\mathcal{P}) = \frac{\exp\{U(\{C_t^{\text{RANK}}(\mathcal{P}), L_t^{\text{RANK}}(\mathcal{P})\})\}}{\int \bar{z}_{h'}^{\frac{\nu}{1+\nu}} \exp\{U(\mathbf{x}_{h'}^0)\} dh'}, \quad (66)$$

where $C_t^{\text{RANK}}(\mathcal{P})$ and $L_t^{\text{RANK}}(\mathcal{P})$ are the equilibrium aggregate consumption and leisure under policy $\mathcal{P}$ in the RANK model. Hence, the efficiency-maximizing policy coincides with the optimal

policy prescriptions in RANK, i.e.,

$$\mathcal{P}^* = \arg \max_{\mathcal{P}} A(\mathcal{P}) = \arg \max_{\mathcal{P}} U \left(\{C_t^{RANK}(\mathcal{P}), L_t^{RANK}(\mathcal{P})\} \right). \quad (67)$$

With access to state-contingent transfers, the efficient allocations are such that the incomplete market wedges are always equal to zero. If, on top of that, labor-income incidence is such that there is a common labor wedge across households, then the efficient allocations are such that aggregate allocations coincide with those in the RANK model. Furthermore, aggregate surplus is a monotone transformation of the representative agent's utility in the RANK model and the efficiency-mandate optimal policy coincides with the optimal policy prescriptions for the RANK model.

Intuitively, in the absence of incomplete market wedges and with a common labor wedge across households, the only sources of inefficiencies in this economy coincide with those in the RANK model, i.e., the price and wage markups. Our efficiency-mandate central bank chooses the policy that best addresses these sources of inefficiency, ignoring the underlying heterogeneity across households. So, the optimal policy exactly coincides with the optimal policy in the RANK model.

Appendix G Shocks to the distribution of productivity

In the baseline model, cyclical changes in the distribution of labor income arise from heterogeneous changes in hours worked. We calibrate this incidence using the worker betas in Guvenen et al. (2017). A different approach allows aggregate shocks to change the cross-sectional distribution of productivity, as in La'O and Morrison (2024). The exogenous inequality shock in McKay and Wolf (2026) also changes relative earnings independently of hours.¹⁸ In this appendix, we study this alternative source of cyclical redistribution and its implications for the efficiency mandate.

We retain the model in Section 2, but allow the cross-sectional distribution of $z_{h,t}$ to vary with aggregate shocks. The joint process for these shocks and household productivities is exogenous to monetary policy. We assume

$$z_{h,t} > 0, \quad \int z_{h,t} dh = 1, \quad (68)$$

at every date and aggregate history. The normalization isolates changes in the distribution of productivity, leaving efficient output at $Y_t^* = Z_t / (1 + \nu)$.

We choose the incidence functions so that there is a common labor wedge across households. With log preferences, this requires

$$\frac{\nu c_{h,t}}{z_{h,t} l_{h,t}} = \frac{\nu Y_t}{L_t}, \quad \text{or equivalently} \quad \frac{z_{h,t} l_{h,t}}{L_t} = \frac{c_{h,t}}{Y_t}. \quad (69)$$

Using the average nominal wage $\tilde{W}_t$ defined alongside (39), household real labor income is

$$\frac{\tilde{W}_t}{P_t} z_{h,t} (1 - l_{h,t}) = \frac{\tilde{W}_t}{P_t} \left[z_{h,t} - L_t \frac{c_{h,t}}{Y_t} \right].$$

Thus productivity shocks change relative earnings even without heterogeneous labor wedges. We distribute profits net of the associated lump-sum taxes in proportion to $z_{h,t}$. As in our local policy analysis, we approximate jointly around the efficient allocation with no risk and zero asset holdings. Financial constraints are locally slack, and available assets span aggregate risk with a regular payoff system at that allocation. Aggregate shocks are publicly observed, and households share the same conditional forecasts of these shocks.

¹⁸An earlier example is Werning (2007), who studies shocks to the distribution of skills in a fiscal-policy model. Bhandari et al. (2021) study heterogeneous productivity exposures in Section 6.4 as an extension of their baseline. McKay and Wolf (2026) also allow relative earnings to depend on aggregate activity. Here we isolate the exogenous component.

Proposition 11 (Aggregate surplus with productivity distribution shocks). *Under the preceding assumptions, the second-order approximation of aggregate surplus is*

$$\log A(\mathcal{P}) \approx \log \mathcal{D} - \frac{(1-\beta)\omega_C}{2} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[(1+\nu^{-1})(\hat{y}_t - \hat{y}_t^*)^2 + \frac{\varepsilon}{\lambda_p} \pi_t^2 + \frac{\psi}{\lambda_w} \pi_{w,t}^2 \right] - \frac{1}{2} E_{\chi_h} \left[\text{Var}_{\beta, \pi} \left(d \log \frac{c_{h,t}}{Y_t} \right) \right]. \quad (70)$$

The incomplete-markets term is independent of the first-order monetary policy choice. Hence, the approximately optimal policy minimizes the same loss in output gaps, price inflation, and wage inflation as in RANK.

To see why productivity dispersion introduces no additional term, use (69) to write period utility as

$$\omega_C \log c_{h,t} + (1-\omega_C) \log l_{h,t} = \log \frac{c_{h,t}}{Y_t} + \omega_C \log Y_t + (1-\omega_C) \log L_t - (1-\omega_C) \log z_{h,t}.$$

The last term is the same in the actual and efficient allocations at the same fundamentals, so it cancels from the compensated comparison. The remaining losses are the standard RANK terms and the incomplete-markets variance, with the weights defined in Proposition 1.

These results contrasts with the heterogeneous incidence of hours in the baseline. Changes in the distribution of productivity generate inequality, but do not themselves create dispersion in labor wedges, and so inefficiencies. Incomplete insurance can still generate losses, but monetary policy cannot reduce those losses locally in this environment.

