Remote Work, Foreign Residents, and the Future of Global Cities*

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August 2023

Abstract

As remote work opportunities expand, more people are seeking residence in foreign destinations. The resulting surge in foreign residents generates capital gains for property owners but negatively impacts renters and creates potentially important production, congestion, and amenities externalities. We study the optimal policy toward foreign residents in a model with key features emphasized in policy discussions. Using this model, we provide sufficient statistics to evaluate the impact of an influx of foreign residents and to calculate the tax/transfer policies required to implement the optimal policy. This policy involves implementing transfers to internalize agglomeration, congestion, and other potential externalities. Importantly, we find that it is not optimal to restrict, tax, or subsidize home purchases by foreign residents.

J.E.L. Classification: H00, J61, R3, R58.
Keywords: remote work, real estate taxes, city design.

*We thank Marios Angeletos, Andy Atkeson, Paul Beaudry, Saki Bigio, Ariel Burstein, Pablo Fajgelbaum, and Jon Vogel for their comments.

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1 Introduction

In 1917, the American composer Cole Porter moved to Paris and acquired an opulent residence built in 1777 for the brother of Louis XIV. There, he hosted luminaries like F. Scott Fitzgerald and Ernest Hemmingway and composed memorable tunes like "Night and Day" and "Anything Goes."

Buying a home in a foreign country was unusual at the beginning of the 20th century but has become increasingly common in recent decades. As remote work opportunities expand (Dingel and Neiman, 2020 and Aksoy, Barrero, Bloom, Davis, Dolls, and Zarate, 2022), many more people are seeking residence in foreign destinations.[1]

The surge in the flow of foreign residents is transforming housing markets in many cities across the globe. These flows generate capital gains for property and land owners but negatively impact renters and create potentially important production, congestion, and amenities externalities.

Many countries have grappled with the question of how to deal with potentially large numbers of foreign residents. The policies adopted so far vary widely, ranging from laissez-faire approaches and incentive programs designed to attract foreign home buyers to special taxes and regulations designed to restrict home purchases by foreigners.[2]

[1] At the same time, higher incomes and reduced air travel costs have greatly increased international tourism flows. According to data compiled by the United Nations World Tourism Organization, international tourist arrivals have grown at an average annual rate of 5.9 percent between 1950 and 2018. See Allen et al. (2020) for a insightful analysis of the effect of tourism on the welfare of the local population.

[2] France and the United States impose no restrictions on foreign home buyers. Greece, Portugal, and Spain offer tax breaks and visa programs to attract foreign buyers. Some Canadian provinces, Hong Kong, Israel, and Singapore levy special taxes on foreign property purchases. The city of Vancouver has imposed taxes on unoccupied homes. Switzerland enforces annual quotas on foreign home sales, and New Zealand has strict foreign real estate investment limitations. In Australia, foreigners are generally prohibited from purchasing established dwellings, but they can invest in new buildings or vacant land. The Philippines and Thailand permit foreign home ownership but prohibit land ownership.

Determining the optimal policy regarding foreign residents is important for three reasons. First, housing is the primary asset in most household portfolios (Cocco, 2005). Second, the availability of affordable housing near the workplace influences commuting times and job choices in ways that can significantly affect worker welfare. Third, most economic activity occurs in cities (Rossi-Hansberg and Wright, 2007).

In this paper, we use a Mirrlees (1971) approach to characterize optimal policy towards foreign residents in a model that embeds key insights from the economic geography literature.[3]

We find that it is optimal to use transfers to internalize externalities. However, it is not optimal to impose restrictions or taxes on home purchases by foreigners. Likewise, it is not optimal to implement programs that subsidize home purchases by foreign residents. We provide a set of sufficient statistics to evaluate the impact of an influx of foreign residents and to calculate the tax/transfer policies required to implement the optimal solution.

Our analysis relates to recent work on using optimal transfers to internalize agglomeration externalities. Prominent examples include Fu and Gregory (2019), Fajgelbaum and Gaubert (2020), and Rossi-Hansberg et al. (2019).

The model has two locations: the center area and the periphery. Each location has a stock of housing and offices that is fixed in the short run. Foreign residents prefer to live in the center and have an outside option: they can always stay in their home country.

Locals can live and work in different locations by incurring commuting costs. Taste shocks, location-specific amenities, and commuting times influence home and work location choices by the locals. In our benchmark model, we assume that the

[3]Important contributions to this literature include Alonso (1964), Mills (1967), Muth (1969), Lucas and Rossi-Hansberg (2002), Rossi-Hansberg (2005), Desmet and Rossi-Hansberg (2013), Ahlfeldt et al. (2015), and Allen et al. (2015).

ownership of houses and office buildings is equally distributed in the population. We revisit this assumption in Section 6, where we consider a model in which property ownership is unequally distributed.

We begin by examining the competitive equilibrium and analyzing the impact of a marginal increase in the number of foreign residents on social welfare. We identify two effects of this increase. The first relates to the agglomeration or production externality emphasized by Jacobs (1969), Lucas (1988), Lucas (2001), and Lucas and Rossi-Hansberg (2002). This effect can be negative if the arrival of foreigners leads to the relocation of locals from high- to low-productivity locations. The second effect pertains to the housing capital gains that accrue to the locals. We call this effect the foreign resident surplus, and it is always positive.

Next, we study the policy toward foreign residents that maximizes the social welfare of the local population. We assume that the planner operates within a Mirrleesian environment. In this environment, the planner faces an information constraint: it cannot observe taste shocks that influence the choice of where the locals choose to live and work. We characterize this second-best optimum.

We expand our model to incorporate three additional effects discussed in policy circles. First, we introduce congestion externalities by assuming that the cost of commuting increases with the number of commuters. In this case, the optimal policy requires three types of transfers to internalize the agglomeration externality, the congestion externality, and their interaction. This interaction arises because increased commuting time results in decreased agglomeration externalities.

Second, we consider the scenario where foreigners have a negative impact on the value that locals attach to amenities in the city center. It is optimal to correct these externalities by imposing a lump-sum tax on foreigners, similar to the per diem or per night tax levied by an increasing number of cities.

Third, we explore the case where foreigners place value on authenticity, deriving utility from having locals live and work in the city center. At first sight, we might think that this effect does not affect the social optimum. After all, the planner does not include the utility of foreigners in the social welfare function. However, it is optimal to internalize this externality by providing transfers to locals who live and work in the city center. The rationale for this policy is that the externality affects the participation constraint of foreigners and influences their decision to relocate.

Our model provides some insights into the implications of an inflow of foreign residents for optimal long-run city design. By the long run, we mean a timeframe where offices can be converted into houses and vice versa in both the city center and the periphery. In our model, it is optimal to convert offices into houses in the city center to meet the increased demand for housing. However, the optimal solution for the periphery is ambiguous. On the one hand, more locals reside in the periphery, raising the marginal value of houses in that area. On the other hand, more people work in the periphery, increasing the value of offices.

Finally, we consider two important extensions of our model. In the first extension, property ownership is unequally distributed. In this situation, once externalities are corrected, it is feasible to implement transfers to redistribute the capital gains so that ex-ante (before taste shocks are realized) all locals benefit from the influx of foreigners. In the second extension, the local labor force can choose to work in the office or remotely. In this case, the optimal policy entails higher transfers for office workers compared to those working from home.

The structure of the paper is as follows. Section 2 introduces the model, characterizes the competitive equilibrium, and assesses the impact of a marginal increase in the number of foreign residents on social welfare. Section 3 outlines the optimal second-best policy. In Section 4, we explore three additional factors: traffic congestion, amenity effects, and the possibility of foreign residents valuing authenticity. Section 5 discusses how the influx of foreign residents affects long-run city design. Section 6 examines a variant of the model that incorporates unequal property holdings. Section 7 considers an economy where the local population can choose between working in an office or remotely. Section 8 concludes.

2 The competitive equilibrium

There are two locations in the model: the center and the periphery. Both locations produce a single tradable good by combining labor and a type of capital that we refer to as office buildings.

The index ℓ takes the value c or p depending on whether a person chooses to live in the center or the periphery. Similarly, the index j takes the value c or p depending on whether a person chooses to work in the center or the periphery.

Each local person i draws a taste shock, $\xi_{i,\ell,j}$, with respect to living in location ℓ and working in location j. Following McFadden (1973), we assume that this shock is governed by a Gumbel $(0,\eta^{-1})$ distribution.[4] These shocks eliminate corner solutions with respect to location choices and make the analysis tractable because the maximum of n i.i.d. Gumbel variables follows a Gumbel distribution.

Locals who live in location ℓ and work in location j derive utility from housing services ($h_{\ell,j}$) and from consuming a single tradable good ($c_{\ell,j}$). They supply exogenously one unit of labor, which they allocate to working and commuting.

In this version of the model, local people have an equal endowment of houses and office buildings. We relax this assumption in Section 6.

Location choices The utility that local person i derives from living in location ℓ and working in location j has two components:

$$\xi_{i,\ell,j} + u_{\ell,j}.$$

The first is the taste shock, $\xi_{i,\ell,j}$. The second is given by

$$u_{\ell,j} = \overline{u}_{\ell} + c_{\ell,j} + v\left(h_{\ell,j}\right).$$

[4] The mean of this distribution is not zero, but this value does not influence the comparative evaluations individuals make between different locations.

We refer to $u_{\ell,j}$ as "common utility" because it is common to all who live in location ℓ and work in location j. The variable $c_{\ell,j}$ denotes consumption, $h_{\ell,j}$ housing services, and $\overline{u}_\ell$ the utility that locals derive from the amenities in location ℓ.

Person i maximizes utility subject to the budget constraint

$$c_{\ell,j} + r_\ell h_{\ell,j} = w_j \left(1 - t_{\ell,j}\right) + T.$$

The variable $t_{\ell,j}$ denotes the time it takes to commute from a home in location ℓ to work at an office in location j. For those who live and work in the same location, commuting costs are zero ($t_{\ell,\ell} = 0$). The variable r_ℓ denotes the cost of renting a unit of housing in location ℓ and T denotes the housing and office rents, which are given by

$$T = \sum_\ell r_\ell \overline{H}_\ell + \sum_\ell r_\ell^K \overline{K}_\ell.$$

The variable r_ℓ^K denotes the rental rate of office buildings in location ℓ. In Section 6, we consider a version of the model in which people are heterogeneous with respect to their ownership of offices and houses.

The first-order conditions for this problem are

$$v'\left(h_{\ell,j}\right) = r_\ell,$$

$$c_{\ell,j} = w_j \left(1 - t_{\ell,j}\right) + T - r_\ell h_{\ell,j}.$$

Note that all locals living in locatio ℓ have the same housing consumption, i.e., $h_{\ell,j} = h_\ell$ for all j. The resulting common utility is

$$u_{\ell,j} = \overline{u}_\ell + w_j \left(1 - t_{\ell,j}\right) + T - r_\ell h_{\ell,j} + v\left(h_{\ell,j}\right).$$

A person lives in ℓ and works in j if

$$u_{\ell,j} + \xi_{i,\ell,j} = \max_{\ell',j'} \{u_{\ell',j'} + \xi_{i,\ell',j'}\}.$$

The share of people who live in ℓ and work in j is

$$\pi_{\ell,j} = \mathbb{P}\left[u_{\ell,j} + \xi_{\ell,j} = \max_{\ell',j'}\{u_{\ell',j'} + \xi_{\ell',j'}\}\right] = \mathbb{P}\left[x_{\ell,j} \geq x_{\ell',j'} \forall \{\ell', j'\}\right],$$

where $x_{\ell,j} = u_{\ell,j} + \xi_{i,\ell,j}$ for $\ell, j = c, p$. The cdf of $x_{\ell,j}$, is

$$G_{\ell,j}(x) = \mathbb{P}\left[x_{\ell,j} < x\right] = \mathbb{P}\left[\xi_{\ell,j} < x - u_{\ell,j}\right] = F\left(x - u_{\ell,j}\right) = e^{-e^{-\eta\left(x - u_{\ell,j}\right)}}.$$

The corresponding pdf is

$$g_{\ell,j}(x) = \eta e^{-\eta\left(x - u_{\ell,j}\right)} e^{-e^{-\eta\left(x - u_{\ell,j}\right)}}$$

The share of local people living in ℓ and working in j is given by

$$\pi_{\ell,j} = \int_{-\infty}^{\infty} g_{\ell,j}(x) \prod_{\{\ell',j'\} \neq \{\ell,j\}} G_{\ell',j'}(x) dx$$

Here, $g_{\ell,j}(x)$ denotes the mass of people with valuation x for the pair of location choices (ℓ, j) and $G_{\ell',j'}(x)$ is the fraction of people with valuations lower than x for (ℓ', j'). This expression can be rewritten as:

$$\pi_{\ell,j} = \frac{e^{\eta u_{\ell,j}}}{\sum_{\ell',j'} e^{\eta u_{\ell',j'}}}$$

Foreign residents To simplify, we assume that foreign residents are not subject to taste shocks and prefer to live in the city center. Their problem is to choose consumption (c_f) and housing in the center (h_f) so as to maximize their utility,

$$\overline{u}_f + c_f + v\left(h_f\right),$$

where $\overline{u}_f$ is the value attached by foreign residents to the amenities in the center. Foreigners bring a fixed endowment of the tradable good (y_f) that they use to pay for consumption, housing services, and any potential taxes. We assume that these taxes are zero in the competitive equilibrium. The foreign residents' budget constraint is:

$$c_f + r_c h_f = y_f.$$

The first-order conditions for this problem are

$$v'\left(h_f\right) = r_c,$$

$$c_f = y_f - r_c h_f.$$

These conditions imply that foreign residents choose the same housing consumption as locals who live in the center.

