The Role of Consumption Taxes Under Incomplete Factor Taxation*

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June 2016

Abstract

The literature on the optimal taxation of capital has shown that when natural restrictions are imposed on the available taxes, the Chamley-Judd result may not hold. The analysis has however disregarded alternative instruments to complete the tax system. What is the role of consumption taxes in a world of incomplete factor taxation? To answer this question I consider different tax restrictions for both representative and heterogeneous agent economies. The consumption tax can serve as a specific income tax and overcome those restrictions, as long as it can act independently on the extra margin. When that is the case, the use of a consumption tax recovers the asymptotic zero tax on capital of Chamley and Judd and provides a welfare improvement.

Keywords: Capital taxation; Incomplete factor taxation; Consumption taxes

JEL Codes: E60; E61; E62; H21

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*E-mail: joao.guerreiro@ucp.pt, This paper was part of my thesis at Catolica Lisbon SBE. I am very grateful to my advisor Pedro Teles for his constant support and advice.

1 Introduction

Judd (1985) and Chamley (1986) showed that the optimal tax on capital income is zero in the steady state. Government revenue should be collected by taxing the remaining factors. This result has, however, been challenged on the basis that there could be restrictions on the taxes available. In fact, when the planner does not have enough instruments to affect each income source separately, the result may no longer hold.

The literature has provided examples under which this is true. Correia (1996) and Jones, Manuelli and Rossi (1997) focus, respectively, on a factor that is not taxed and two labor types taxed at the same rate. These restrictions imply a non-zero capital tax, even in the steady state. Reis (2011) also considers the case under which the government may not be able to distinguish, in proprietary's income, the compensation for managerial labor from that of capital ownership, and, therefore, both must be taxed at the same rate. This sort of restrictions can also be thought of as examples of incomplete sets of tax instruments, as defined by Chari and Kehoe (1999). The implementability set is no longer described solely by the resource constraints and the implementability condition.

In an actual, more complex, economy, it is very likely that policymakers are unable to perfectly differentiate every income source. These models would then suggest that capital taxes should be used. The prescription hinges, however, on the assumption that only capital and labor income taxes are available, and ignores a role for other instruments, such as the consumption tax. What is the role of consumption taxes in a world of incomplete factor taxation? Can it help overcome these restrictions?

This paper shows that if the policymaker is able to levy a consumption tax, he may think of it as a specific factor tax. The intuition is straightforward: being a tax on the use of income, it can be used to target a specific income source, leaving a complementary role for the remaining instruments. If the consumption tax is able to independently drive the needed wedges, then it will be able to overcome the tax restriction and recover the Chamley-Judd result. It is shown that whenever the relevant restrictions are such that the second labor type is not taxed or must be taxed at the same rate as capital, the consumption tax provides an independent instrument and recovers the zero-tax on capital. Moreover, by overcoming the restriction, the planner is able to generate a welfare improvement.

Instead, when the relevant restriction is such that the two labor types are taxed at the same rate, the introduction of the consumption tax does not provide an independent instrument. Therefore, even with consumption taxes the planner may choose to distort the intertemporal margin, which can be done through the use of capital taxes. Whether capital is optimally taxed in this case depends on two conditions: the first is that taxing capital is actually able to affect differently the allocations in labor types and the second is that the planner would actually want to set different taxes, thus that the restriction is relevant. Absent these two conditions, capital should not be taxed. These results are shown to be consistent across representative agent and heterogeneous agents economies, where each agent supplies a different labor input.

Recently, Straub and Werning (2015) have shown that the result of Chamley and Judd will not hold generally. In particular, they have shown that the restriction that capital cannot be taxed at more than 100% may be binding forever, so that there will be permanent full capital taxation. Chari, Nicolini and Teles (2016) show that the use of alternative taxes, such as consumption and dividend taxes, recover the Chamley-Judd result. They show that Straub and Werning's results are driven by initial confiscatory intents and that, absent these effects, the optimal wedges are determined solely by the relevant elasticities. Once these elasticities are constant, as in an interior steady state, the intertemporal margins should not be distorted and, therefore, capital should not be taxed. Backed by these results, in this paper it will be assumed that an interior steady state exists and that initial confiscatory intents have been dealt with during the transition.