Appendix H Optimal simple policy rules

Following Woodford (1999), we restrict the policymaker to allocations that are affine functions of current aggregate shocks, with time-invariant coefficients. We consider one source of nominal rigidity at a time. Write $z_t \equiv \hat{z}_t = \hat{y}_t^*$ for the productivity (and efficient-output) log deviation in this section. The demand shock, natural-output shock, and relevant cost-push shock follow mutually independent AR(1) processes, with persistence parameters ρ_δ , ρ_z , and ρ_u , respectively. The cost-push shock is $u_{p,t}$ in the sticky-price case and $u_{w,t}$ in the sticky-wage case. Each process starts from zero prehistory and receives mean-zero innovations from date 0 onward. The status-quo policy is also centered, with no deterministic aggregate distortion.

H.1 Sticky-price theory

With flexible wages and homogeneous labor wedges, the incomplete-market loss is locally orthogonal to monetary policy. The simple-rule problem therefore coincides with its RANK counterpart.

Proposition 12. *Consider the sticky-price model with flexible wages, no wage cost-push shocks, and homogeneous labor wedges with simple policy rules of the form:*

$$\hat{y}_t = \phi_y + \phi_{y,\delta}\delta_t + \phi_{y,z}z_t + \phi_{y,u}u_{p,t} \quad (71)$$

$$\hat{\pi}_t = \phi_\pi + \phi_{\pi,\delta}\delta_t + \phi_{\pi,z}z_t + \phi_{\pi,u}u_{p,t}. \quad (72)$$

Then, the optimal simple policy rules in HANK coincide with those in the RANK model and satisfy:

- i. Zero average inflation and output gap: $\phi_y = \phi_y^{RANK} \equiv 0$ and $\phi_\pi = \phi_\pi^{RANK} \equiv 0$.
- ii. Full accommodation of demand shocks: $\phi_{y,\delta} = \phi_{y,\delta}^{RANK} \equiv 0$ and $\phi_{\pi,\delta} = \phi_{\pi,\delta}^{RANK} \equiv 0$.
- iii. Optimal response to natural output shocks: $\phi_{y,z} = \phi_{y,z}^{RANK} \equiv 1$ and $\phi_{\pi,z} = \phi_{\pi,z}^{RANK} \equiv 0$.
- iv. Leaning against cost-push shocks:

$$\phi_{y,u} = \phi_{y,u}^{RANK} \equiv -\frac{\varepsilon}{\kappa_p \varepsilon + (1 - \beta \rho_u)^2} \quad \text{and} \quad \phi_{\pi,u} = \phi_{\pi,u}^{RANK} \equiv \frac{(1 - \beta \rho_u)}{\kappa_p \varepsilon + (1 - \beta \rho_u)^2},$$

where $\kappa_p \equiv \lambda_p \{1 + v^{-1}\}$.

H.1.1 Proof

In the sticky-price model, wages are flexible, $u_{w,t} = 0$, and $\gamma_h = 0$ for all h . By Proposition 3, the incomplete-market term is locally orthogonal to monetary policy and the reduced-form policy problem coincides with the sticky-price RANK problem. Thus it is enough to solve the RANK simple-rule problem.

Let $\tilde{y}_t \equiv \hat{y}_t - z_t$ denote the output gap and let

$$\kappa_p \equiv \lambda_p(1 + \nu^{-1}).$$

In the sticky-price model the price Phillips curve reduces to

$$\pi_t = \kappa_p \tilde{y}_t + \beta \mathbb{E}_t[\pi_{t+1}] + u_{p,t}.$$

The relevant loss is

$$\frac{1}{2} \mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \left[(1 + \nu^{-1}) \tilde{y}_t^2 + \frac{\varepsilon}{\lambda_p} \pi_t^2 \right].$$

Under the proposed rules,

$$\tilde{y}_t = \phi_y + \phi_{y,\delta} \delta_t + (\phi_{y,z} - 1) z_t + \phi_{y,u} u_{p,t}, \quad \pi_t = \phi_\pi + \phi_{\pi,\delta} \delta_t + \phi_{\pi,z} z_t + \phi_{\pi,u} u_{p,t}.$$

The shocks are mean zero and mutually orthogonal, so the problem separates across the constant, demand-shock, natural-output, and cost-push components. Substituting the rules into the Phillips curve and using $\mathbb{E}_t[\vartheta_{t+1}] = \rho_\vartheta \vartheta_t$ for $\vartheta \in \{\delta, z, u_p\}$ gives the coefficient restrictions

$$\begin{aligned} (1 - \beta) \phi_\pi &= \kappa_p \phi_y, \\ (1 - \beta \rho_\delta) \phi_{\pi,\delta} &= \kappa_p \phi_{y,\delta}, \\ (1 - \beta \rho_z) \phi_{\pi,z} &= \kappa_p (\phi_{y,z} - 1), \\ (1 - \beta \rho_u) \phi_{\pi,u} &= \kappa_p \phi_{y,u} + 1. \end{aligned}$$

For the first three components, the feasible choice that sets both inflation and the output gap to zero is available and uniquely minimizes the quadratic loss. Hence

$$\phi_y = \phi_\pi = 0, \quad \phi_{y,\delta} = \phi_{\pi,\delta} = 0, \quad \phi_{y,z} = 1, \quad \phi_{\pi,z} = 0.$$

For the cost-push component, write $a_u \equiv 1 - \beta\rho_u$. The Phillips-curve restriction gives

$$\phi_{y,u} = -\frac{1 - a_u\phi_{\pi,u}}{\kappa_p}.$$

The cost-push part of the loss is proportional to

$$\frac{\varepsilon}{\lambda_p}\phi_{\pi,u}^2 + (1 + \nu^{-1}) \left(\frac{1 - a_u\phi_{\pi,u}}{\kappa_p} \right)^2.$$

The first-order condition is

$$\kappa_p\varepsilon\phi_{\pi,u} = a_u(1 - a_u\phi_{\pi,u}),$$

so

$$\phi_{\pi,u} = \frac{a_u}{\kappa_p\varepsilon + a_u^2}, \quad \phi_{y,u} = -\frac{\varepsilon}{\kappa_p\varepsilon + a_u^2}.$$

Substituting back $a_u = 1 - \beta\rho_u$ gives the expressions in the proposition.