Foreigners can stay in their own country and receive utility u_f^*. They only migrate if their participation constraint is satisfied:

$$\overline{u}_f + c_f + v\left(h_f\right) \geq u_f^*.$$

Firms' problem Each location has a measure one of identical firms. Firms in location j produce output (y_j) by combining offices (k_j) and labor (l_j) according to a Cobb-Douglas production function:

$$y_j = A\left(L_j\right) k_j^{\alpha} l_j^{1-\alpha}.$$

The function $A(L_j)$ represents an agglomeration or production externality. Locations with more workers tend to be more productive because there are more opportunities for workers to learn from each other. We assume that the function $A(L_j)$ takes the form:

$$A\left(L_j\right) = L_j^{\gamma}$$

The problem of a representative firm in location j is to maximize its profits (π_j):

$$\pi_j = A\left(L_j\right) k_j^{\alpha} l_j^{1-\alpha} - w_j l_j - r_j^K k_j.$$

The first-order condition for the firms' problem are:

$$w_j = (1-\alpha)\, A\left(L_j\right) \left(\frac{k_j}{l_j}\right)^{\alpha},$$

$$r_j^K = \alpha A\left(L_j\right) \left(\frac{k_j}{l_j}\right)^{\alpha-1}.$$

Equilibrium conditions The goods market clearing condition is:

$$\sum_{\ell,j} \pi_{\ell,j} c_{\ell,j} + N_f c_f = \sum_j A\left(L_j\right) L_j^{1-\alpha} \overline{K}_j^{\alpha} + N_f y_f.$$

On the right-hand side of this equation we have the sum of the locals' consumption across all living and working locations ($\sum_{\ell,j} \pi_{\ell,j} c_{\ell,j}$) and the total consumption by foreign residents, $N_f c_f$, where N_f denotes the total amount of foreign residents. On the left-hand side we have the production in the center and periphery $\left(\sum_j A\left(L_j\right) L_j^{1-\alpha} \overline{K}_j^{\alpha}\right)$ and the endowment of goods brought by the foreigners $N_f y_f$.

The labor market clearing condition is

$$l_j = L_j,$$

where L_j, the amount of labor available in location j. This variable is equal to the time supplied by all the people who work at location j net of commuting costs

$$L_j = \sum_{\ell} \pi_{\ell,j} \left(1 - t_{\ell,j}\right).$$

The market clearing condition for office buildings in location j is

$$k_j = \overline{K}_j.$$

Finally, the housing market clearing conditions for the center and the periphery are

$$\pi_{c,c} h_{c,c} + \pi_{c,p} h_{c,p} + N_f h_f = \overline{H}_c,$$

$$\pi_{p,c} h_{p,c} + \pi_{p,p} h_{p,p} = \overline{H}_p.$$

The welfare impact of a marginal increase in the number of foreign residents We define social welfare as the sum of the utility of all local people.

$$\mathcal{W} = \int_0^1 \max\left\{u_{\ell,j} + \xi_{i,\ell,j}\right\} di$$

In the appendix, we show that social welfare is given by:

$$\mathcal{W} = \frac{\log\left(\sum_{\ell,j} e^{\eta u_{\ell,j}}\right)}{\eta} + \frac{1}{\eta}\int_0^\infty [-\log(y)\, e^{-y}]dy,$$

where $\int_0^\infty [-\log(y)\, e^{-y}]dy$ is the Euler-Mascheroni constant.

We assume that the foreign residents' participation constraint is satisfied:

$$\overline{u}_f + c_f + v\left(h_f\right) \geq u_f^*,$$

and that the function $v(h)$ takes the form

$$v(h) = \chi h^{1-\sigma} / (1-\sigma).$$

The following proposition provides sufficient statistics to evaluate the impact of an influx of foreign residents on social welfare.

Proposition 1. *The change in social welfare from a marginal increase in the number of foreign residents is*

$$d\mathcal{W} = \sum_{\ell,j} \pi_{\ell,j} du_{\ell,j}.$$

This welfare change has two components, $d\mathcal{W} = \mathcal{PE} + \mathcal{FS}$. *The production or agglomeration externality,* $\mathcal{PE}$, *is given by*

$$\mathcal{PE} \equiv \gamma \times \mathbb{COV}\left(\frac{Y_j}{L_j}\left(1 - t_{\ell,j}\right), \frac{d\pi_{\ell,j}}{\pi_{\ell,j}}\right),$$

with

$$\mathbb{COV}\left(\frac{Y_j}{L_j}\left(1 - t_{\ell,j}\right), \frac{d\pi_{\ell,j}}{\pi_{\ell,j}}\right) = \sum_{\ell,j} \pi_{\ell,j} \frac{Y_j}{L_j}\left(1 - t_{\ell,j}\right) d\pi_{\ell,j} / \pi_{\ell,j}.$$

The foreign residents surplus, $\mathcal{FS}$, *is given by*

$$\mathcal{FS} \equiv \sigma \frac{N_f}{\Pi_c + N_f} r_c h_f \left(d\Pi_c + dN_f\right).$$

where $\Pi_c \equiv \pi_{c,c} + \pi_{c,p}$

See appendix A.3 for the proof.

The interpretation of the production or agglomeration externality, $\mathcal{PE}$, is as follows. If, on average, people leave higher productivity locations, $\mathbb{COV}$ is negative, and there is a welfare loss. Three pertinent comments about this component of the change in welfare are as follows. If foreigners choose the same distribution of locations as locals and $\sigma = 1$, then $d\pi_{j,\ell} = 0$ and $\mathbb{COV} = 0$ (see Appendix C). So, there is no welfare loss from the production externality. Second, the production externalities would be more important in a model with multiple peripheries because workers who move from the center would scatter across different peripheriess. Third, the ability of locals to work from home reduces production externalities. We discuss the issue of working from home in Section 7.

The foreign resident surplus is equal to the capital gains realized on the houses sold to foreigners. Foreigners replace some of the locals who live in the center ($d\Pi_c < 0, dN_f > 0$). In addition, people in the center reduce housing consumption, making space for additional foreign residents. As a result, the number of people who live in the center increases ($d\Pi_c + dN_f > 0$). Since everybody who lives in the center consumes the same amount of housing, per capita housing consumption falls. Rents rise, resulting in an increase in rental income obtained from foreigners. This effect is the foreign resident surplus.

The foreign resident surplus is similar to the immigration surplus discussed in the immigration literature (e.g., Borjas, 1995 and Guerreiro, Rebelo, and Teles, 2020). This surplus is the net benefit of immigration that results from increases in income to non-labor factors such as land.

3 Optimal policy

It is natural to assume that the planner does not observe taste shocks but has information about individuals' residential and work locations. We compute a second-best optimal solution in which the planner can only choose utilities and allocations that are contingent on location choices.

Our analysis of the impact of foreign residents on the competitive equilibrium suggests two questions. First, when the foreign resident surplus is lower than the production externality, is it optimal to restrict home purchases by foreigners? Second, when $\mathbb{COV} < 0$, is it optimal to tax home purchases by foreigners to internalize the production externality? We will show that the answer to both of these questions is no.

Location decisions must be privately optimal given the allocations chosen by the planner. In other words, incentive compatibility requires that two local people who live in the same location and work in the same location have the same common utility, $u_{\ell,j}$. It follows that person i chooses to live in location ℓ and work in location j if

$$u_{\ell,j} + \xi_{i,\ell,j} = \max\{u_{\ell',j'} + \xi_{i,\ell',j'}\}.$$

We can show that the incentive compatibility constraints imply that

$$\pi_{\ell,j} = \frac{e^{\eta u_{\ell,j}}}{\sum_{\ell',j'} e^{\eta u_{\ell',j'}}}.$$

The planner maximizes

$$\mathcal{W} = \frac{\log\left(\sum_{\ell,j} e^{\eta u_{\ell,j}}\right)}{\eta}$$

subject to the resource constraints for goods,

$$\sum_{\ell,j} \pi_{\ell,j} c_{\ell,j} + N_f c_f \leq \sum_j A(L_j) L_j^{1-\alpha} \overline{K}_j^{\alpha} + N_f y_f,$$

the adding-up contraints for housing in the center and in the periphery,

$$\sum_j \pi_{c,j} h_{c,j} + N_f h_f \leq \overline{H}_c,$$

$$\sum_j \pi_{p,j} h_{p,j} \leq \overline{H}_p,$$

the location-decisions constraints,

$$\pi_{\ell,j} = \frac{e^{\eta u_{\ell,j}}}{\sum_{\ell',j'} e^{\eta u_{\ell',j'}}}$$

and the foreign resident participation constraints,

$$\overline{u}_f + c_f + v\left(h_f\right) \geq u_f^*.$$

We write the Lagrangean for this optimization problem as follows,

$$\begin{aligned}
\mathcal{L} = & \frac{\log\left(\sum_{\ell,j} e^{\eta u_{\ell,j}}\right)}{\eta} \\
& + \lambda_c \left(L_c^{\gamma+1-\alpha}\overline{K}_c^{\alpha} + N_f\left(y_f - c_f\right) + L_p^{\gamma+1-\alpha}\overline{K}_p^{\alpha} - \sum_{\ell,j} \pi_{\ell,j} c_{\ell,j} \right) \\
& + \lambda_{h,c}\left(\overline{H}_c - \sum_j \pi_{c,j} h_{c,j} - N_f h_f\right) + \lambda_{h,p}\left(\overline{H}_p - \sum_j \pi_{p,j} h_{p,j}\right) \\
& + \sum_{\ell,j} \lambda_{\ell,j}^{\text{loc}} \left(\pi_{\ell,j} - \frac{e^{\eta u_{\ell,j}}}{\sum_{\ell',j'} e^{\eta u_{\ell',j'}}}\right) \\
& + N_f \lambda_f \left(\overline{u}_f + c_f + v\left(h_f\right) - u_f^*\right)
\end{aligned}$$

The first-order conditions for this problem can be written as

$$1 - \eta \lambda_{\ell,j}^{\text{loc}} + \eta \sum_{\ell',j'} \lambda_{\ell',j'}^{\text{loc}} \pi_{\ell',j'} = \lambda_c$$

$$v'\left(h_{\ell,j}\right)\left(1 - \eta \lambda_{\ell,j}^{\text{loc}} + \eta \sum_{\ell',j'} \lambda_{\ell',j'}^{\text{loc}} \pi_{\ell',j'}\right) = \lambda_{h,\ell}$$

$$\lambda_f = \lambda_c$$

$$\lambda_f v'\left(h_f\right) = \lambda_{h,c}$$

$$\lambda_c\left(y_f - c_f\right) - \lambda_{h,c} h_f = 0$$

$$\lambda_c \left\{ (1+\gamma-\alpha) \frac{Y_j}{L_j} \left(1 - t_{\ell,j}\right) - c_{\ell,j} \right\} - \lambda_{h,\ell} h_{\ell,j} = \lambda^{\text{loc}}_{\ell,j}$$

Combining the first-order conditions for $c_{\ell,j}$ we find $\lambda_c = 1$. Because utility is quasi-linear, the social marginal value of consumption goods is equal to one. In addition,

$$\lambda^{\text{loc}}_{c,c} = \lambda^{\text{loc}}_{c,p} = \lambda^{\text{loc}}_{p,c} = \lambda^{\text{loc}}_{p,p}.$$

The planner equalizes the marginal value of people across all locations. We also find that the marginal rates of substitution across houses are equated for locals living in the same location, i.e., $v'(h_{\ell,\ell}) = v'(h_{\ell,j})$ for all $\ell \neq j$.

With quasi-linear preferences, welfare is independent of the distribution of consumption. Only aggregate consumption matters. The planner can engineer any distribution of consumption to provide incentives without affecting aggregate consumption. Second-best aggregates coincide with first-best ones, and social welfare is the same in the two solutions.