The paper proceeds as follows: Section 2 develops the representative agent framework and considers optimal fiscal policy, under the different tax system restrictions. Section 3 shows that this also holds for an heterogeneous agents economy. Finally, Section 4 concludes.

2 Representative Agent Economy

Consider a deterministic neoclassical growth model modified to include a third input of production, which is referred to as a second labor type. The preferences of the household are defined over consumption, c_t, the first type of labor, n_t, and the second labor type, l_t,

$$U = \sum_{t=0}^{\infty} \beta^t u(c_t, n_t, l_t),$$

where $u_{ct} > 0$, $u_{nt} < 0$, $u_{lt} < 0$, $u_{cc,t} < 0$, $u_{nn,t} > 0$ and $u_{ll,t} > 0$.

Government spending, g_t, is exogenous and the production technology is CRS, using both labor types and capital, k_t. The resource constraints are given by

$$c_t + g_t + k_{t+1} = f(k_t, n_t, l_t) + (1-\delta)k_t, \tag{1}$$

where δ is the depreciation rate of capital.

The available tax instruments are capital income taxes, τ_t^k, labor taxes, τ_t^n, and consumption taxes τ_t^c. A tax on the second labor, τ_t^l, will also be considered, and must verify some restriction depending on the situation.

The household must verify the flow of funds constraints, defined as

$$(1+\tau_t^c)c_t + k_{t+1} + b_{t+1} = (1+r_t)b_t + [1+(1-\tau_t^k)(u_t-\delta)]k_t + (1-\tau_t^n)w_t^n n_t + (1-\tau_t^l)w_t^l l_t,$$

where b_t defines bonds issued by the government in period $t-1$, that yield the return r_t. u_t is the rental rate on capital and w_t^i defines the wage rate for each labor type.

The household maximizes utility subject to the flow of funds constraints and a no-Ponzi games condition. This agent equates the marginal rates of substitution to the ratio of prices

$$\frac{u_{ct}}{\beta u_{ct+1}} = \frac{1+\tau_t^c}{1+\tau_{t+1}^c}(1+r_{t+1}), \tag{2}$$

$$-\frac{u_{nt}}{u_{ct}} = \frac{(1-\tau_t^n)w_t^n}{1+\tau_t^c}, \tag{3}$$

$$-\frac{u_{lt}}{u_{ct}} = \frac{(1-\tau_t^l)w_t^l}{1+\tau_t^c}. \tag{4}$$

An arbitrage condition between the return on bonds and that on capital must be verified

$$1 + r_{t+1} = 1 + (1-\tau_{t+1}^k)(u_{t+1}-\delta). \tag{5}$$

The combination of this, the flow of funds and the transversality condition allows us to write the intertemporal budget constraint

$$\sum_{t=0}^{\infty} Q_t \big[(1+\tau_t^c)c_t - (1-\tau_t^n)w_t^n n_t - (1-\tau_t^l)w_t^l l_t\big] = \mathcal{W}_0, \tag{6}$$

where $Q_t \equiv \left(\prod_{s=1}^{t}(1+r_s)\right)^{-1}$, $Q_0 \equiv 1$ and $\mathcal{W}_0 \equiv (1+r_0)b_0 + [1+(1-\tau_0^k)(U_0-\delta)]k_0$.

The representative firm maximizes profits subject to the production function, implying it equates the remuneration of each factor to its marginal productivity, i.e. $u_t = f_{kt}$, $w_t^n = f_{nt}$ and $w_t^l = f_{lt}$. Profits are zero.