H.2 Sticky-wage theory

With flexible prices and heterogeneous labor incidence, the simple-rule problem retains the labor-dispersion and lifetime-income covariance terms. These modify the relative weight on output stabilization. The following proposition states the coefficients; its proof keeps these terms separate from the incomplete-market interaction.

Proposition 13 (Simple rules in a sticky-wage HANK model). *Consider the sticky-wage model with flexible prices and no price cost-push shocks with policy rules of the form:*

$$\hat{y}_t = \phi_y + \phi_{y,\delta}\delta_t + \phi_{y,z}z_t + \phi_{y,u}u_{w,t} \tag{73}$$

$$\hat{\pi}_{w,t} = \phi_\pi + \phi_{\pi,\delta}\delta_t + \phi_{\pi,z}z_t + \phi_{\pi,u}u_{w,t}. \tag{74}$$

Then, the optimal policy rules in HANK satisfy:

- i. Zero average inflation and output gap: $\phi_y = \phi_y^{\text{RANK}} \equiv 0$ and $\phi_\pi = \phi_\pi^{\text{RANK}} \equiv 0$.
- ii. Full accommodation of demand shocks: $\phi_{y,\delta} = \phi_{y,\delta}^{\text{RANK}} \equiv 0$ and $\phi_{\pi,\delta} = \phi_{\pi,\delta}^{\text{RANK}} \equiv 0$.
- iii. Optimal response to natural output shocks: $\phi_{y,z} = \phi_{y,z}^{\text{RANK}} \equiv 1$ and $\phi_{\pi,z} = \phi_{\pi,z}^{\text{RANK}} \equiv 0$.

iv. *Leaning against cost-push shocks:*

$$\phi_{y,u} = -\frac{\tilde{\psi}}{\kappa_w \tilde{\psi} + (1 - \beta \rho_u)^2}, \quad \text{and} \quad \phi_{\pi,u} = \frac{(1 - \beta \rho_u)}{\kappa_w \tilde{\psi} + (1 - \beta \rho_u)^2}.$$

where $\kappa_w \equiv \lambda_w \{1 + \nu^{-1}\} > 0$ and

$$\tilde{\psi} \equiv \frac{\psi}{1 + \frac{1}{1+\nu^{-1}} \left[\nu^{-1} \text{Var}_{\chi_h}(\gamma_h) + \frac{(1-\beta)(1-\beta\rho_u^2)\text{Cov}_{\chi_h}(\gamma_h, \gamma_h^Y)}{(1-\beta\rho_u)^2(1+\beta)} \right]} > 0.$$

If $\text{Var}_{\chi_h}(\gamma_h) > 0$ and $\text{Cov}_{\chi_h}(\gamma_h, \gamma_h^Y) \geq 0$, then $\tilde{\psi} < \psi$ and the optimal policy puts more weight on output stabilization relative to inflation stabilization than in the RANK model, i.e., $\phi_{y,u} > \phi_{y,u}^{\text{RANK}}$ and $\phi_{\pi,u} > \phi_{\pi,u}^{\text{RANK}}$.

H.2.1 Proof

Let $\tilde{y}_t \equiv \hat{y}_t - z_t$ and

$$V_\gamma \equiv \text{Var}_{\chi_h}(\gamma_h), \quad C_{\gamma Y} \equiv \text{Cov}_{\chi_h}(\gamma_h, \gamma_h^Y), \quad \kappa_w \equiv \lambda_w(1 + \nu^{-1}).$$

In the sticky-wage model the wage Phillips curve is

$$\pi_{w,t} = \kappa_w \tilde{y}_t + \beta \mathbb{E}_t[\pi_{w,t+1}] + u_{w,t}.$$

The policy-dependent part of the second-order loss is proportional to

$$\mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t \left[\frac{\psi}{\lambda_w} \pi_{w,t}^2 + \left(1 + \nu^{-1} + \nu^{-1} V_\gamma\right) \tilde{y}_t^2 \right] + (1 - \beta) C_{\gamma Y} \mathbb{E}_{-1} \left(\sum_{t=0}^{\infty} \beta^t \tilde{y}_t \right)^2.$$

The linear incomplete-market term does not affect the optimal time-invariant simple rules. Indeed,

$$(1 - \beta) \sum_{t=0}^{\infty} \beta^t \text{Cov}_{\chi_h} \left(d \log \frac{c_{h,t}}{\bar{c}_h}, \gamma_h \right) = 0$$

by the definition of $\bar{c}_h$, and the aggregate shocks entering the simple rules have zero unconditional means.

Under the proposed rules,

$$\tilde{y}_t = \phi_y + \phi_{y,\delta} \delta_t + (\phi_{y,z} - 1) z_t + \phi_{y,u} u_{w,t}, \quad \pi_{w,t} = \phi_\pi + \phi_{\pi,\delta} \delta_t + \phi_{\pi,z} z_t + \phi_{\pi,u} u_{w,t}.$$

Substituting these rules into the wage Phillips curve gives

$$\begin{aligned}(1 - \beta)\phi_\pi &= \kappa_w \phi_y, \\ (1 - \beta\rho_\delta)\phi_{\pi,\delta} &= \kappa_w \phi_{y,\delta}, \\ (1 - \beta\rho_z)\phi_{\pi,z} &= \kappa_w(\phi_{y,z} - 1), \\ (1 - \beta\rho_u)\phi_{\pi,u} &= \kappa_w \phi_{y,u} + 1.\end{aligned}$$

The constant, demand-shock, and natural-output components are minimized by setting both wage inflation and the output gap to zero. Therefore

$$\phi_y = \phi_\pi = 0, \quad \phi_{y,\delta} = \phi_{\pi,\delta} = 0, \quad \phi_{y,z} = 1, \quad \phi_{\pi,z} = 0.$$