Transfers to individuals living in location ℓ and working in location j are defined as

$$T_{\ell,j} \equiv c_{\ell,j} + v'(h_{\ell,j}) h_{\ell,j} - w_j (1 - t_{\ell,j}), \tag{1}$$

where $w_j \equiv (1-\alpha) A(L_j) (\overline{K}_j / L_j)^{\alpha}$ denotes the marginal productivity of labor in location j. The following proposition provides sufficient statistics to calculate the tax/transfer policies required to implement the optimal solution.

Proposition 2. *In the optimal solution, the transfers to locals have two key features.*

1. *Absent externalities, rents on houses and offices are equally distributed among locals.*

2. *The planner corrects the production externality by giving higher (lower) transfers to location-pairs higher (lower) than average labor income.*

The total transfers implemented by the planner are:

$$T_{\ell,j} = \underbrace{\alpha \sum_j Y_j + \sum_\ell r_\ell \overline{H}_\ell}_{\text{Rents on houses and offices}} + \underbrace{\frac{\gamma}{1-\alpha} \left\{ w_j \left(1 - t_{\ell,j}\right) - \sum_{\ell,j} \pi_{\ell,j} \times w_j \left(1 - t_{\ell,j}\right) \right\}}_{\text{Externality correction}}.$$

If there are no externalities, private location decisions are socially optimal and so the planner does not have an incentive to distort these decisions. It follows that the optimal transfers simply redistribute the rents on houses and offices equally across the population. But, in the presence of agglomeration externalities, these location decisions turn out to be suboptimal from a social standpoint (see also Rossi-Hansberg et al., 2019, and Fajgelbaum et al., 2019). It follows that the planner needs to change transfers in order to incentivize people to move to the locations where their contribution to these externalities is highest. So, the planner gives a relatively higher subsidy for people who choose location pairs with higher than average contribution to production and gives a relatively lower subsidy to people who choose location pairs with lower than average contribution to production.

Turning to the optimal treatment of foreigners, we say that the economy features quotas on foreign entry if the utility obtained by foreigners exceeds their outside option. In this case, the government would have to impose restrictions on entry, as more individuals would be willing to enter. In addition, we say that houses are taxed if the marginal rate of substitution between houses and consumption is higher for foreigners than for locals. We define the house tax (or wedge) as

$$\tau_h \equiv \frac{v'(h_f)}{v'(h_{c,\ell})} - 1. \tag{2}$$

Finally, we say that foreigners pay an entry fee if their income exceeds their expenditure on consumption and housing goods. We define this fee as

$$T_f \equiv y_f - c_f - v'(h_f) h_f. \tag{3}$$

The following proposition summarizes the optimal treatment of foreigners in this model.

Proposition 3. *In the optimal allocation, house purchases are not taxed, and there is free and unrestricted migration, i.e., the optimal solution features:*

1. *No quotas/restrictions on foreign entry,*

$$\overline{u}_f + c_f + v(h_f) = u_f^*.$$

2. *No taxes on foreign house purchases, $\tau_h = 0$.*

3. *No entry fees, $T_f = 0$.*

We now discuss the three parts of this proposition. Suppose we impose a quota on the number of foreigners who come to reside in the home country. Alternatively, we can impose an entry fee such that the number of foreigners is the same as under the quota system. It is always better to impose an entry fee than a quota because the former generates revenue that can be rebated to the locals.

Second, it is not optimal to tax home purchases by foreign residents. This result follows from standard public-finance principles: it is better to use a discriminatory lump-sum tax than to distort the purchases of goods.

Third, the optimal number of foreigners is such that the entry fee is exactly zero. This result can be interpreted as the optimality of production efficiency (Diamond and Mirrlees, 1971). Foreigners can be interpreted as a technology that transforms houses into consumption goods, and, in that sense, no entry fees correspond to production efficiency. Production efficiency is optimal despite the presence of externalities because these externalities are corrected using transfers to locals. When there are other externalities to which foreigners contribute directly, production efficiency is not optimal (see Section 4).

In sum, in this model, the optimal policy with respect to foreigners is laissez-faire: no taxes on house purchases by foreign residents and no entry fees. Agglomeration externalities are corrected using location-based transfers to locals.

Relation to the Optimal Trade-Tax literature We can interpret the sales of houses to foreigners as exports that are paid for in units of the tradable consumption good. So, it is natural to relate our findings to standard results in the trade literature.

In Appendix D, we use an optimal trade-tax model to discuss this relation. It is useful to describe our results in two parts. First, for a given number of foreigners, N_f, it is not optimal to tax foreign home purchases. Second, the optimal value of N_f obtains when the entry fee imposed on foreign residents is zero.

In a trade context, the first part states that the optimal trade tax is zero. This implication apparently contradicts the classical result that it is optimal to manipulate the terms-of-trade. This apparent contradiction emerges because, unlike in the standard trade literature, we impose no exogenous restrictions on the set of policy instruments available to the home country. In particular, in our model the government can impose a lump-sum tax on foreigners. As we discuss in Appendix D, when this type of lump-sum instrument is available, the optimal policy is to set the trade tax to zero and instead charge a right-to-trade fee. This fee extracts the gains from trade of foreign countries. This set-up resembles a monopolist who uses a two-part tariff: it sets the price equal to the marginal cost and charges a fixed fee that extracts all the consumer surplus. Similarly, in our model, it is optimal to leave house purchases by foreigners untaxed and instead impose a lump-sum tax on foreigners.

In the context of a trade model, the second part of our result is that the optimal number of trading partners obtains when the rights-to-trade fee is zero. We prove this result for the trade model in the appendix. The intuition is that the optimal number of trading partners maximizes the total value of exports, while preserving incentives for foreigners to participate.

4 Congestion, amenity, and authenticity effects

In this section, we add three additional effects that are often mentioned in policy discussions: traffic congestion, negative externalities exerted by foreign residents on the value that the locals place on amenities, and the possibility that foreign residents derive utility from authenticity effects created by the presence of the locals.

Congestion effects Suppose that commuting time is an increasing function of the number of commuters:

$$t_{\ell,j} = \mathcal{T}_{\ell,j}\left(\pi_{\ell,j}\right),$$

with $\mathcal{T}_{\ell,\ell}\left(\pi_{\ell,\ell}\right) = 0$.

Assume that

$$\frac{\mathcal{T}'_{\ell,j}\left(\pi_{\ell,j}\right)\pi_{\ell,j}}{\mathcal{T}_{\ell,j}\left(\pi_{\ell,j}\right)} = \psi.$$

Consider the effect on social welfare of a marginal increase in the number of foreign residents:

$$d\mathcal{W} = \mathcal{FS} + \mathcal{PE} + \mathcal{CE} + \mathcal{PCE}.$$

The first two effects, the production externality and the foreign resident surplus, are the same as before. In addition, there are two new effects. The first is the commuting externality

$$\mathcal{CE} \equiv -\psi \cdot \mathbb{COV}\left(w_j t_{\ell,j}, \frac{d\pi_{\ell,j}}{\pi_{\ell,j}}\right).$$

The second is the interaction between the production and congestion externalities,

$$\mathcal{PCE} \equiv -\frac{\gamma}{1-\alpha} \cdot \psi \cdot \mathbb{COV}\left(w_j t_{\ell,j}, \frac{d\pi_{\ell,j}}{\pi_{\ell,j}}\right).$$

This interaction arises because the more time people spend commuting, the weaker is the agglomeration/production externality. The transfers that the planner needs to make to implement the second-best optimal solution are given by

$$T_{\ell,j} = \underbrace{\sum_j \alpha Y_j + \sum_\ell r_\ell \overline{H}_\ell}_{\text{Rents on houses and offices}} + \underbrace{\frac{\gamma}{1-\alpha}\left\{ w_j\left(1-t_{\ell,j}\right) - \sum_{\ell,j} \pi_{\ell,j} \times w_j\left(1-t_{\ell,j}\right)\right\}}_{\text{Production externality correction}}$$
$$- \underbrace{\psi\left\{w_j t_{\ell,j} - \sum \pi_{\ell,j} w_j t_{\ell,j}\right\}}_{\text{Congestion externality correction}} - \underbrace{\frac{\gamma}{1-\alpha}\psi\left\{w_j t_{\ell,j} - \sum \pi_{\ell,j} w_j t_{\ell,j}\right\}}_{\text{Externality complementarity correction}}$$

Amenity effects There is a growing literature on the impact of changes in resident composition on amenities. Important contributions include Guerrieri, Hartley, and Hurst (2013), Diamond (2016), and Almagro and Dominguez-Iino (2022).

To study these effects in our model, suppose that foreign residents affect the value attributed by locals to amenities in the city center:

$$\overline{u}_c = \mathcal{U}\left(N_f\right).$$

Assume that

$$\frac{\mathcal{U}'\left(N_f\right) N_f}{\mathcal{U}\left(N_f\right)} = -\phi_{\overline{u}}.$$

The impact of a marginal increase in the number of foreign residents is the same as before plus an additional effect ($\mathcal{AE}$) which results from the amenity externalities. Same effects as before plus an additional effect

$$d\mathcal{W} = \mathcal{FS} + \mathcal{PE} + \mathcal{CE} + \mathcal{PCE} + \mathcal{AE}.$$

The new term $\mathcal{AE}$ is given by

$$\mathcal{AE} \equiv -\phi_{\overline{u}} \Pi_c \overline{u}_c \frac{dN_f}{N_f}.$$

To examine the optimal taxation of foreign residents, consider the first-order condition for N_f

$$\underbrace{\lambda_c\left(y_f - c_f\right)}_{\text{Benefit of goods paid by foreigners}} = \underbrace{\lambda_{h,c} h_f + \phi_{\overline{u}} \frac{\Pi_c}{N_f} \overline{u}_c}_{\text{Cost of providing homes to foreigners}}$$

This condition can be rewritten as

$$y_f = c_f + \lambda_{h,c} h_f + \phi_{\overline{u}} \frac{\Pi_c}{N_f} \overline{u}_c.$$

This equation implies that it is not optimal to tax house purchases by foreigners: $v'(h_f) = \lambda_{h,c} = r_c$. However, it is optimal to charge foreigners a lump-sum tax that corrects the amenity externality

$$T_f = \phi_{\overline{u}} \frac{\Pi_c}{N_f} \overline{u}_c.$$

Authenticity effects Finally, suppose that foreign residents derive utility from authenticity and that this authenticity is fostered by having more locals live and work in the city center. In this case, we can write the foreign resident utility as

$$\overline{u}_f \left(\pi_{c,c}, \pi_{c,p}, \pi_{p,c} \right) + c_f + v \left(h_f \right).$$

The welfare consequences in the competitive equilibrium are the same as in the previous case. However, in the second-best optimum, the planner has a new reason to subsidize living/working in the city center.

The following proposition provides sufficient statistics to implement the optimal transfers.

Proposition 4. *In a model with congestion, amenity, and authenticity effects the optimal* *transfers are given by:*

$$T_{\ell,j} = \underbrace{\sum_j \alpha Y_j + \sum_\ell r_\ell \overline{H}_\ell}_{\text{Rents on houses and offices}} + \underbrace{\frac{\gamma}{1-\alpha}\left\{w_j\left(1-t_{\ell,j}\right) - \sum_{\ell,j}\pi_{\ell,j} \times w_j\left(1-t_{\ell,j}\right)\right\}}_{\text{Production externality correction}}$$
$$- \underbrace{\psi\left\{w_j t_{\ell,j} - \sum \pi_{\ell,j} w_j t_{\ell,j}\right\}}_{\text{Congestion externality correction}} - \underbrace{\frac{\gamma}{1-\alpha}\psi\left\{w_j t_{\ell,j} - \sum \pi_{\ell,j} w_j t_{\ell,j}\right\}}_{\text{Externality complementarity correction}}$$
$$+ \underbrace{N_f\left\{\frac{\partial \overline{u}_f}{\partial \pi_{\ell,j}} - \sum_{\ell,j}\pi_{\ell,j}\frac{\partial \overline{u}_f}{\partial \pi_{\ell,j}}\right\}}_{\text{Foreign-resident externality correction}}.$$

5 Long run: the future of global cities

In this section, we study how an influx of foreign residents affects the optimal long-run city design. To study these effects, we consider the possibility that offices can be converted into homes and vice versa.

Consider first the marginal social welfare effect of converting offices into houses in the city center.

$$d\mathcal{W} = v'\left(\frac{\overline{H}_c}{\Pi_c + N_f}\right) - \alpha\frac{L_c^{1+\gamma-\alpha}}{\overline{K}_c^{1-\alpha}}.$$

Suppose that before the influx of foreign residents, the rental rates of houses and offices are equalized in the center and in the periphery: $r_c = r_c^K$ and $r_p = r_p^K$. The condition $r_c = r_c^K$ can be rewritten as:

$$v'(\overline{H}_c/\Pi_c) - \alpha\frac{L_c^{1+\gamma-\alpha}}{\overline{K}_c^{1-\alpha}} = 0.$$

Foreign home purchases reduce housing consumption in the center ($\frac{\overline{H}_c}{\Pi_c+N_f}$), increasing the utility of additional homes. At the same time, locals move away from the center, reducing labor supply L_c and the rents of office buildings. It is optimal to increase home supply in the city center, decreasing office supply: $d\mathcal{W} > 0$.