The competitive equilibrium can thus be defined by (1), (6) and the conditions

$$\frac{u_{ct}}{\beta u_{ct+1}} = \frac{1+\tau_t^c}{1+\tau_{t+1}^c}(1+(1-\tau_{t+1}^k)(f_{kt+1}-\delta)), \tag{7}$$

$$-\frac{u_{nt}}{u_{ct}} = \frac{(1-\tau_t^n)f_{nt}}{1+\tau_t^c}, \tag{8}$$

$$-\frac{u_{lt}}{u_{ct}} = \frac{(1-\tau_t^l)f_{lt}}{1+\tau_t^c}, \tag{9}$$

$$1+r_{t+1} = 1+(1-\tau_{t+1}^k)(f_{kt+1}-\delta). \tag{10}$$

2.1 Optimal Policy without Restrictions on Taxes

Consider the benchmark case where the tax on the second type of labor is not restricted. It is a free variable which can be used to target that second source of income independently. In this case, the tax system is complete and the Chamley-Judd result is valid. Furthermore, the consumption tax is simply an extra-instrument which needs not be used. Therefore, let us consider $\tau_t^c = 0$.

As in Lucas and Stokey (1983), it is possible to construct the implementability condition using the intertemporal budget constraint of the household. This is given by

$$\sum_{t=0}^{\infty} \beta^t [u_{ct}c_t + u_{nt}n_t + u_{lt}l_t] = u_{c0}\mathcal{W}_0. \tag{11}$$

The implementability set is defined by the resource constraints and (11). All other conditions can be met by using other variables. The equilibrium condition (7) can be satisfied by a tax on capital, τ_{t+1}^k, (8) can be satisfied by τ_t^n and (9) can be satisfied by a tax on the second labor τ_t^l. (10) defines an intertemporal price, r_{t+1}.

The planner maximizes the household's utility subject to the implementability set. To make the problem non-trivial let us set the initial tax on capital to an arbitrary level. Defining μ as the Lagrange multiplier of (11) and $\beta^t\lambda_t$ as the multiplier of the resource constraints, the first order conditions for consumption and capital accumulation can be written as

$$u_{ct}[1+\mu(1-\sigma_t^{cc}-\sigma_t^{nc}-\sigma_t^{lc})]=\lambda_t, \tag{12}$$

$$\lambda_t=\beta\lambda_{t+1}(f_{kt+1}+1-\delta), \tag{13}$$

where $\sigma_t^{ij}\equiv -\frac{u_{ij,t}}{u_{jt}}i_t$ for $i,j=c,n,l$.

The intertemporal marginal condition for the Ramsey planner can be written as

$$\frac{u_{ct}}{\beta u_{ct+1}}=\frac{1+\mu(1-\sigma_{t+1}^{cc}-\sigma_{t+1}^{nc}-\sigma_{t+1}^{lc})}{1+\mu(1-\sigma_t^{cc}-\sigma_t^{nc}-\sigma_t^{lc})}(1-\delta+f_{kt+1}). \tag{14}$$

As shown in Chari et al. (2016), since the elasticities are constant in an interior steady state, the intertemporal margin should not be distorted and thus the tax on capital should be set to zero.

Whenever the specific income taxes are free, they can be used to tax each income source. Consumption taxes in such an environment are redundant instruments, which need not be used. What if there are natural restrictions on those income taxes? Can the consumption tax help overcome those restrictions?

The paper proceeds by considering three possible restrictions: when the second labor is not taxed, when it is taxed at the same rate as capital and when it is taxed at the same rate as the first labor.

2.2 Optimal Policy when $\tau_t^l=0$

Suppose that the restriction is such that the second labor is not taxed, i.e. $\tau_t^l=0$. The intuition for how the consumption taxes enter in this problem is straightforward. Even if the third income source is not taxed, the planner will be able to use the consumption tax to target the use of all income. Then, it can use the remaining taxes to affect differently the other margins.