It remains to solve for the cost-push coefficients. Let $a_u \equiv 1 - \beta\rho_u$. The wage Phillips curve implies

$$\phi_{y,u} = -\frac{1 - a_u \phi_{\pi,u}}{\kappa_w}.$$

For the AR(1) cost-push experiment with zero prehistory and new innovations each quarter,

$$\frac{\mathbb{E}_{-1} \left(\sum_{t=0}^{\infty} \beta^t u_{w,t} \right)^2}{\mathbb{E}_{-1} \sum_{t=0}^{\infty} \beta^t u_{w,t}^2} = \frac{1 - \beta\rho_u^2}{(1 - \beta\rho_u)^2(1 + \beta)},$$

which follows from the fact that since:

$$\sum_{t=0}^{\infty} \beta^t u_{w,t} = \sum_{t=0}^{\infty} \beta^t \sum_{s=0}^t \rho^{t-s} \varepsilon_s = \sum_{s=0}^{\infty} \varepsilon_s \sum_{t=s}^{\infty} \beta^t \rho^{t-s} = \frac{1}{1 - \beta\rho} \sum_{s=0}^{\infty} \beta^s \varepsilon_s,$$

then

$$\begin{aligned}\mathbb{E} \left(\sum_{t=0}^{\infty} \beta^t u_{w,t} \right)^2 &= \frac{\sigma_u^2}{(1 - \beta\rho)^2} \sum_{s=0}^{\infty} \beta^{2s} = \frac{\sigma_u^2}{(1 - \beta\rho)^2(1 - \beta^2)} \\ \mathbb{E} \sum_{t=0}^{\infty} \beta^t u_{w,t}^2 &= \sigma_u^2 \sum_{t=0}^{\infty} \beta^t \sum_{j=0}^t \rho^{2j} = \sigma_u^2 \sum_{t=0}^{\infty} \beta^t \frac{1 - \rho^{2(t+1)}}{1 - \rho^2} = \frac{\sigma_u^2}{(1 - \beta)(1 - \beta\rho^2)}.\end{aligned}$$

Hence the cost-push component of the loss is proportional to

$$\frac{\psi}{\lambda_w} \phi_{\pi,u}^2 + \left[1 + \nu^{-1} + \nu^{-1} V_\gamma + \frac{(1 - \beta)(1 - \beta\rho_u^2)C_{\gamma Y}}{(1 - \beta\rho_u)^2(1 + \beta)} \right] \phi_{y,u}^2.$$

Equivalently, using $\kappa_w = \lambda_w(1 + \nu^{-1})$, this is

$$\frac{\psi}{\lambda_w} \phi_{\pi,u}^2 + (1 + \nu^{-1}) \kappa_{\gamma Y} \phi_{y,u}^2, \quad \kappa_{\gamma Y} \equiv 1 + \frac{1}{1 + \nu^{-1}} \left[\nu^{-1} V_\gamma + \frac{(1 - \beta)(1 - \beta \rho_u^2) C_{\gamma Y}}{(1 - \beta \rho_u)^2 (1 + \beta)} \right].$$

Thus the sticky-wage HANK simple-rule problem is isomorphic to the RANK problem with the wage-inflation coefficient ψ replaced by

$$\tilde{\psi} \equiv \frac{\psi}{\kappa_{\gamma Y}}.$$

The first-order condition for $\phi_{\pi,u}$ is

$$\kappa_w \tilde{\psi} \phi_{\pi,u} = a_u (1 - a_u \phi_{\pi,u}),$$

so

$$\phi_{\pi,u} = \frac{a_u}{\kappa_w \tilde{\psi} + a_u^2}, \quad \phi_{y,u} = -\frac{\tilde{\psi}}{\kappa_w \tilde{\psi} + a_u^2}.$$

Whenever $\kappa_{\gamma Y} > 1$, we have $\tilde{\psi} < \psi$. Therefore $\phi_{y,u}$ is less negative than the corresponding RANK coefficient and $\phi_{\pi,u}$ is larger than the corresponding RANK coefficient. In the aligned-incidence calibration, $C_{\gamma Y} = V_\gamma > 0$, so this condition is satisfied. Substituting $a_u = 1 - \beta \rho_u$ gives the proposition.

H.2.2 Interpretation and compensatory transfers

Under the sign conditions in Proposition 13, $\tilde{\psi} < \psi$: HANK permits more wage inflation and a smaller output-gap contraction than RANK in response to a cost-push shock. The ratio of the absolute output-gap and inflation responses is

$$\frac{|\phi_{y,u}|}{|\phi_{\pi,u}|} = \frac{\tilde{\psi}}{1 - \beta \rho_u}.$$

This distinction matters because persistence enters the Phillips-curve trade-off as well as the lifetime-income term.

Corollary 8 (First-order compensation under simple rules). *Under the centered-shock and status-quo assumptions above, implementing either optimal simple rule requires no first-order ex-ante compensatory transfers: $dT_h = 0$ for every household.*

Both rules have zero intercepts and zero ex-ante mean aggregate reform paths. The linear compensation map in (64) therefore gives $dT_h = \mathcal{T}_h[0] = 0$. This statement concerns

first-order compensation; it does not assert that the exact compensatory transfers vanish at higher orders.