Consider now the effect on social welfare of converting houses into offices in the periphery,

$$d\mathcal{W} = \alpha \frac{L_p^{1+\gamma-\alpha}}{\overline{K}_p^{1-\alpha}} - v'\left(\frac{\overline{H}_p}{\Pi_p}\right).$$

Locals move to the periphery, reducing per-capita housing consumption $(\frac{\overline{H}_p}{\Pi_p})$. At the same time, the labor supply increases in the periphery. As a result, the total marginal effect on social welfare is ambiguous: $d\mathcal{W} \lesseqgtr 0$.

Proposition 5. *In response to an influx of foreign residents, it is optimal to convert offices into houses in the city center. In contrast, the welfare effect of converting offices into houses in the periphery is ambiguous.*

6 Heterogeneous property ownership

In this section, we extend the model to allow for heterogeneous ownership of houses and office buildings. We assume that each individual i belongs to one of a finite number of groups: $g \in \{1, ..., G\}$. This formulation allows us to use the law of large numbers in computing the welfare of each group.

The mass of group g is given by $\chi_g \geq 0$, which satisfies the adding-up condition $\sum_g \chi_g = 1$. Each member of group g is endowed with a share $s_g \geq 0$ of houses and $s_g^k \geq 0$ of office buildings. These shares are defined as the housing (office buildings) holdings of a person in group g divided by the per capita housing stock (office building stock). In groups whose members own more houses than the per capita housing stock, $s_g \geq 1$.

The non-labor income of a person in group g is

$$T_g = s_g \sum_j r_j \overline{H}_j + s_g^K \sum_j r_j^K \overline{K}_j,$$

where $\sum_g \chi_g s_g = 1$ and $\sum_g \chi_g s_g^K = 1$.

The equilibrium conditions are given by

$$
\begin{aligned}
h_{g,\ell,j} &= h_\ell, \quad \text{given by } v(h_\ell) = r_\ell \\
c_{\ell,j} &= w_j(1-t_{\ell,j}) + T_g - r_\ell h_\ell \\
u_{g,\ell,j} &= \overline{u}_{\ell,j} + w_j(1-t_{\ell,j}) + T_g - r_\ell h_\ell + v(h_\ell) \\
\pi_{g,\ell,j} &= \frac{e^{\eta u_{g,\ell,j}}}{\sum_{\ell',j'} e^{\eta u_{g,\ell',j'}}} = \pi_{\ell,j}.
\end{aligned}
$$

Because preferences are quasi-linear, heterogeneity in property holdings does not affect housing purchases or location choices.

We define the welfare of group g as the average utility across group members. We can write the welfare of group g as:

$$
\mathcal{W}_g = \frac{\log\left(\sum_{\ell,j} e^{\eta u_{g,\ell,j}}\right)}{\eta} + \frac{\text{Euler-Mascheroni constant}}{\eta}.
$$

We investigate the impact of home purchases by foreigners on individuals with different holdings of houses and office buildings.

Proposition 6. *The change in group-g welfare is given by*

$$
d\mathcal{W}_g = \mathcal{PE} + \left(s_g^K - 1\right)\mathcal{CG}^K + s_g \mathcal{FS} + \left(s_g - 1\right)\mathcal{CG}^{H,L},
$$

where

$$
\mathcal{CG}^{H,L} \equiv \sigma r_c \frac{\overline{H}_c - N_f h_f}{\Pi_c + N_f}\left(d\Pi_c + dN_f\right) + \sigma r_p \frac{\overline{H}_p}{\Pi_p} d\Pi_p,
$$

denotes the capital gains on houses purchased by locals and

$$
\mathcal{CG}^K \equiv \sum_j \alpha Y_j \left\{\gamma + (1-\alpha)\right\} \frac{dL_j}{L_j}.
$$

denotes the capital gains on office buildings.

To understand the expression for the change in welfare in Proposition 6, note that people in group g benefit from the foreign-resident surplus in proportion to the share of houses they own, s_g. To the extent that $s_g \neq 1$, they may also gain or lose from the fact that houses purchased by locals become more expensive, $\mathcal{CG}^{H,L}$.

People with $s_g = 0$ have to pay higher rents but do not benefit from housing capital gains. More generally, if $s_g < 1$, their capital gains are lower than the increase in housing costs. People who own more shares than average, $s_g > 1$, receive capital gains that exceed the rise in housing costs.

The change in wage income of people in group g is given by

$$\sum_j (\gamma - \alpha)(1-\alpha) Y_j \frac{dL_j}{L_j},$$

and the change in their capital income is given by

$$s_g^K \sum_j \alpha Y_j \{\gamma + (1-\alpha)\} \frac{dL_j}{L_j}.$$

Adding these two effects, we obtain

$$\gamma \sum_j \frac{dL_j}{L_j} Y_j + \left(s_g^K - 1\right) \sum_j \alpha Y_j \{\gamma + (1-\alpha)\} \frac{dL_j}{L_j} = \mathcal{PE} + \left(s_g^K - 1\right) \mathcal{CG}^K.$$

So, $\mathcal{PE}$ has two components: the change in wages and the changes in rents to office buildings. Implicitly, $\mathcal{PE}$ is defined as if offices are equally distributed among the population. The term $\left(s_g^K - 1\right) \mathcal{CG}^K$ corrects $\mathcal{PE}$ for the fact that the change in office rents are unequally distributed among the population. When $s_g^K = 0$, people in group g receive no capital income. So, the production externality effect must be subtracted by the change in office rents in order to obtain only the change in wage income.

6.1 Optimal policy

We compute the second-best optimal allocation in which the planner cannot observe taste shocks. The planner has information about people's residential and workplace choices as well as their holdings of houses and office buildings, so it can make allocations contingent on these factors.

The planner chooses the share of people in each location, $\pi_{g,\ell,j}$. These shares must satisfy

$$\pi_{g,\ell,j} = \frac{e^{\eta u_{g,\ell,j}}}{\sum_{\ell',j'} e^{\eta u_{g,\ell',j'}}}.$$

In order to characterize the welfare possibilities frontier, the planner maximizes the welfare of group 1,

$$\max \mathcal{W}_1 = \frac{\log\left(\sum_{1,\ell,j} e^{\eta u_{1,\ell,j}}\right)}{\eta},$$

subject to achieving specific welfare levels for other groups

$$\frac{\log\left(\sum_{\ell,j} e^{\eta u_{g,\ell,j}}\right)}{\eta} \geq \overline{u}_g^p,$$

the resource constraints for goods,

$$\sum_g \chi_g \sum_{\ell,j} \pi_{g,\ell,j} c_{\ell,j} + N_f c_f \leq L_c^{\gamma+1-\alpha} \overline{K}_c^{\alpha} + L_p^{\gamma+1-\alpha} \overline{K}_p^{\alpha} + N_f y_f,$$

the adding-up constraints for housing in the center and in the periphery,

$$\sum_g \chi_g \sum_j \pi_{g,c,j} h_{g,c,j} + N_f h_f \leq \overline{H}_c,$$

$$\sum_g \chi_g \sum_j \pi_{g,p,j} h_{g,p,j} \leq \overline{H}_p,$$

the incentive compatibility constraints,

$$\pi_{g,\ell,j} = \frac{e^{\eta u_{g,\ell,j}}}{\sum_{\ell',j'} e^{\eta u_{g,\ell',j'}}},$$

and the foreign resident participation constraints,

$$\overline{u}_f + c_f + v\left(h_f\right) \geq u_f^*.$$

By varying the parameters $\overline{u}_g^p$ we can trace out the full group welfare frontier.

The first-order conditions for the planner's problem are given by

$$\lambda_g^u - \eta\lambda_{g,\ell,j}^{\text{loc}} + \eta \sum_{g,\ell',j'} \lambda_{g,\ell',j'}^{\text{loc}} \pi_{g,\ell',j'} = \lambda_c$$

$$v'\left(h_{g,\ell,j}\right)\left(1 - \eta\lambda_{g,\ell,j}^{\text{loc}} + \eta \sum_{g,\ell',j'} \lambda_{g,\ell',j'}^{\text{loc}} \pi_{g,\ell',j'}\right) = \lambda_{h,\ell},$$

$$\lambda_f = \lambda_c,$$

$$\lambda_f v'\left(h_f\right) = \lambda_{h,c},$$

$$\lambda_c\left(y_f - c_f\right) - \lambda_{h,c} h_f = 0,$$

$$\lambda_c\left\{(1+\gamma-\alpha)\frac{Y_j}{L_j}\left(1-t_{\ell,j}\right) - c_{g,\ell,j}\right\} - \lambda_{h,\ell} h_{g,\ell,j} = \lambda_{g,\ell,j}^{\text{loc}},$$

where $\lambda_1^u \equiv 1$. The first-order conditions with respect to consumption imply that $\lambda_c = \lambda_g^u = \lambda_1^u = 1$. In addition, $\lambda_{g,\ell,j}^{\text{log}} = \lambda_g^{\text{log}}$. These results imply that housing purchases are equal for all individuals in a given location independently of group membership or work location

$$v'(h_{g,\ell,j}) = \lambda_{h,\ell} \Rightarrow h_{g,\ell,j} \equiv h_\ell = [v']^{-1}(\lambda_{h,\ell}).$$

Optimal location choices $\pi_{g,\ell,j}$ are constant across groups. The transfers to individuals in group g who live in location ℓ and work in location j are given by

$$T_{g,\ell,j} = T_g + \frac{\gamma}{1-\alpha}\left\{\frac{w_j}{L_j}\left(1-t_{\ell,j}\right) - \sum_{\ell,j}\pi_{\ell,j}\frac{w_j}{L_j}(1-t_{\ell,j})\right\},$$

where T_g are group-specific transfers which are a function of the parameters $\overline{u}_g^p$ and satisfy

$$\sum_g \chi_g T_g = \alpha\sum_j Y_j + \sum_\ell r_\ell \overline{H}_\ell \equiv T.$$

By varying the elements of the vector that comprises the welfare of the different groups, $\{\overline{u}_g\}$, we can calculate the set of transfers for each group that satisfies this equation. It is always possible to find a distribution of welfare across groups $\{\overline{u}^p_\ell\}$ such that the second-best solution does not involve redistributing the rental income received by different groups in the competitive equilibrium.

The key result in this section is summarized by the following proposition.

Proposition 7. *Given an initial distribution of property ownership, it is possible to implement a transfer and tax policy such that, ex-ante, before taste shocks materialize, all groups gain from the influx of foreign residents.*

In the model, capital gains can be redistributed through lump-sum taxes and transfers. In practice, this redistribution can be implemented by taxing capital gains on housing and making transfers to those with property holdings below average. In a static model like ours, this tax does not distort the decisions of individuals. In a dynamic setting, capital gain taxes are also not distorting as long as investment expenses can be deducted from the tax base (see Abel, 2007).

7 Remote work

In this section, we consider a version of our model in which local workers can either work onsite at an office, or at home.[5]

The production function of the representative firm in location j is given by

$$y_j = A\left(L_{j,o}\right)\left(l_{j,o}^{1-\alpha}k_j^{\alpha} + A_h l_{j,h}\right),$$

where $l_{j,o}$ and $l_{j,h}$ denote the number of people working for the firm in the office and at home, respectively. The agglomeration or production externality, $A\left(L_{j,o}\right)$,

[5]See Monte et al., 2023 for a dynamic theory of remote work and city structure in which agglomeration forces can generate multiple equilibria.

depends on the total number of people who work in offices in location j, $L_{j,o}$. This externality benefits the productivity of all the workers. We assume that

$$A\left(L_{j,o}\right) = L_{j,o}^{\gamma}.$$

Location choices People's choices are influenced by Gumbel-distributed taste shocks, $\xi_{\ell,j,e}$, about the location of their residence, workplace, and remote versus onsite work.

The subscript e indexes labor arrangements. It takes the value o and h depending on whether the individual works onsite or at home, respectively. Consider a local individual who chooses to live in location ℓ, work in location j, and use work arrangement e. The optimal consumption and housing services for this individual are those that maximize common utility, that is, the utility net of taste shocks

$$\overline{u}_{\ell} + c_{\ell,j,e} + v\left(h_{\ell,j,e}\right),$$

subject to the budget constraint

$$c_{\ell,j,e} + r_{\ell} h_{\ell,j,e} = w_{j,e}\left(1 - t_{\ell,j,e}\right) + T.$$

Here $t_{\ell,j,e}$ is the cost of commuting from ℓ to j for someone working at e (home or office). This commuting cost is zero for those who work in an office in the same location as their residence, $t_{\ell,\ell,e} = 0$, and for those who work from home, $t_{\ell,j,h} = 0$ for all ℓ, j.