We construct the implementability condition using the intertemporal budget constraint of the household. This is given by

$$\sum_{t=0}^{\infty}\beta^t[u_{ct}c_t+u_{nt}n_t+u_{lt}l_t]=-\frac{u_{l0}}{f_{l0}}\mathcal{W}_0. \tag{15}$$

The implementability set is defined by the resource constraints and (15). All other conditions can be met by using other variables. The equilibrium condition (7) can be satisfied by finding a tax on capital, τ^k_{t+1}, (8) can be satisfied by τ^n_t and (9) by a consumption tax τ^c_t. (10) defines an intertemporal price, r_{t+1}.

To make the problem interesting, restrict the initial tax on capital to an arbitrary level, possibly zero. Define μ to be the multiplier of (15) and $\beta^t\lambda_t$ to be the multiplier of the resource constraints. The first order conditions with respect to consumption, the second type of labor and capital are given by

$$u_{ct}[1+\mu(1-\sigma^{cc}_t-\sigma^{nc}_t-\sigma^{lc}_t)] = \lambda_t, \tag{16}$$

$$u_{lt}[1+\mu(1-\sigma^{cl}_t-\sigma^{nl}_t-\sigma^{ll}_t)] = -\lambda_t f_{lt}, \tag{17}$$

$$\lambda_t = \beta\lambda_{t+1}(1-\delta+f_{kt+1}). \tag{18}$$

where $\sigma^{ij}_t \equiv -\frac{u_{ij,t}}{u_{jt}} i_t$ for $i,j = c,n,l$.

This then implies that the optimal wedges in the intertemporal and the second labor's intratemporal margin can be seen through

$$-\frac{u_{lt}}{u_{ct}} = \frac{1+\mu(1-\sigma^{cc}_t-\sigma^{nc}_t-\sigma^{lc}_t)}{1+\mu(1-\sigma^{cl}_t-\sigma^{nl}_t-\sigma^{ll}_t)} f_{lt}, \tag{19}$$

$$\frac{u_{ct}}{\beta u_{ct+1}} = \frac{1+\mu(1-\sigma^{cc}_{t+1}-\sigma^{nc}_{t+1}-\sigma^{lc}_{t+1})}{1+\mu(1-\sigma^{cc}_t-\sigma^{nc}_t-\sigma^{lc}_t)}(1-\delta+f_{kt+1}). \tag{20}$$

Evaluating these conditions at an interior steady state, where all these elasticities are constant, implies that the marginal rate of intertemporal substitution should not be distorted and the intra-temporal margin (19) should have a constant distortion.

How does the implementation look like? (19) implies that $\tau^c_t \to \tau^c$. Then, since a constant consumption tax does not give rise to an intertemporal distortion, the capital tax should be zero in the limit. Consumption taxes and labor taxes can both be used to drive different wedges on each intratemporal margin. If it was the case that the two labor types were perfect substitutes for the agent, the benevolent planner would only use the consumption tax to finance government expenditures, as the optimal wedges on the intratemporal margins would be the same.

Furthermore, since the introduction of the consumption tax makes the restriction irrelevant, it generates a welfare improvement in this economy. This can be easily seen from the fact that the Ramsey problem requires one less constraint.

2.3 Optimal Policy when $\tau_t^l = \tau_t^k$

Let us now assume that the third factor has to be taxed at the capital tax rate, i.e. $\tau_t^l = \tau_t^k$. The literature has motivated this case by an inability to distinguish managerial labor's income apart from returns to capital ownership, therefore justifying the same tax rate.

This problem is significantly different from the previous one, in the sense that the planner will now have to choose a tax to cover two different sources of income. Taxes do not target specifically only one factor. Nevertheless, also under these sort of restrictions the consumption tax provides an independent instrument, recovering the Chamley-Judd result.