H.3 Numerical implementation of optimal simple rules in the sticky-wage HANK model

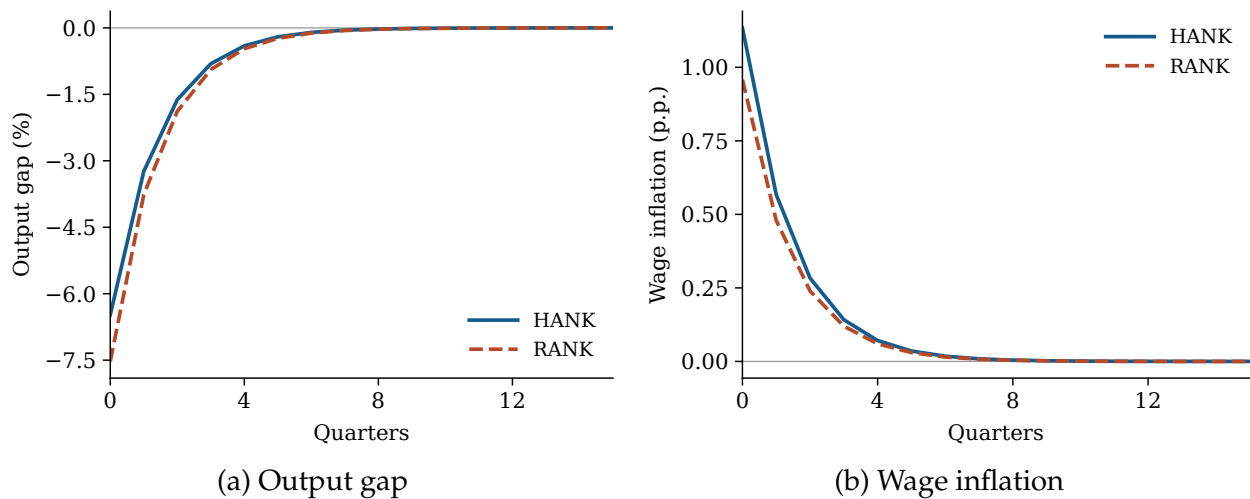
The problem under commitment can be complex to implement in practice. A natural question is whether the key insights carry over to simple policy rules that a central bank could more easily communicate and follow. We therefore consider the rules in Proposition 13, under which the output gap and wage inflation are linear functions of the current shocks.

The HANK simple rule produces a smaller output-gap contraction and a larger wage-inflation response than the RANK rule. In this calibration,

$$\frac{\phi_{y,u}}{\phi_{y,u}^{RANK}} = 0.8602, \quad \frac{\phi_{\pi,u}}{\phi_{\pi,u}^{RANK}} = 1.1833.$$

Thus the response to a given cost-push shock has a 13.98% smaller output-gap contraction and 18.33% more wage inflation. Since both variables are proportional to the shock under these rules, the same percentage differences apply to their impact and cumulative responses.

Figure 7: Optimal policy – simple-rules response to cost-push shock



Notes: Responses under optimal simple rules to the same cost-push innovation as in Figure 3, with persistence 0.5. Output gaps are in percent and wage inflation in quarterly percentage points.

Figure 7 plots the responses to the same cost-push innovation as in the commitment solution. In both models, simple rules produce higher impact inflation and a larger initial output contraction than commitment. The greater weight on output stabilization under HANK remains evident.

Appendix I Utility transformations and SWF-optimal policy

This appendix extends Section 7 by applying a common monotone transformation to household utility. Preferences and equilibrium behavior are unchanged, but the transformation affects the curvature of social welfare and, together with welfare weights, its preferred wealth distribution. We derive the resulting policy problem, present the numerical results, and then provide the welfare proof and exact supporting-weight conditions.

We use the illustrative economy of Section 7: complete markets, no idiosyncratic risk, $\nu > 0$, constant productivity, flexible goods prices, sticky wages, and homogeneous incidence. There are no profits or cost-push shocks. The reference allocation has efficient aggregate quantities and inherited wealth shares χ_h^0 . Reference utilities and inherited nominal claims remain fixed across policy and demand-shock counterfactuals.

I.1 Transformed utility and the welfare approximation

Let U_h^0 denote household h 's utility at the no-shock status quo and write $B \equiv (1 - \beta)^{-1}$. Define

$$\tilde{U}_h \equiv \frac{\left(1 + \frac{U(\mathbf{x}_h) - U_h^0}{1 - \beta}\right)^\rho}{\rho}, \quad \mathcal{W}_\rho \equiv \int \bar{\chi}_h \tilde{U}_h dh, \quad (75)$$

where $\rho \leq 1$ controls the additional curvature relative to log. The bracketed utility index is restricted to be positive; for $\rho = 0$, define $\tilde{U}_h = \log(1 + [U(\mathbf{x}_h) - U_h^0]/(1 - \beta))$. The transformation first recenters this index at one at the no-shock status quo and then applies the curvature transformation once.¹⁹ At $\rho = 1$, this objective is a positive affine transformation of the baseline SWF in Section 7. Every member of this class represents the same underlying preferences.

To separate the preferred distribution from the strength of distributional concerns, let $\chi_h^{\text{FB}} > 0$ denote the desired wealth shares in the quadratic approximation. We choose welfare weights according to

$$\bar{\chi}_h = \frac{(\chi_h^{\text{FB}})^{\kappa_\rho} (\chi_h^0)^{1 - \kappa_\rho}}{\int (\chi_j^{\text{FB}})^{\kappa_\rho} (\chi_j^0)^{1 - \kappa_\rho} dj}, \quad (76)$$

where $\kappa_\rho \equiv 1 + (1 - \rho)/(1 - \beta) \geq 1$. The next proposition derives the second-order approximation of the SWF, extending that in Proposition 8.

¹⁹The factor $1/(1 - \beta)$ expresses utility gains as discounted sums.

Proposition 14 (Second-order approximation of social welfare). *In the illustrative model of Section 7, the SWF in (75), with weights (76), can be approximated by*

$$\begin{aligned} \frac{\mathcal{W}_\rho - \mathcal{W}_\rho^{\text{FB}}}{\bar{\Lambda}} \approx & - \overbrace{\frac{(1-\beta)\omega_C}{2} \sum_{t=0}^{\infty} \beta^t \left[(1+\nu^{-1})\tilde{y}_t^2 + \frac{\psi}{\lambda_w} \pi_{w,t}^2 \right]}^{\text{RANK loss}} \\ & - \underbrace{\frac{\kappa_\rho}{2} \text{Var}_{\chi_h^0}(\log \chi_h - \log \chi_h^{\text{FB}})}_{\text{Redistributional loss}}, \end{aligned} \quad (77)$$

where $\bar{\Lambda} = B$ and the variance uses fixed reference expenditure weights χ_h^0 . For fixed ρ and β , the error is third-order jointly in shocks, policies, and deviations of welfare weights from reference shares.