The first-order conditions for this problem are,

$$v'\left(h_{\ell,j,e}\right) = r_{\ell},$$

$$c_{\ell,j,e} = w_{j,e}\left(1 - t_{\ell,j,e}\right) + T - r_{\ell} h_{\ell,j,e},$$

$$u_{\ell,j,e} = \overline{u}_{\ell} + w_{j,e}\left(1 - t_{\ell,j,e}\right) + T - r_{\ell} h_{\ell,j,e} + v\left(h_{\ell,j,e}\right),$$

All individuals residing in ℓ choose the same housing consumption.

$$h_{\ell,j,e} = h_{\ell}.$$

Optimal location and work arrangement choices are given by

$$\{\ell, j, e\} = \arg\max \left\{ u_{\ell,j,e} + \xi_{\ell,j,e} \right\}.$$

The share of individuals living in ℓ and working in j with employment type e is

$$\pi_{\ell,j,e} = \frac{e^{\eta u_{\ell,j,e}}}{\sum_{\ell',j',e'} e^{\eta u_{\ell',j',e'}}}.$$

Even when the wage for remote work is low, there are always people who choose this option because of their taste shock. Note that individuals who live and work in different places are relatively more likely to work remotely, since,

$$\frac{\pi_{j,j,o}}{\pi_{j,j,h}} = e^{\eta(w_{j,o} - w_{j,h})} > e^{\eta(w_{j,o}(1-t_{\ell,j}) - w_{j,h})} = \frac{\pi_{\ell,j,o}}{\pi_{\ell,j,h}},$$

for $\ell \neq j$. This result suggests that the influx of foreign home buyers may increase telecommuting within the city by relocating local workers to the periphery.

Foreign resident problem The foreign resident problem is the same as in the benchmark model.

Firms' problem A firm in location j maximizes profits, given by

$$\pi_j = A\left(L_{j,o}\right)\left(l_{j,o}^{1-\alpha} k_j^{\alpha} + A_h l_{j,h}\right) - w_{j,o} l_{j,0} - w_{j,h} l_{j,h} - r_j k_j.$$

The first-order conditions for this problem are,

$$w_{j,h} = A\left(L_{j,o}\right) A_h,$$

$$w_{j,o} = (1-\alpha) A\left(L_{j,o}\right) l_{j,o}^{-\alpha} k_j^{\alpha},$$

$$r_j = \alpha A\left(L_{j,o}\right) l_{j,o}^{1-\alpha} k_j^{\alpha-1}.$$

Equilibrium conditions There are two labor market clearing conditions. The first is for office workers in location j:

$$l_{j,o} = L_{j,o} = \sum_{\ell} \pi_{\ell,j,o} \left(1 - t_{\ell,j,o}\right).$$

The second is for remote workers employed by firms in location j is

$$l_{j,h} = L_{j,h} = \sum_{\ell} \pi_{\ell,j,h}.$$

The market clearing conditions for office buildings in location j is

$$k_j = \overline{K}_j.$$

The goods market clearing condition is

$$\sum_{\ell,j,e} \pi_{\ell,j,e} c_{\ell,j,e} + N_f c_f = \sum_j A\left(L_{j,o}\right)\left(L_{j,o}^{\alpha} \overline{K}_j^{1-\alpha} + A_h L_{j,h}\right) + N_f y_f.$$

The housing market clearing conditions for the center and the periphery are

$$\Pi_c h_c + N_f h_f = \overline{H}_c,$$

$$\Pi_p h_p = \overline{H}_p,$$

where

$$\Pi_c \equiv \sum_{j,e} \pi_{c,j,e},$$

$$\Pi_p \equiv \sum_{j,e} \pi_{p,j,e}.$$

Rents on housing and office buildings are distributed equally across locals. These rents are given by

$$T = \sum_{\ell} r_{\ell} \overline{H}_{\ell} + \sum_j r_j^K \overline{K}_j.$$

Social welfare Social welfare is the average utility across the local population,

$$\mathcal{W} = \frac{\log\left(\sum_{\ell,j,e} e^{\eta u_{\ell,j,e}}\right)}{\eta} + \frac{\text{Euler-Mascheroni constant}}{\eta}.$$

The welfare impact of a marginal increase in the number of foreign residents We assume that the foreign residents' participation constraint is satisfied. The marginal change in social welfare from an influx of foreign residents is

$$d\mathcal{W} = \sum_{\ell,j,e} \pi_{\ell,j,e} du_{\ell,j,e}.$$

The change in common utility is

$$u_{\ell,j,e} = \overline{u}_\ell + w_{j,e}\left(1 - t_{\ell,j,e}\right) + T - r_\ell h_{\ell,j,e} + v\left(h_{\ell,j,e}\right),$$

$$du_{\ell,j,e} = dw_{j,e}\left(1 - t_{\ell,j,e}\right) + dT + \sigma v'\left(h_\ell\right) dh_\ell.$$

The change in the wages paid to onsite and remote workers are

$$\frac{dw_{j,o}}{w_{j,o}} = (\gamma - \alpha)\frac{dL_{j,o}}{L_{j,o}},$$

$$\frac{dw_{j,h}}{w_{j,h}} = \gamma\frac{dL_{j,o}}{L_{j,o}},$$

where the change in the number of office workers in location j is

$$\frac{dL_{j,o}}{L_{j,o}} = \sum_j \frac{\pi_{j,\ell,o}}{L_{j,o}}\left(1 - t_{j,\ell,o}\right)\frac{d\pi_{j,\ell,o}}{\pi_{j,\ell,o}}.$$

The changes in housing consumption are

$$\frac{dh_p}{h_p} = -\frac{d\Pi_p}{\Pi_p},$$

and

$$\frac{dh_c}{h_c} = -\left(\frac{\Pi_c}{\Pi_c + N_f}\frac{d\Pi_c}{\Pi_c} + \frac{N_f}{\Pi_c + N_f}\frac{dN_f}{N_f}\right),$$

where $\Pi_\ell = \sum_{j,e} \pi_{\ell,j,e}$. The change in the rents on houses and office buildings are:

$$\begin{aligned} dT &= -\sum_\ell \sigma \overline{H}_\ell v'\left(h_\ell\right) \frac{dh_\ell}{h_\ell} + \sum_\ell \alpha Y_\ell \left\{\gamma + (1-\alpha)\right\} \frac{dL_\ell}{L_\ell} \\ &= \sigma \overline{H}_c v'\left(h_c\right) \left(\frac{\Pi_c}{\Pi_c + N_f} \frac{d\Pi_c}{\Pi_c} + \frac{N_f}{\Pi_c + N_f} \frac{dN_f}{N_f} \right) \\ &+ \sigma \overline{H}_p v'\left(h_p\right) \frac{d\Pi_p}{\Pi_p} + \sum_\ell \alpha Y_{\ell,o} \left\{\gamma + (1-\alpha)\right\} \frac{dL_\ell}{L_\ell}. \end{aligned}$$

Using these expressions, we can compute the change in utility for those who live in the center and the periphery and add them to compute the change in social welfare. The result is the following expression

$$\begin{aligned} d\mathcal{W} &= \frac{\gamma}{1-\alpha} \sum_{j,\ell} \pi_{\ell,j,o} w_{j,o} \left(1 - t_{\ell,j,o}\right) \frac{d\pi_{\ell,j,o}}{\pi_{\ell,j,o}} \\ &+ \gamma \sum_{j,\ell} \pi_{\ell,j,o} w_{j,h} \left(1 - t_{\ell,j,o}\right) \frac{d\pi_{\ell,j,o}}{\pi_{\ell,j,o}} \\ &+ \sigma v'\left(h_c\right) N_f h_f \left(\frac{\Pi_c}{\Pi_c + N_f} \frac{d\Pi_c}{\Pi_c} + \frac{N_f}{\Pi_c + N_f} \frac{dN_f}{N_f} \right). \end{aligned}$$

Proposition 8. *The change in social welfare caused by a marginal increase in the number of foreign residents has two components. The first is the foreign resident surplus,*

$$\mathcal{FS} \equiv \sigma \frac{N_f}{\Pi_c + N_f} r_c h_f \left(d\Pi_c + dN_f\right).$$

The second is the production externality

$$\mathcal{PE} = \gamma \mathbb{COV} \left(\left(Y_{j,o} + Y_{j,h}\right) \frac{1 - t_{\ell,j,o}}{L_{j,o}}, \frac{d\pi_{\ell,j,o}}{\pi_{\ell,j,o}} \right) + \gamma \left(\sum_j Y_{j,h} \right) d\Pi_o^e.$$

The production externality includes the impact of an increase in foreign residents on the productivity of onsite and remote workers. This impact operates through two channels. The first is changes in the spatial distribution of office workers. If office workers move to locations where they contribute less to the externality, then the covariance in $\mathcal{PE}$ is negative because overall productivity falls. The second channel is changes in the overall number of people working onsite. If people move to the periphery but work remotely in the center, the number of office workers falls, reducing the external effect.

Second-best solution In the second best, we have that the planner assigns utilities for each triplet comprised of residence location, work location, and work arrangement. The share of agents that chooses the triplet (ℓ, j, e) is given by

$$\pi_{\ell,j} = \frac{e^{\eta u_{\ell,j,e}}}{\sum_{\ell',j',e'} e^{\eta u_{\ell',j',e'}}}$$

and so welfare is

$$\mathcal{W} = \frac{\log\left(\sum_{\ell,j,e} e^{\eta u_{\ell,j,e}}\right)}{\eta}.$$

It is useful to define $u_{\ell,j,e} = \overline{u}_{\ell,j,e} + c_{\ell,j,e} + v\left(h_{\ell,j,e}\right)$ and $L_{j,e} = \sum_{\ell} \pi_{\ell,j,e}\left(1 - t_{\ell,j,e}\right)$.

The second-best problem is to maximize

$$\frac{\log\left(\sum_{\ell,j} e^{\eta u_{\ell,j}}\right)}{\eta},$$

subject to

$$\sum_{\ell,j,e} \pi_{\ell,j,e} c_{\ell,j,e} + \pi_p c_p + N_f c_f = \sum_j A\left(L_{j,o}\right)\left(L_{j,o}^{\alpha} \overline{K}_j^{1-\alpha} + A_h L_{j,h}\right) + N_f y_f,$$

$$\sum_{j,e} \pi_{c,j,e} h_{c,j,e} + N_f h_f \leq \overline{H}_c,$$

$$\sum_{j,e} \pi_{p,j,e} h_{p,j,e} \leq \overline{H}_p,$$

$$\pi_{\ell,j,e} = \frac{e^{\eta u_{\ell,j,e}}}{\sum_{\ell',j',e'} e^{\eta u_{\ell',j',e'}}},$$

$$c_f + v\left(h_f\right) \geq \overline{u}_f.$$

The Lagrangean for this problem is

$$\begin{aligned}
\mathcal{L} = & \frac{\log\left(\sum_{\ell,j} e^{\eta u_{\ell,j,e}}\right)}{\eta} + \lambda_c\left(\sum_j A\left(L_{j,o}\right)\left(L_{j,o}^{\alpha}\overline{K}_j^{1-\alpha} + A_h L_{j,h}\right) + N_f\left(y_f - c_f\right) - \sum_{\ell,j,e}\pi_{\ell,j,e}c_{\ell,j,e}\right) \\
& + \lambda_{h,c}\left(\overline{H}_c - \sum_{j,e}\pi_{c,j,e}h_{c,j,e} - N_f h_f\right) + \lambda_{h,p}\left(\overline{H}_p - \sum_{j,e}\pi_{p,j,e}h_{p,j,e}\right) \\
& + \sum_{\ell,j,e}\lambda_{\ell,j,e}^{\text{loc}}\left(\pi_{\ell,j,e} - \frac{e^{\eta u_{\ell,j,e}}}{\sum_{\ell',j',e'} e^{\eta u_{\ell',j',e'}}}\right) \\
& + N_f\lambda_f\left(c_f + v\left(h_f\right) - \overline{u}_f\right).
\end{aligned}$$

The first-order conditions for consumption and housing services of locals and foreign residents can be written as

$$1 + \eta\left(\sum_{\ell',j',e'}\lambda_{\ell',j',e'}^{\text{loc}}\pi_{\ell',j',e'} - \lambda_{\ell,j,e}^{\text{loc}}\right) = \lambda_c,$$

$$v'\left(h_{\ell,j}\right)\left\{1 + \eta\left(\sum_{\ell',j',e'}\lambda_{\ell',j',e'}^{\text{loc}}\pi_{\ell',j',e'} - \lambda_{\ell,j,e}^{\text{loc}}\right)\right\} = \pi_{\ell,j}\lambda_{\ell,h},$$

$$\lambda_f = \lambda_c,$$

$$\lambda_f v'\left(h_f\right) = \lambda_{h,c}.$$

Summing the first equation over ℓ, j we conclude that $\lambda_c = 1$ and $\lambda_{\ell,j,e}^{\text{loc}} = \lambda^{\text{loc}}$ for all ℓ, j. These results imply that the multipliers on the location-decision constraints take on the same value. This property means that the social welfare in the second-best allocation is the same as in the first-best allocation. Because utility is linear in consumption, social welfare depends only on the aggregate level of consumption. Since the distribution of consumption does not affect social welfare, the planner can always redistribute consumption without impacting social welfare to incentivize individuals to choose a certain residential and working location or working arrangement.