Under this scenario the set of implementable allocations can be defined by the resource constraints and the implementability condition, which is given by

$$\sum_{t=0}^{\infty} \beta^t [u_{ct} c_t + u_{nt} n_t + u_{lt} l_t] = -\frac{u_{l0}}{(1-\tau_0^k) f_{l0}} \mathcal{W}_0. \tag{21}$$

All other conditions can be met by using other variables. As in the previous case, the equilibrium condition (7) can be satisfied by a tax on capital, τ_{t+1}^k, (8) can be satisfied by τ_t^n and (9) can be satisfied by a consumption tax τ_t^c. (10) defines an intertemporal price, r_{t+1}.

Just like the previous case, the asymptotic tax on consumption converges to a constant and therefore it does not distort the intertemporal margin. The asymptotic tax on capital income is zero.

2.4 Optimal Policy when $\tau_t^l = \tau_t^n$

When the relevant restriction is such that two different labor inputs must be taxed at the same rate, the long-run capital tax may be non-zero. The restriction is $\tau_t^l = \tau_t^n$.

Unlike the previous cases, the introduction of a consumption tax in this framework provides no improvement. Intuitively, this is because the consumption tax cannot be used to target independently one of the margins, as it did in the previous cases. If we consider the marginal rate of substitution between n and l it can be understood that no tax instrument is available to optimally affect it,

$$u_{nt} f_{lt} = u_{lt} f_{nt}. \tag{22}$$

Since this condition must now be added as an additional restriction to the planner's problem, it is not characterized by a complete set of instruments.

The implementability is defined by the resource constraints, (22) and the implementability condition,

$$\sum_{t=0}^{\infty} \beta^t [u_{ct} c_t + u_{nt} n_t + u_{lt} l_t] = \frac{u_{c0}}{(1+\tau_0^c)} \mathcal{W}_0. \tag{23}$$

All other conditions can be met by other variables. The equilibrium condition (7) can be satisfied by a consumption tax in $t+1$, given some level in period t. (8) can be satisfied by τ_t^n. Finally, (10) defines an intertemporal price, r_{t+1}.

This implementation of the Ramsey solution does not use the capital tax. Under this scenario, it is simply a redundant extra tool. Given this, it can always be set to zero, $\tau_t^k = 0$, as time-varying consumption taxes can replace its role. The same characteristic is found in Chari et al. (2016). As in their paper, capital taxation is to be understood as an optimal wedge on the intertemporal margin.

To make the problem interesting, restrict the initial taxes to an arbitrary level, possibly zero. Define $\beta^t \eta_t$ as the multiplier of (22), μ to be the multiplier of (23) and $\beta^t \lambda_t$ as the multiplier of the resource constraints. The first order conditions for this case are given by

$$u_{ct}[1 + \mu(1 - \sigma_t^{cc} - \sigma_t^{nc} - \sigma_t^{lc})] + \eta_t [u_{nc,t} f_{lt} - u_{lc,t} f_{nt}] = \lambda_t, \tag{24}$$

$$u_{nt}[1+\mu(1-\sigma_t^{cn}-\sigma_t^{nn}-\sigma_t^{ln})]+\eta_t[u_{nn,t}f_{lt}+u_{n,t}f_{ln,t}-u_{ln,t}f_{nt}-u_{l,t}f_{nn,t}] = -\lambda_t f_{nt}, \tag{25}$$

$$u_{lt}[1 + \mu(1 - \sigma_t^{cl} - \sigma_t^{nl} - \sigma_t^{ll})] + \eta_t [u_{nl,t} f_{lt} + u_{nt} f_{ll,t} - u_{ll,t} f_{nt} - u_{lt} f_{nl,t}] = -\lambda_t f_{lt}, \tag{26}$$

$$\lambda_t = \beta \lambda_{t+1} \Big[1 - \delta + f_{kt+1} + \eta_{t+1} u_{nt+1} f_{lt+1} \Big(\frac{f_{nk,t+1}}{f_{nt+1}} - \frac{f_{lk,t+1}}{f_{lt+1}} \Big) \Big]. \tag{27}$$