The first term is denoted by $\mathcal{H}_{\text{RANK}}$ below. Section I.4 provides the proof.

We compare three welfare criteria. *Egalitarian preferences* keep the desired shares equal, $\chi_h^{\text{FB}} = 1$, while *Negishi weights* keep the target at the inherited shares, $\chi_h^{\text{FB}} = \chi_h^0$. An *equally weighted utilitarian* SWF instead holds $\bar{\chi}_h = 1$. Equal weights on transformed utility do not generally imply equal desired wealth shares. Inverting (76), the utilitarian target in the quadratic approximation is

$$\chi_h^{\text{FB}} = \frac{(\chi_h^0)^{1-1/\kappa_\rho}}{\int (\chi_j^0)^{1-1/\kappa_\rho} dj}. \quad (78)$$

At $\rho = 1$, the utilitarian and egalitarian criteria coincide with the equal-weight case in Section 7. Lowering ρ increases the penalty on unequal utility gains. With equal welfare weights, this also moves the preferred shares toward the inherited distribution. Egalitarian preferences instead continue to favor equal shares, while Negishi weights continue to favor the inherited shares.

The target-weight mapping above concerns the quadratic approximation. Section I.5 derives exact supporting weights and the restrictions imposed by positivity of the recentered utility index.

I.2 Monetary redistribution and optimal policy

Let $V \equiv \sum_{t=0}^{\infty} q_t w_t$ denote the present value of the common time endowment. The wealth map in Section 7 implies

$$\chi_h = 1 + \frac{a_{h,-1}}{P_0 V} = 1 + (\chi_h^0 - 1)e^{-S}, \quad S \equiv \log \frac{P_0 V}{P_0^0 V^0}.$$

Define $E_0[x_h] \equiv \int \chi_h^0 x_h dh$, $r_h \equiv \log(\chi_h / \chi_h^0)$, and $b_h \equiv (\chi_h^0 - 1) / \chi_h^0$. Let Ξ be the first-order part of S and ϵ denote the perturbation size. Then $E_0[b_h] = 0$ and $r_h = -b_h \Xi + O(\epsilon^2)$. Write

$$D \equiv \text{Var}_{\chi_h^0}(b_h), \quad K \equiv \text{Cov}_{\chi_h^0}(b_h, \log \chi_h^0).$$

Substituting this share response into (77), and omitting policy-independent constants, gives

$$\mathcal{L}_c = \mathcal{H}_{\text{RANK}} - C_c \Xi + \frac{\kappa_\rho D}{2} \Xi^2, \quad C_E = \kappa_\rho K, \quad C_U = K, \quad C_N = 0, \quad (79)$$

where E, U, and N denote egalitarian, utilitarian, and Negishi criteria. The utilitarian coefficient follows from (78): the initial log-share gap equals $\log \chi_h^0 / \kappa_\rho$ up to a common constant.

For the numerical solution, collect inflation over T quarters in π . Price and wage inflation coincide in this economy, and the wage Phillips curve is $\pi_t = \beta \pi_{t+1} + \lambda_w(1 + \nu^{-1})\tilde{y}_t$. Eliminating output gives $\mathcal{H}_{\text{RANK}} = \pi' Q \pi / 2$, with Q positive definite. The wealth revaluation satisfies

$$\Xi = \pi_0 + \tilde{y}_0 - (1 - \beta) \sum_{t=0}^{T-1} \beta^t \tilde{y}_t + G \delta_0 \equiv g' \pi + G \delta_0, \quad G \equiv \sum_{t=0}^{T-1} \beta^{t+1} (0.9)^t, \quad (80)$$

for a demand shock $\delta_t = \delta_0 (0.9)^t$. Let $z_\pi \equiv Q^{-1}g$ and $M \equiv g' Q^{-1}g$. The first-order condition for (79) gives

$$\pi_c^*(\delta_0) = \frac{C_c - \kappa_\rho D G \delta_0}{1 + \kappa_\rho D M} z_\pi, \quad (81)$$

$$\pi_c^*(\delta_0) - \pi_c^*(0) = -\frac{\kappa_\rho D G \delta_0}{1 + \kappa_\rho D M} z_\pi. \quad (82)$$

Without shocks, utilitarian inflation is $1/\kappa_\rho$ times egalitarian inflation, and Negishi inflation is zero. The incremental response to a demand shock is identical across the three criteria because C_c affects only the no-shock optimum. The same statements hold for the output gap by the wage Phillips curve.

I.3 Numerical implementation and results

Figures 8 and 9 use the same inherited asset distribution and calibration as Figure 4: $\beta = 0.99$, $\nu = 0.75$, $\omega_C = 4/7$, $\psi = 6$, and $\lambda_w = 0.05$. We use $T = 500$, with zero terminal inflation and output gap, and retain the value of the time endowment after the horizon. The demand innovation is $\delta_0 = 0.01$. The distribution gives $K = 0.00496991$ and $D = 0.00435166$, while $M = 59.45503$ and $G = 9.08257$.