The first-order conditions also imply that $\lambda_f = 1$. Combining these results, we obtain

$$v'\left(h_{c,j,e}\right) = v'\left(h_f\right) = v'\left(\frac{\overline{H}_c}{\sum_{j,e} \pi_{c,j,e} + N_f}\right),$$
$$v'\left(h_{p,j,e}\right) = v'\left(\frac{\overline{H}_p}{\sum_{j,e} \pi_{p,j,e}}\right).$$

These equations imply that all individuals who live in the same location consume the same housing services,

The first-order condition for N_f. is

$$\lambda_c\left(y_f - c_f\right) - \lambda_{h,c} h_f = 0 \Leftrightarrow y_f = c_f + v'\left(h_f\right) h_f.$$

This equation implies that it is not optimal to tax foreign residents or restrict in any way their home purchases.

The first-order conditions for the optimal shares of people in offices can be written as:

$$(1 + \gamma - \alpha) \frac{Y_{j,o}}{L_j}\left(1 - t_{\ell,j}\right) + \gamma \frac{Y_{j,h}}{L_{j,o}}\left(1 - t_{\ell,j}\right) - c_{\ell,j,o} - \lambda_{h,\ell} h_{\ell,j,o} = -\lambda^{\mathrm{loc}}.$$

The first-order conditions for the shares of people working from home are

$$A\left(L_{j,o}\right) A_h - c_{\ell,j,h} - \lambda_{h,\ell} h_{\ell,j,h} = -\lambda^{\mathrm{loc}}.$$

The key results are summarized in the following proposition.

Proposition 9. *The optimal solution involves transfers to office and remote workers. The optimal transfer to office workers is*

$$T_{\ell,j,o} = \alpha \sum_j Y_{j,o} + \sum_\ell \lambda_{h,\ell} \overline{H}_\ell + \gamma \left\{ \frac{Y_{j,o} + Y_{j,h}}{L_{j,o}}\left(1 - t_{\ell,j}\right) - \sum_{\ell,j} \pi_{\ell,j,o} \frac{Y_{j,o} + Y_{j,h}}{L_{j,o}}\left(1 - t_{\ell,j}\right) \right\}.$$

The optimal transfer to remote workers is

$$T_{\ell,j,h} = \alpha \sum_j Y_{j,o} + \sum_\ell \lambda_{h,\ell} \overline{H}_\ell - \gamma \sum_j \left(Y_{j,o} + Y_{j,h}\right).$$

The optimal transfer to office workers has two components. The first is the rent from houses and office buildings. These rents are equally distributed among locals. The second is transfers that internalize the production externality. Office workers receive higher transfers if they work in a location where productivity is higher than average.

The two components of the optimal transfer to remote workers are as follows. The first, which is positive, is the rental income from houses and office buildings. The second, which is negative, reflects the fact that remote workers do not contribute to the production externality.

8 Conclusion

Many nations and urban areas are grappling with the challenge of devising policies to ensure that the local population benefits from a potentially large influx of foreign residents.

We show that an optimal approach involves internalizing externalities through the implementation of transfers to local individuals based on their residential and occupational locations. Once these externalities are internalized, the marginal impact of additional foreign residents is positive. There is no marginal impact of foreign residents on external effects, and there exists a positive surplus resulting from capital gains on housing. Consequently, restricting property purchases by foreigners or imposing taxes on those purchases is not optimal.

In situations where there is an unequal distribution of housing and office buildings ownership in the population, it can be optimal to implement transfers that redistribute the capital gains produced by the influx of foreign residents.

In scenarios where the local population can choose whether to work from home or at the office, it is optimal to implement transfers targeted toward office workers to internalize production or agglomeration externalities.

Looking toward the future, it is optimal in the long-run to convert office spaces in the city center into residential units and relocate production facilities to the periphery. This urban design mirrors the one adopted in Paris. In the 19th century, Napoleon III granted Baron Hausmann broad powers to remodel Paris. The result was the monumental city we know today, with wide boulevards, impressive squares, and views of the Eiffel Tower that are not obstructed by towering skyscrapers. Office buildings, production structures, and residential complexes, where a majority of the local population resides, were shifted to La Defense and other peripheral areas. The ability of Paris to accommodate foreign residents impressed Ernest Hemingway who wrote that "There are only two places in the world where we can live happy—at home and in Paris."

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A Appendix to Section 2

A.1 Location shares

Define $x_{i,\ell,j} = u_{\ell,j} + \xi_{i,\ell,j}$ for $\ell, j = c, p$. The cross-sectional cumulative density function of $x_{i,\ell,j}$ is given by

$$G_{\ell,j}(x) = \mathbb{P}\left[x_{i,\ell,j} \leq x\right] = F\left(x - u_{\ell,j}\right) = e^{-e^{-\eta\left(x-u_{\ell,j}\right)}},$$

and the associated probability density function is given by

$$g_{\ell,j}(x) = \eta e^{-\eta\left(x-u_{\ell,j}\right)} e^{-e^{-\eta\left(x-u_{\ell,j}\right)}}$$

Then, by the law of large numbers

$$\begin{aligned}
\pi_{\ell,j} = \mathbb{P}\left[x_{i,\ell,j} = \max_{\ell',j'} x_{i,\ell',j'}\right] &= \int_{-\infty}^{\infty} g_{\ell,j}(x) \prod_{(\ell',j')\neq(\ell,j)} G_{\ell',j'}(x)dx \\
&= \int_{-\infty}^{\infty} \eta e^{-\eta\left(x-u_{\ell,j}\right)} e^{-e^{-\eta\left(x-u_{\ell,j}\right)}} \prod_{(\ell',j')\neq(\ell,j)} e^{-e^{-\eta\left(x-u_{\ell',j'}\right)}} dx \\
&= \frac{e^{\eta u_{\ell,j}}}{\sum_{\ell',j'} e^{\eta u_{\ell',j'}}} \int_{-\infty}^{\infty} \eta e^{-\eta x}\left(\sum_{\ell',j'} e^{\eta u_{\ell',j'}}\right) e^{-e^{-\eta x}\left(\sum_{\ell',j'} e^{\eta u_{\ell',j'}}\right)} dx \\
&= \frac{e^{\eta u_{\ell,j}}}{\sum_{\ell',j'} e^{\eta u_{\ell',j'}}}.
\end{aligned}$$

A.2 Social welfare

By the law of large numbers

$$\mathcal{W} \equiv \int_0^1 \max\left\{u_{\ell,j} + \xi_{i,\ell,j}\right\} di = \mathbb{E}\left[\max_{\ell,j}\left\{u_{\ell,j} + \xi_{i,\ell,j}\right\}\right].$$

Let $x^* \equiv \max_{\ell,j}\left\{u_{\ell,j} + \xi_{i,\ell,j}\right\}$. The cumulative distribution function of x^* is given by:

$$F^*(x) = \mathbb{P}\left[x^* \leq x\right] = \mathbb{P}\left[\xi_{\ell,j} \leq x - u_{\ell,j}, \quad \forall(\ell,j)\right] = \prod_{\ell,j} e^{-e^{-\eta\left(x-u_{\ell,j}\right)}} = e^{-e^{-\eta x}\sum_{\ell,j} e^{\eta u_{\ell,j}}}$$

and the probability density function is given by:

$$f^{*}(x) = \eta e^{-\eta x} \left(\sum_{\ell,j} e^{\eta u_{\ell,j}} \right) e^{-e^{-\eta x} \sum_{\ell,j} e^{\eta u_{\ell,j}}}$$

So, social welfare is give by

$$\mathcal{W} = \mathbb{E}\left[\max\left\{u_{\ell,j} + \xi_{\ell,j}\right\}\right] = \int_{-\infty}^{\infty} x \eta e^{-\eta x} \left(\sum_{\ell,j} e^{\eta u_{\ell,j}} \right) e^{-e^{-\eta x} \sum_{\ell,j} e^{\eta u_{\ell,j}}} dx$$

It is useful to do a change of variables: $y = e^{-\eta x} \sum_{\ell,j} e^{\eta u_{\ell,j}}$. Then,

$$x = -\frac{1}{\eta} \log \left(\frac{y}{\sum_{\ell,j} e^{\eta u_{\ell,j}}} \right),$$

$dx = -\frac{1}{\eta}\frac{dy}{y}$, $\lim_{x\to\infty} y = 0$ and $\lim_{x\to-\infty} y = \infty$. We can rewrite social welfare as follows:

$$\begin{aligned} \mathcal{W} &= \int_{\infty}^{0} \left(-\log \left(\frac{y}{\sum_{\ell,j} e^{\eta u_{\ell,j}}} \right) \right) y e^{-y} \left(-\frac{1}{\eta} \frac{dy}{y} \right) \\ &= \frac{1}{\eta} \int_{0}^{\infty} \left(\log \left(\sum_{\ell,j} e^{\eta u_{\ell,j}} \right) - \log(y) \right) e^{-y} dy \\ &= \frac{\log \left(\sum_{\ell,j} e^{\eta u_{\ell,j}} \right)}{\eta} + \frac{1}{\eta} \underbrace{\int_{0}^{\infty} [-\log(y)] e^{-y} dy}_{\text{Euler-Mascheroni Constant}} \end{aligned}$$

A.3 The welfare impact of increasing foreign residents

Using the fact that $r_\ell = v'(h_\ell)$, we can write common utility as

$$u_{\ell,j} = \overline{u}_{\ell,j} + w_j \cdot (1 - t_{\ell,j}) + T - v'(h_\ell) \cdot h_\ell + v(h_\ell),$$

where h_ℓ denote the quantity of housing purchased by people who live in location ℓ.

Then,

$$du_{\ell,j} = dw_j \cdot (1 - t_{\ell,j}) + dT - v''(h_\ell) \cdot h_\ell \cdot dh_\ell$$

Assuming that $v(h) = h^{1-\sigma}/(1-\sigma)$ we can write

$$du_{\ell,j} = dw_j \cdot (1 - t_{\ell,j}) + dT + \sigma \cdot r_\ell \cdot dh_\ell.$$

Note that, because

$$h_c = \frac{\overline{H}_c}{\Pi_c + N_f} \Rightarrow \frac{dh_c}{h_c} = -\left(\frac{\Pi_c}{\Pi_c + N_f}\frac{d\Pi_c}{\Pi_c} + \frac{N_f}{\Pi_c + N_f}\frac{dN_f}{N_f}\right),$$
$$h_p = \frac{\overline{H}_p}{\Pi_p} \Rightarrow \frac{dh_p}{h_p} = -\frac{d\Pi_p}{\Pi_p}.$$

Furthermore, since $w_j = L_j^\gamma \left(\frac{\overline{K}_j}{L_j}\right)^\alpha (1-\alpha)$ and $L_j = \sum_\ell \pi_{\ell,j}(1 - t_{\ell,j})$ then

$$\frac{dw_j}{w_j} = (\gamma - \alpha)\frac{dL_j}{L_j} = (\gamma - \alpha)\sum_\ell \frac{\pi_{\ell,j}}{L_j}\left(1 - t_{\ell,j}\right)\frac{d\pi_{\ell,j}}{\pi_{\ell,j}}.$$

Finally, the change in total rents is given by

$$\begin{aligned}
dT =& d\left(\sum_\ell v'\left(h_\ell\right)\overline{H}_\ell + \sum_\ell \alpha A\left(L_\ell\right)L_\ell^{1-\alpha}\overline{K}_\ell^\alpha\right) \\
=& \sum_\ell v''\left(h_\ell\right)\overline{H}_\ell dh_\ell + \sum_\ell \alpha A\left(L_\ell\right)L_\ell^{1-\alpha}\overline{K}_\ell^\alpha\left\{\frac{A'\left(L_\ell\right)}{A\left(L_\ell\right)}L_\ell + (1-\alpha)\right\}\frac{dL_\ell}{L_\ell} \\
=& -\sum_\ell \sigma\overline{H}_\ell v'\left(h_\ell\right)\frac{dh_\ell}{h_\ell} + \sum_\ell \alpha Y_\ell\left\{\gamma + (1-\alpha)\right\}\frac{dL_\ell}{L_\ell} \\
=& \sigma\overline{H}_c v'\left(h_c\right)\left(\frac{\Pi_c}{\Pi_c + N_f}\frac{d\Pi_c}{\Pi_c} + \frac{N_f}{\Pi_c + N_f}\frac{dN_f}{N_f}\right) + \sigma\overline{H}_p v'\left(h_p\right)\frac{d\Pi_p}{\Pi_p} \\
&+ \sum_\ell \alpha Y_\ell\left\{\gamma + (1-\alpha)\right\}\frac{dL_\ell}{L_\ell}.
\end{aligned}$$