Suppose the allocations converge to an interior steady state in c, n and l. Then the multipliers λ_t and η_t also converge to some constant value. In such a steady state, the intertemporal margin for consumption is simply β^{-1}. Condition (27) implies that capital should, in general, be taxed, even at a steady state,

$$1 - \delta + f_k = \frac{1}{\beta} - \eta u_n f_l \Big(\frac{f_{nk}}{f_n} - \frac{f_{lk}}{f_{lt}} \Big). \tag{28}$$

However, for capital taxes to be optimally used two conditions must hold. As was noted by Jones et al., a Cobb-Douglas production function will verify that $\frac{f_{nk}}{f_n} - \frac{f_{lk}}{f_{lt}} = 0$. In general, any production function that is weakly separable in labor types, $f(k, g(n,l))$, will have that a unitary increase in the capital stock results in the same proportionate growth of each labor's productivity. If such is the case, capital taxation is not optimal. This is the case under which a distortion of the steady state's optimal amount of capital does not affect differently the allocations in each labor type. Unable to affect labor types differently, the optimal policy is not to distort the intertemporal margin.

Another condition for capital taxes to be optimally used is that $\eta \neq 0$. Suppose that the utility function is weakly separable and homothetic in labor types. Under this assumption, the planner would optimally choose to set the same tax on each labor type, regardless of their productive characteristics. Therefore, restricting taxes to be equal becomes a non-binding restriction, $\eta_t = 0$. A special case occurs when the labor types are perfect substitutes in the utility function, $u(c_t, n_t + l_t)$.

Absent these conditions, the intertemporal margin should be distorted, and thus capital should be taxed. A possible implementation will keep the consumption tax and the labor tax constant in the steady state, and use the capital tax to drive the needed wedge in the intertemporal margin.

3 Heterogeneous Agents Economy

This section shows that the same ideas hold in a model with two heterogeneous agents. It is very close to the framework of Judd (1985), but allowing for unrestricted government debt and endogenous decisions on labor and consumption for each agent. Let us keep Judd's assumption that only agent 2, the capitalist, is able to save. The preferences over consumption c_t^i and labor n_t^i for each household i are assumed to be represented by

$$U^i = \sum_{t=0}^{\infty} \beta^t u^i(c_t^i, n_t^i). \tag{29}$$

The technology is equivalent to the previous setting, given by

$$c_t^1 + c_t^2 + g_t + k_{t+1} = f(k_t, n_t^1, n_t^2) + (1-\delta)k_t. \tag{30}$$

Household 1 faces a per-period budget constraint, given by

$$(1+\tau_t^c)c_t^1 \leq (1-\tau_t^n)w_t^1 n_t^1 + \mathbb{T}_t, \tag{31}$$

where τ_t^n denotes the tax on labor income and $\mathbb{T}_t$ denotes a transfer given to that agent. The solution to this problem must verify that

$$-\frac{u_{nt}^1}{u_{ct}^1} = \frac{(1-\tau_t^n)w_t^1}{1+\tau_t^c}, \tag{32}$$

and (31) holds in equality.

Household 2, on the other hand, owns the capital in this economy. Every period it is subject to the flow of funds constraint

$$(1+\tau_t^c)c_t^2 + k_{t+1} + b_{t+1} = b_t(1+r_t) + [1+(1-\tau_t^k)(u_t-\delta)]k_t + (1-\tau_t^2)w_t^2 n_t^2,$$

where b_t defines bonds issued by the government in period $t-1$, that yield the return r_t. u_t is the rental rate on capital and w_t^2 defines the wage rate for the labor of this household. τ_t^2 denotes a tax on household 2's wage income, which must verify a restriction depending on the situation.