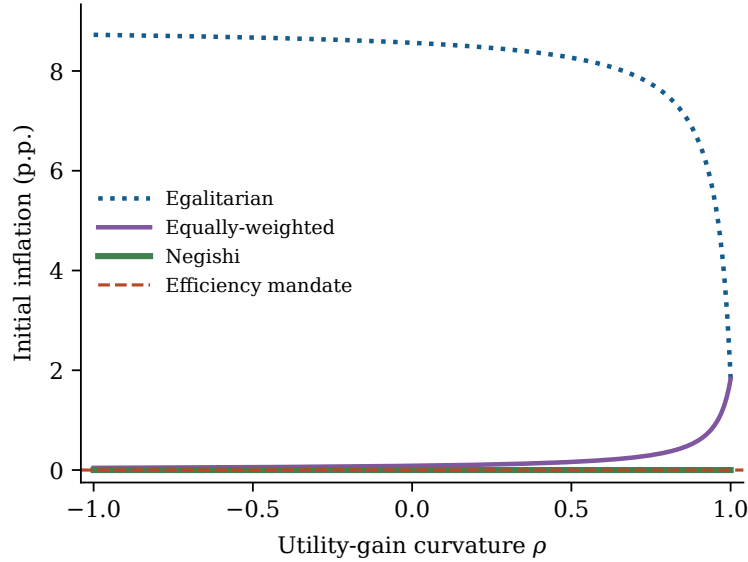
The figures vary ρ from -1 to 1 , with $\rho = 0$ interpreted as the logarithmic transformation. Since $B = 100$, even a change from $\rho = 1$ to $\rho = 0.99$ doubles κ_ρ from one to two. The utility scale is therefore material: applying the power transformation to $1 + U(\mathbf{x}_h) - U_h^0$ without the factor B would instead give curvature $2 - \rho$.

The calculations solve the unconstrained quadratic policy problem, as in Section 7. Locally, maximizing second-order welfare delivers first-order optimal policy (Baqae et al., 2026). Large responses for some values of ρ are formal implications of this quadratic problem, not numerical optima of the exact shifted-utility SWF. The calculations do not enforce nonlinear household feasibility or positivity of the recentered utility index. These qualifications matter particularly for the egalitarian target (Section I.5).

Figure 8 shows initial inflation in the absence of shocks. Under egalitarian preferences, lowering ρ increases the inflationary bias: moving from $\rho = 1$ to $\rho = 0$ raises quarterly inflation from 1.83 to 8.57 percentage points. Under equal utilitarian weights, the same change lowers inflation to 0.085 percentage points, capturing the fact that the first-best wealth shares also change as ρ changes and $\bar{\chi}_h$ is held fixed at 1. As expected, Negishi weights prescribe zero inflation throughout.

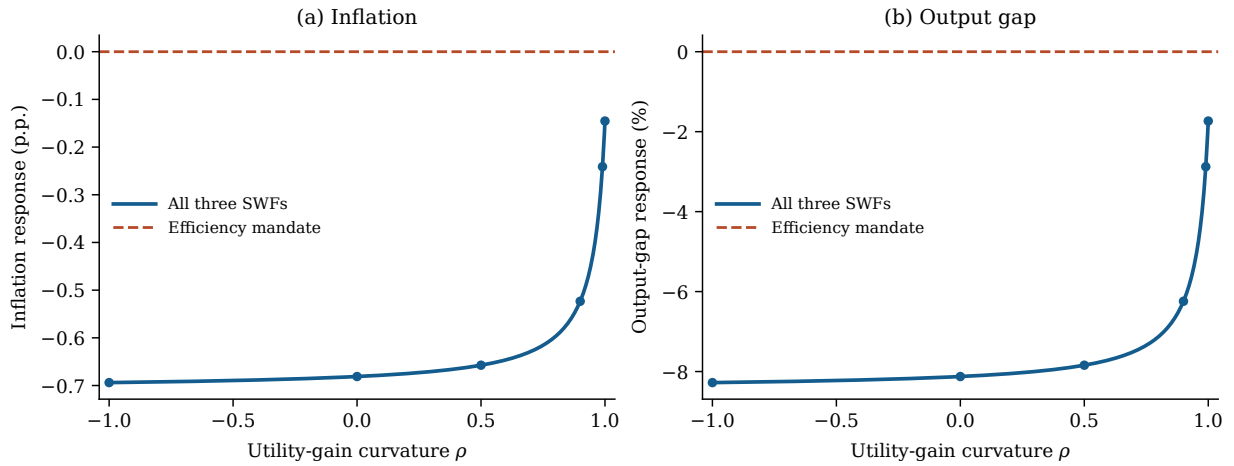
Figure 9 shows the impact response to a contractionary demand shock, relative to each criterion's own no-shock optimum. The responses are the same for all criteria, since they put the same curvature on deviations from their respective desired wealth distribution. As ρ falls, the redistributive incentive becomes stronger, widening the gap relative to full stabilization of the demand shock. Moving from $\rho = 1$ to $\rho = 0$ increases the initial output contraction from 1.73% to 8.13%. As discussed before, the efficiency mandate fully stabilizes output and inflation independently of the cardinalization of utility.

Figure 8: Utility curvature and inflation in the absence of shocks



Notes: Initial inflation under the quadratic criterion in Proposition 14. Egalitarian preferences target equal shares, utilitarian weights set $\bar{\chi}_h = 1$, and Negishi weights target inherited shares. The blue dotted curve is the egalitarian case. Negishi and efficiency mandate inflation coincide at zero. Calibration and units are as in Figure 4; $\kappa_\rho = 1 + 100(1 - \rho)$.

Figure 9: Utility curvature and the response to a demand shock



Notes: Impact responses to a one-percentage-point contractionary demand innovation with persistence 0.9, relative to each criterion's own no-shock optimum. Egalitarian, utilitarian, and Negishi responses coincide. Inflation is in quarterly percentage points and the output gap in percent. The efficiency mandate fully stabilizes both variables. Calibration is as in Figure 4.

These results show how welfare weights and utility cardinalization affect optimal policy without changing household behavior. The efficiency mandate remains invariant to these choices because it evaluates efficiency without choosing a desired wealth distribution.