Putting everything together, we find that

$$\begin{aligned} d\mathcal{W} &= (\gamma-\alpha)\, L_c w_c \frac{dL_c}{L_c} + (\gamma-\alpha)\, L_p w_p \frac{dL_p}{L_p} \\ &\quad + \sigma v'\left(h_c\right)\left(\overline{H}_c - \Pi_c h_c\right)\left(\frac{\Pi_c}{\Pi_c+N_f}\frac{d\Pi_c}{\Pi_c} + \frac{N_f}{\Pi_c+N_f}\frac{dN_f}{N_f}\right) \\ &\quad + \sigma v'\left(h_p\right)\left(\overline{H}_p - \Pi_p h_p\right)\frac{d\Pi_p}{\Pi_p} \\ &\quad + \sum_j \alpha Y_j \left\{\gamma + (1-\alpha)\right\}\frac{dL_j}{L_j}. \end{aligned}$$

Using the fact that $\overline{H}_c = \Pi_c h_c + N_f h_f$, $\overline{H}_p = \Pi_p h_p$, and $w_j = (1-\alpha)\frac{Y_j}{L_j}$, we can write

$$d\mathcal{W} = \gamma \sum_{\ell,j} \pi_{\ell,j} \frac{Y_j}{L_j}\left(1 - t_{\ell,j}\right)\frac{d\pi_{\ell,j}}{\pi_{\ell,j}} + \sigma \frac{N_f}{\Pi_c+N_f} r_c h_f \left(d\Pi_c + dN_f\right),$$

or, equivalently,

$$d\mathcal{W} = \frac{\gamma}{1-\alpha}\sum_{\ell,j} \pi_{\ell,j} w_j \left(1 - t_{\ell,j}\right)\frac{d\pi_{\ell,j}}{\pi_{\ell,j}} + \sigma \frac{N_f}{\Pi_c+N_f} r_c h_f \left(d\Pi_c + dN_f\right),$$

B Appendix to Section 3

B.1 Second-best problem and incentive compatibility

Let $c(\boldsymbol{\xi})$, $h(\boldsymbol{\xi})$, $\ell(\boldsymbol{\xi})$ and $j(\boldsymbol{\xi})$ denote, respectively, the consumption, housing, living location, and working location of a person with idiosyncratic location preferences $\boldsymbol{\xi} = [\xi_{c,c}, \xi_{c,p}, \xi_{p,c}, \xi_{p,p}]$.

The utility net of taste shocks of this person is given by

$$U(\boldsymbol{\xi}) \equiv \overline{u}_{\ell(\boldsymbol{\xi}),j(\boldsymbol{\xi})} + c(\boldsymbol{\xi}) + v\left(h(\boldsymbol{\xi})\right)$$

The incentive compatibility constraints of the direct revelation mechanism can be written as

$$U(\boldsymbol{\xi}) + \xi_{\ell(\boldsymbol{\xi}),j(\boldsymbol{\xi})} \geq U(\boldsymbol{\xi'}) + \xi_{\ell(\boldsymbol{\xi'}),j(\boldsymbol{\xi'})} \tag{4}$$

for all $\boldsymbol{\xi}$ and $\boldsymbol{\xi}'$.

It follows from (4) that if two people have the same location choices, then they must have the same level of common utility, i.e., assuming $(\ell(\boldsymbol{\xi}), j(\boldsymbol{\xi})) = (\ell(\boldsymbol{\xi}'), j(\boldsymbol{\xi}'))$, then

$$U(\boldsymbol{\xi}) = U(\boldsymbol{\xi}'). \tag{5}$$

Let $u_{\ell,j}$ denote the level of common utility attained by individuals with location choices ℓ, j.

Note that now incentive compatibility can now be equivalently written as

$$\{\ell(\boldsymbol{\xi}), j(\boldsymbol{\xi})\} = \arg\max\{u_{\ell,j} + \xi_{\ell,j}\}, \tag{6}$$

and $U(\boldsymbol{\xi}) = u_{\ell(\boldsymbol{\xi}),j(\boldsymbol{\xi})}$.

Using the properties of the Gumbel distribution, these equations imply that the share of individuals with location choices ℓ, j is given by

$$\pi_{\ell,j} = \frac{e^{\eta u_{\ell,j}}}{\sum_{\ell',j'} e^{\eta u_{\ell',j'}}}, \tag{7}$$

and, furthermore, the social welfare function is

$$\mathcal{W} = \frac{\log\left(\sum_{\ell,j} e^{\eta u_{\ell,j}}\right)}{\eta} + \frac{\text{Euler-Mascheroni Constant}}{\eta}. \tag{8}$$

Note that these are the only restrictions on the aggregate shares and social welfare implied by incentive compatibility. This means that if the planner chooses common utility levels $u_{\ell,j}$, location shares $\pi_{\ell,j}$, and welfare $\mathcal{W}$ which satisfy (7) and (8), then we can always find individual location choices which are consistent with incentive compatibility.

Note, furthermore, that because utility is concave in housing and all attain the same level of common utility then the optimal plan must always feature equal housing consumption for all people with the same location choices. It also follows that consumption is the same for all individuals with the same location choices.

Second-best solution

We write the Lagrangean for this optimization problem as follows,

$$\mathcal{L} = \frac{\log\left(\sum_{\ell,j} e^{\eta u_{\ell,j}}\right)}{\eta} + \lambda_c\left(L_c^{\gamma+1-\alpha}\overline{K}_c^{\alpha} + N_f\left(y_f - c_f\right) + L_p^{\gamma+1-\alpha}\overline{K}_p^{\alpha} - \sum_{\ell,j}\pi_{\ell,j}c_{\ell,j}\right)$$
$$+ \lambda_{h,c}\left(\overline{H}_c - \sum_j \pi_{c,j}h_{c,j} - N_f h_f\right) + \lambda_{h,p}\left(\overline{H}_p - \sum_j \pi_{p,j}h_{p,j}\right)$$
$$+ \sum_{\ell,j}\lambda_{\ell,j}^{\text{loc}}\left(\pi_{\ell,j} - \frac{e^{\eta u_{\ell,j}}}{\sum_{\ell',j'} e^{\eta u_{\ell',j'}}}\right) + N_f\lambda_f\left(\overline{u}_f + c_f + v\left(h_f\right) - u_f^*\right)$$

The first-order conditions for this problem can be written as

$$[c_{\ell,j}] \quad 1 - \eta\lambda_{\ell,j}^{\text{loc}} + \eta\sum_{\ell',j'}\lambda_{\ell',j'}^{\text{loc}}\pi_{\ell',j'} = \lambda_c \tag{9}$$

$$[h_{\ell,j}] \quad v'\left(h_{\ell,j}\right)\left(1 - \eta\lambda_{\ell,j}^{\text{loc}} + \eta\sum_{\ell',j'}\lambda_{\ell',j'}^{\text{loc}}\pi_{\ell',j'}\right) = \lambda_{h,\ell} \tag{10}$$

$$[c_f] \quad \lambda_f = \lambda_c \tag{11}$$

$$[h_f] \quad \lambda_f v'\left(h_f\right) = \lambda_{h,c} \tag{12}$$

$$[N_f] \quad \lambda_c\left(y_f - c_f\right) - \lambda_{h,c}h_f = 0 \tag{13}$$

$$[\pi_{\ell,j}] \quad \lambda_c\left\{(1+\gamma-\alpha)\frac{Y_j}{L_j}\left(1 - t_{\ell,j}\right) - c_{\ell,j}\right\} - \lambda_{h,\ell}h_{\ell,j} = \lambda_{\ell,j}^{\text{loc}} \tag{14}$$

and all constraints bind with equality.

Note that averaging across (9) for different ℓ, j we obtain

$$\sum_{\ell,j}\pi_{\ell,j}[1 - \eta\lambda_{\ell,j}^{\text{loc}} + \eta\sum_{\ell',j'}\lambda_{\ell',j'}^{\text{loc}}\pi_{\ell',j'}] = \sum_{\ell,j}\pi_{\ell,j}\lambda_c$$
$$\Leftrightarrow 1 - \eta\sum_{\ell,j}\pi_{\ell,j}\lambda_{\ell,j}^{\text{loc}} + \eta\sum_{\ell',j'}\lambda_{\ell',j'}^{\text{loc}}\pi_{\ell',j'} = \lambda_c \Leftrightarrow 1 = \lambda_c.$$

Using this finding back in (9) we find that

$$\lambda_{\ell,j}^{\text{loc}} = \sum_{\ell',j'}\lambda_{\ell',j'}^{\text{loc}}\pi_{\ell',j'} = \lambda^{\text{loc}}$$

is constant across location choices adn where $Y_j = A(L_j)\overline{K}_j^{\alpha} L_j^{1-\alpha}$.

Then, note that these first order conditions can be simplified to

$$[c_{\ell,j}] \quad 1 = \lambda_c \tag{15}$$

$$[h_{\ell,j}] \quad v'(h_{\ell,j}) = \lambda_{h,\ell} \tag{16}$$

$$[c_f] \quad \lambda_f = 1 \tag{17}$$

$$[h_f] \quad \lambda_f v'\left(h_f\right) = \lambda_{h,c} \tag{18}$$

$$[N_f] \quad \left(y_f - c_f\right) - \lambda_{h,c} h_f = 0 \tag{19}$$

$$[\pi_{\ell,j}] \quad \left\{(1+\gamma-\alpha)\frac{Y_j}{L_j}\left(1-t_{\ell,j}\right) - c_{\ell,j}\right\} - \lambda_{h,\ell} h_{\ell,j} = \lambda^{\text{loc}} \tag{20}$$

Note that these imply that

$$v'(h_{c,c}) = v'(h_{c,p}) = v'(h_f)$$

and

$$v'(h_{p,c}) = v'(h_{p,p}).$$

Note that these also imply that there are no marginal distortions in house purchases by foreigners. Furthermore, equation (19) shows that

$$y_f = c_f + v'(h_f)h_f,$$

which also shows that, in the optimum, foreigner spending is equal to their income, i.e., there should be no taxes on foreigners.

Now, averaging equation (20) across ℓ, j we find that

$$(1+\gamma-\alpha)\sum_j Y_j - \sum_{\ell,j} \pi_{\ell,j} c_{\ell,j} - \sum_{\ell,j} \pi_{\ell,j} \lambda_{h,\ell} h_{\ell,j} = \lambda^{\text{loc}}$$

Using the fact that $\sum_{\ell,j} \pi_{\ell,j} c_{\ell,j} = \sum Y_j + N_f(y_f - c_f) = \sum Y_j + N_f \lambda_{h,c} h_f$ we can rewrite that equation as

$$(\gamma-\alpha)\sum_j Y_j - \sum_{\ell} \lambda_{h,\ell} \overline{H}_{\ell} = \lambda^{\text{loc}}$$

Replacing λ^{loc} in equation (20), we find

$$c_j + \lambda_{\ell,j} = (1-\alpha)\frac{Y_j}{L_j}(1-t_{\ell,j}) + \gamma\frac{Y_j}{L_j}(1-t_{\ell,j}) + (\alpha-\gamma)\sum_j Y_j + \sum_\ell \lambda_{h,\ell}\overline{H}_\ell$$

$$c_j + \lambda_{\ell,j}h_{\ell,j} = (1-\alpha)\frac{Y_j}{L_j}(1-t_{\ell,j}) + \gamma\left[\frac{Y_j}{L_j}(1-t_{\ell,j}) - \sum_{\ell',j'}\frac{Y_{j'}}{L_{j'}}(1-t_{\ell',j'})\right] + \sum_j \alpha Y_j + \sum_\ell \lambda_{h,\ell}\overline{H}_\ell.$$

The decentralized equilibrium features the price of housing $r_\ell = \lambda_{h,\ell}$, the wage $w_j = (1-\alpha)Y_j/L_j$ and the rent of capital $r_j^K = \alpha Y_j/\overline{K}_j$. It follows that the transfer to individuals with location choices ℓ, j is given by

$$T_{\ell,j} = \gamma\left[\frac{Y_j}{L_j}(1-t_{\ell,j}) - \sum_{\ell',j'}\frac{Y_{j'}}{L_{j'}}(1-t_{\ell',j'})\right] + \sum_j r_j^K\overline{K}_j + \sum_\ell r_\ell\overline{H}_\ell. \tag{21}$$

C What if foreigners had the same spatial distribution as locals?

In the baseline model, we assume that foreigners only live in the city center. However, for the purposes of our results, all that is needed is that foreigners disproportionally seek to live in the city center relative to locals. In this appendix, we show that if $\sigma = 1$ and influx of foreigners chooses the same geographical spread as the incumbent population, then the production externality term is zero.

Suppose that foreigners enter both locations and let $N_{f,\ell}$ denote the number of foreigners that live in location ℓ. Due to the quasi-linearity of preferences, each individual in location ℓ consumes

$$h_\ell = \frac{\overline{H}_\ell}{\Pi_\ell + N_{f,\ell}} \tag{22}$$

houses, where $\Pi_\ell = \sum_j \pi_{\ell,j}$. Locals in location ℓ who work in j obtain common utility

$$u_{\ell,j} = \overline{u}_\ell + w_j(1-t_{\ell,j}) + T - r_\ell h_\ell + v(h_\ell), \tag{23}$$

where $r_\ell = v'(h_\ell)$. Their location choices satisfy

$$\pi_{\ell,j} = \frac{e^{\eta u_{\ell,j}}}{\sum_{\ell',j'} e^{\eta u_{\ell',j'}}}. \tag{24}$$

Suppose that there is an influx of foreigners which locates in proportion to the incumbent distribution. We denote with superscript the new equilibrium. Suppose that $N'_f > N_f$ and

$$N'_{f,\ell} = N_{f,\ell} + \frac{\Pi_\ell + N_{f,\ell}}{1 + N_f}(N'_f - N_f). \tag{25}$$

We prove via guess and verification that equilibrium location choices are unchanged. In each place, housing consumption falls proportionally

$$h'_\ell = \frac{\overline{H}_\ell}{\Pi_\ell + N'_{f,\ell}} = h_\ell \frac{1 + N_f}{1 + N'_f} < h_\ell.$$

Furthermore, note that since $\pi_{\ell,j}$ are unchanged then $w'_j = w_j$. Furthermore, with $\sigma = 1$

$$\begin{aligned}
u'_{\ell,j} =& \overline{u}_\ell + w_j(1 - t_{\ell,j}) + T' - \underbrace{v'(h'_\ell)h'_\ell}_{=1} + v(h'_\ell) \\
=& \overline{u}_\ell + w_j(1 - t_{\ell,j}) + T - \underbrace{v'(h_\ell)h_\ell}_{=1} + v(h_\ell) + \log\left(\frac{1 + N_f}{1 + N'_f}\right) + (T' - T) \\
=& u_{\ell,j} + \log\left(\frac{1 + N_f}{1 + N'_f}\right) + (T' - T).
\end{aligned}$$

Finally, using this expression we see that

$$\pi'_{\ell,j} = \frac{e^{\eta u_{\ell,j} + \eta\left[\log\left(\frac{1+N_f}{1+N'_f}\right) + (T'-T)\right]}}{\sum_{\ell',j'} e^{\eta u_{\ell',j'} + \eta\left[\log\left(\frac{1+N_f}{1+N'_f}\right) + (T'-T)\right]}} = \pi_{\ell,j},$$

confirming the guess.

D Relation to the Optimal Trade-Tax Literature

We can interpret the sales of houses to foreigners as exports that are paid for in units of the tradable consumption good. So, there is a connection between our results and those in the trade literature (see, e.g., Dixit, 1985, Caliendo and Parro, 2022, and references therein). In this appendix, we discuss this relation using a simple trade model.

Consider a world with a home country and $n \in \mathbb{R}$ identical foreign countries. Countries are endowed with two consumption goods, 1 and 2. The home country has y_1 units of good 1 and y_2 units of good 2. Each foreign country has y_1^* and y_2^* units of goods 1 and 2, respectively (throughout, we use stars to denote foreign-country variables). The representative agent of the home country has utility $u(c_1, c_2)$ and the representative agent of each foreign country has utility $u^*(c_1^*, c_2^*)$.

Abstracting from location choices and goods production, this model is analogous to our main model if we interpret one good as houses and the other as consumption.

D.1 Why is the optimal tax on houses bought by foreigners zero?

To compute the optimal trade tax, we assume that the home country can unilaterally impose a proportional tax τ on imports (or, equivalently, a subsidy to exports). The resulting tax revenue, T, is rebated back to the households of the home country. The budget constraints of home and foreign consumers are given by

$$c_1 - y_1 + (1 + \tau)p(c_2 - y_2) - T = 0, \tag{26}$$

$$c_1^* - y_1^* + p(c_2^* - y_2^*) = 0, \tag{27}$$

where p denotes the relative price of good 2. The equilibrium in this economy is described by two first-order conditions,

$$\frac{u_2}{u_1} = (1+\tau)p, \tag{28}$$

$$\frac{u_2^*}{u_1^*} = p, \tag{29}$$

the budget constraints, and the resource constraints

$$c_1 + nc_1^* = y_1 + ny_1^*, \tag{30}$$

$$c_2 + nc_2^* = y_2 + ny_2^*. \tag{31}$$

We compute the optimal tariff using the primal approach developed by Lucas and Stokey (1983). This approach involves choosing $\{c_1, c_2, c_1^*, c_2^*\}$ to maximize the utility in the home country subject to the resource constraints (30) and (31), the implementability condition

$$u_1^*(c_1^* - y_1^*) + u_2^*(c_2^* - y_2^*) = 0, \tag{32}$$

and a participation constraint for the foreign countries.[6]

$$u^*(c_1^*, c_2^*) \geq \overline{u}^*. \tag{33}$$

This constraint could potentially reflect the existence of un-modelled alternatives to trading with the home country which guarantee a level of utility of $\overline{u}^*$.

Theorem 1. *Let ϕ and λ_p denote the Lagrange multipliers associated with (32) and (33), respectively. The optimal tariff is given by*

$$\tau = \varphi \frac{\left(\frac{u_{22}^*}{u_2^*} - \frac{u_{21}^*}{u_1^*}\right)\left\{c_2^* - y_2^*\right\} - \left(\frac{u_{11}^*}{u_1^*} - \frac{u_{12}^*}{u_2^*}\right)\left\{c_1^* - y_1^*\right\}}{\lambda_p + \varphi\left[1 + \frac{u_{11}^*}{u_1^*}\left\{c_1^* - y_1^*\right\} + \frac{u_{21}^*}{u_1^*}\left\{c_2^* - y_2^*\right\}\right]} \neq 0. \tag{34}$$

[6] We can find the relative price, p and the policy variables, τ and T that satisfy all other equilibrium conditions. Equation (26) determines T, (28) determines τ, and (29) determines p. Using these values for p, τ and T, (27) is satisfied as long as (32) is also satisfied.

Suppose that $u^*(c_1^*, c_2^*) = [(c_1^*)^{1-\sigma} + (c_2^*)^{1-\sigma}]/(1-\sigma)$, then the optimal trade tax takes the form

$$\tau = \sigma\varphi \frac{\left\{\frac{c_1^*-y_1^*}{c_1^*}\right\} - \left\{\frac{c_2^*-y_2^*}{c_2^*}\right\}}{\lambda_p + \varphi\left[1 - \sigma\left\{\frac{c_1^*-y_1^*}{c_1^*}\right\}\right]}.$$

Suppose $\varphi > 0$. If foreigners export good 2, then $c_1^* > y_1^*$ and $c_2^* < y_2^*$. It follows that the optimal tariff is positive, i.e., $\tau > 0$. If foreigners export good 1, then $c_1^* < y_1^*$ and $c_2^* > y_2^*$. It follows that the optimal tariff is negative, i.e., $\tau < 0$.

This is the classical result that a country has an incentive to unilaterally tax imports or subsidize exports to manipulate terms-of-trade and obtain monopolistic rents. In our baseline model, the home country exports houses and imports traded goods. So, why do we find that taxing the houses purchased by foreigners is not optimal?

Note that, in deriving the optimal trade tax, we have assumed away the possibility of levying a lump-sum tax on foreigners. This possibility is not precluded in our main model, since the home country can impose an entry fee on foreign residents. Suppose that the home country can charge foreign countries a fee for the right to trade. The foreigners' budget constraint is

$$c_1^* - y_1^* + p(c_2^* - y_2^*) + T^* = 0. \tag{35}$$

It follows that we do not need to impose the implementability condition (32). So, the planning problem is to maximize the welfare of the home country subject to (30), (31), and (33).

Proposition 10. *Suppose that the home country can impose a right-to-trade fee, T^*. Then, the optimal trade tax is zero*

$$\tau = 0. \tag{36}$$

The right-to-trade fee is set so that foreign countries are indifferent between trading and not trading:

$$u^*(c_1^*, c_2^*) = \overline{u}^*. \tag{37}$$

When a lump-sum instrument is available, it is always better to use it to extract the gains from trade from foreign countries than to impose a distortionary tax on trade. The reason is as follows. A zero trade tax maximizes the gains from trade. These gains are then taxed away by the home country using the lump-sum instrument. This scheme resembles the optimal use of a two-part tariff by a monopolist. It is optimal for the monopolist to set the price equal to marginal cost and use a fixed fee to extract all the consumer surplus.

In our model, we impose no exogenous restrictions on the set of available instruments. Instead, the set of feasible instruments is determined by the primitive informational constraints faced by the planner or government. Since the planner can observe the country of origin, it can design a tax system that features a lump-sum tax on foreigners. The result above implies that it is not optimal to tax houses.

In our model, for any fixed number of foreign countries N_f, it is optimal for the home country to choose a non-zero entry fee $T_f \neq 0$ to extract the gains of foreign countries relative to their outside option.

D.2 Why is a zero entry fee optimal in our model?

The third part of proposition 3 states that in our main model, the optimal entry fee is zero. This result reflects the fact that the planner can choose the optimal number of foreigners, N_f.

To discuss the optimal entry fee using the trade model presented in this section, we allow the home country to choose the number of trading partners, n. Let λ_1 and λ_2 denote the Lagrange multipliers on resource constraints for good 1 and 2, respectively. The first-order condition for n is[7]

$$\lambda_1(y_1^* - c_1^*) + \lambda_2(y_2^* - c_2^*) = 0. \tag{38}$$

This equation equates marginal benefits with marginal costs. The marginal benefit

[7] We assume throughout that the solution is interior.

of an additional trading partner is the value of the goods they bring to the table $\lambda_1 y_1^* + \lambda_2 y_2^*$. The marginal cost is the value of goods that they consume $\lambda_1 c_1^* + \lambda_2 c_2^*$.

Combining (38) with the implementability condition (32), we find that

$$\frac{\lambda_1(y_1^* - c_1^*)}{u_1^*(y_1^* - c_1^*)} = \frac{\lambda_2(y_2^* - c_2^*)}{u_2^*(y_2^* - c_2^*)} \Leftrightarrow \frac{u_2}{u_1} = \frac{\lambda_2}{\lambda_1} = \frac{u_2}{u_1}. \tag{39}$$

It follows that if the home country cannot levy a lump-sum tax, T^*, then the optimal number of trading partners is such that $\tau = 0$.

In case the home country can choose $T^* \neq 0$, then we already know that $\tau = 0$ and $p = u_2^*/u_1^* = \lambda_2/\lambda_1$. It then follows from (38) that

$$(y_1^* - c_1^*) + \frac{u_2^*}{u_1^*}(y_2^* - c_2^*) = 0 \Leftrightarrow (y_1^* - c_1^*) + p(y_2^* - c_2^*) = 0 \Leftrightarrow T^* = 0. \tag{40}$$

So, even if the home country can levy a lump-sum tax, the optimal number of trading partners is such that $T^* = 0$.

These results are summarized in the following proposition, which echoes the results in Proposition 3.

Proposition 11. *Suppose that the home country can choose the number of trading partners, n. Then, the optimal number of trading partners is such that:*

1. *If the home country cannot impose a right-to-trade fee, then the optimal trade tax is zero, $\tau = 0$.*
2. *If the home country can impose a right-to-trade fee, then the optimal fee is zero, $T_f = 0$.*

It follows that the optimal number of trading partners is the same as in a laissez-faire solution. To explain why, let's say we start with too few trading partners. As we increase n, each trading partner receives a smaller portion of the home country's exports. The relative price of the exported good rises, and the home country benefits more from exports.[8] In order to satisfy the participation constraint, the home country

[8] Note also that the home country also exports more in total, so it consumes a lower amount of the exported good and more of the imported good.

must reduce the rights-to-trade fee. The benefit from increasing the value of exports is strictly higher than the reduction in fee revenue.

For analogous reasons, in our model, optimizing the number of foreigners N_f, requires setting the entry fee, T_f, to zero.