I.4 Proof of the welfare approximation

To prove Proposition 14, use complete markets and the labor assignment to write $c_{h,t} = \chi_h Y_t$ and $l_{h,t} = \chi_h L_t$, with $\int \chi_h dh = 1$. Define aggregate utility

$$A \equiv (1 - \beta) \sum_{t=0}^{\infty} \beta^t \frac{\log Y_t + \nu \log L_t}{1 + \nu}.$$

We write deterministic paths, as in the numerical experiments. Under uncertainty, $U(\mathbf{x}_h)$ in (75) is expected lifetime utility as defined in Section 2; the transformation is applied after taking this expectation. Let A^* denote aggregate utility at the efficient reference and $a \equiv A - A^*$. Recall that $r_h = \log(\chi_h / \chi_h^0)$. Substitution into household utility gives

$$U(\mathbf{x}_h) = A + \log \chi_h, \quad \tilde{U}_h = f_\rho(1 + B(a + r_h)). \quad (83)$$

Here $f_\rho(x) = x^\rho / \rho$ for $\rho \neq 0$ and $f_0(x) = \log x$, with $x > 0$. The aggregate first-order welfare effect vanishes at the efficient reference. The same output-gap and wage-dispersion calculation as in Proposition 8 gives

$$a = -\mathcal{H}_{\text{RANK}} + O(\epsilon^3), \quad \mathcal{H}_{\text{RANK}} \equiv \frac{(1 - \beta)\omega_C}{2} \sum_{t=0}^{\infty} \beta^t \left[(1 + \nu^{-1}) \tilde{y}_t^2 + \frac{\psi}{\lambda_w} \pi_{w,t}^2 \right], \quad (84)$$

Here $\omega_C = 1/(1 + \nu)$.

In addition to E_0 , define the welfare-weighted average $E_{\tilde{\chi}}[x_h] \equiv \int \tilde{\chi}_h x_h dh$. Define the proportional share change $e_h \equiv (\chi_h - \chi_h^0) / \chi_h^0$. Feasibility gives $E_0[e_h] = 0$, while $r_h = e_h - e_h^2/2 + O(\epsilon^3)$. Since $f'_\rho(1) = 1$ and $f''_\rho(1) = \rho - 1$, expanding (75) around $\mathcal{W}_\rho^0 = f_\rho(1)$ yields

$$\begin{aligned} \frac{\mathcal{W}_\rho - \mathcal{W}_\rho^0}{B} &= a + E_{\tilde{\chi}}[r_h] + \frac{(\rho - 1)B}{2} E_{\tilde{\chi}}[r_h^2] + O(\epsilon^3) \\ &= -\mathcal{H}_{\text{RANK}} + E_{\tilde{\chi}}[e_h] - \frac{\kappa_\rho}{2} E_{\tilde{\chi}}[e_h^2] + O(\epsilon^3). \end{aligned} \quad (85)$$

Terms involving ar_h or a^2 are of order at least three. Equation (85) holds for arbitrary

fixed welfare weights, provided the share change in its linear term includes its second-order component.

For the first-best-relative expression, write $\bar{\chi}_h = \chi_h^0(1 + v_h)$, with $v_h = O(\epsilon)$ and $E_0[v_h] = 0$. This is the joint expansion in welfare weights, policies, and shocks used in Proposition 8. Then

$$\frac{\mathcal{W}_\rho - \mathcal{W}_\rho^0}{B} = -\mathcal{H}_{\text{RANK}} + E_0[v_h e_h] - \frac{\kappa_\rho}{2} E_0[e_h^2] + O(\epsilon^3). \quad (86)$$

Since $\kappa_\rho > 0$, the first best sets the aggregate loss to zero and $e_h^{\text{FB}} = v_h/\kappa_\rho + O(\epsilon^2)$. Completing the square and using $\log(\bar{\chi}_h/\chi_h^0) = v_h + O(\epsilon^2)$ gives

$$\frac{\mathcal{W}_\rho - \mathcal{W}_\rho^{\text{FB}}}{B} = -\mathcal{H}_{\text{RANK}} - \frac{\kappa_\rho}{2} \text{Var}_{\chi_h^0} \left(\log \frac{\chi_h}{\chi_h^0} - \frac{1}{\kappa_\rho} \log \frac{\bar{\chi}_h}{\chi_h^0} \right) + O(\epsilon^3). \quad (87)$$

Substitution of (76) leaves $\log \chi_h - \log \chi_h^{\text{FB}}$ inside the variance, up to a common constant. This proves Proposition 14. The first best allows unrestricted redistribution of shares; monetary policy generally cannot implement it.

I.5 Exact supporting weights and domain restrictions

Equation (76) holds the desired distribution fixed inside the quadratic criterion. To support a target $s_h > 0$, $\int s_h dh = 1$, in the exact nonlinear problem, the first-order condition at aggregate efficiency instead requires

$$\frac{\bar{\chi}_h}{s_h} \left[1 + B \log \frac{s_h}{\chi_h^0} \right]^{\rho-1} = \zeta, \quad (88)$$

where $\zeta > 0$ is common across households. Thus the exact supporting weights are

$$\bar{\chi}_h^{\text{exact}} = \frac{s_h [1 + B \log(s_h/\chi_h^0)]^{1-\rho}}{\int s_j [1 + B \log(s_j/\chi_j^0)]^{1-\rho} dj}, \quad (89)$$

provided the bracket is positive for every household and the denominator is finite. For $\rho \leq 1$, the objective is strictly concave in shares on this domain, so these weights support an interior feasible target. Their first-order expansion agrees with (76).

Negishi weights, $\bar{\chi}_h = \chi_h^0$, support the inherited distribution exactly: every household has a recentered utility index of one and $\tilde{U}_h = f_\rho(1)$ there, and its marginal social value of wealth is $B\bar{\chi}_h/\chi_h^0 = B$. The egalitarian target requires a separate qualification. At equal

shares, the recentered utility index is $1 - B \log \chi_h^0$, which is positive only if $\chi_h^0 < \exp(1/B)$ for every household. The calibrated distribution violates this condition. The egalitarian target used in the numerical results therefore describes the quadratic criterion, rather than an admissible exact first best of the positive-domain objective. More generally, the first-best-relative approximation requires target shares and welfare weights to be near their reference values for fixed ρ and β .