The household solves the problem of maximizing utility subject to these constraints and a no-Ponzi games condition. The optimality conditions are given by

$$\frac{u_{ct}^2}{\beta u_{ct+1}^2} = \frac{1+\tau_t^c}{1+\tau_{t+1}^c}(1+r_{t+1}), \tag{33}$$

$$-\frac{u_{nt}^2}{u_{ct}^2} = \frac{(1-\tau_t^2)w_t^2}{1+\tau_t^c}. \tag{34}$$

An arbitrage condition between the return to bonds and returns on capital investment must be verified

$$1+r_{t+1} = 1+(1-\tau_{t+1}^k)(u_{t+1}-\delta). \tag{35}$$

The combination of this, the flow of funds and the transversality conditions allows us to write the intertemporal budget constraint for this household

$$\sum_{t=0}^{\infty} Q_t\big[(1+\tau_t^c)c_t^2 - (1-\tau_t^2)w_t^2 n_t^2\big] = \mathcal{W}_0^2, \tag{36}$$

where $Q_t \equiv \left(\prod_{s=1}^{t}(1+r_s)\right)^{-1}$, $Q_0 \equiv 1$ and $\mathcal{W}_0^2 \equiv b_0(1+r_0)+k_0[1+(1-\tau_0^k)(U_0-\delta)]$.

The representative firm maximizes profits subject to the CRS production function, implying it equates the remuneration of each factor to its marginal productivity, i.e. $u_t = f_{kt}$, $w_t^1 = f_{n1,t}$ and $w_t^2 = f_{n2,t}$.

The competitive equilibrium can thus be defined by (30), (31) in equality, (36) and the conditions

$$\frac{u_{ct}^2}{\beta u_{ct+1}^2} = \frac{1+\tau_t^c}{1+\tau_{t+1}^c}(1+(1-\tau_{t+1}^k)(f_{kt+1}-\delta)), \tag{37}$$

$$\frac{u_{nt}^1}{u_{ct}^1} = -\frac{(1-\tau_t^n)f_{n1,t}}{1+\tau_t^c}, \tag{38}$$

$$\frac{u_{nt}^2}{u_{ct}^2} = -\frac{(1-\tau_t^2)f_{n2,t}}{1+\tau_t^c}, \tag{39}$$

$$1+r_{t+1} = 1+(1-\tau_{t+1}^k)(f_{kt+1}-\delta). \tag{40}$$

3.1 Optimal Policy without Restrictions on Taxes

Very similarly to the representative agent model, whenever all taxes are unrestricted the consumption tax does not need to be used. Furthermore, the Chamley-Judd result holds.

We think of the planner as maximizing a weighted average of utilities, where ω_i denotes the weight given to agent i, such that $\omega_i \geq 0$ and $\omega_1 + \omega_2 = 1$. The implementability set under this scenario is described by (30) and the implementability condition

$$\sum_{t=0}^{\infty} \beta^t [u_{ct}^2 c_t^2 + u_{nt}^2 n_t^2] = u_{c0}^2 \mathcal{W}_0^2. \tag{41}$$

It can be understood that all other conditions can be satisfied by other variables. The equilibrium condition (31) can be satisfied by a transfer to the worker, (37) can be met by a capital income tax τ_{t+1}^k, (38) by a labor tax τ_t^n, and (39) can be satisfied by a tax on the capitalist's labor income, τ_t^2. Finally, (40) defines a price r_{t+1}.

The marginal rate of intertemporal substitution for the Ramsey planner is given by

$$\frac{u_{ct}^2}{\beta u_{ct+1}^2} = \frac{\omega_2 + \mu(1-\sigma_{2,t+1}^{cc} - \sigma_{2,t+1}^{nc})}{\omega_2 + \mu(1-\sigma_{2,t}^{cc} - \sigma_{2,t}^{nc})}[1-\delta+f_{kt+1}]. \tag{42}$$

Also in the case with agent heterogeneity, the result of Chari et al. (2016) holds. The characteristics of taxation are determined by the relevant elasticities. Whenever these are constant, as in an interior steady state, the intertemporal margin should not be distorted and thus capital taxes should be zero.

Just as in the representative agent scenario, when the specific income taxes are free, they can be used to tax each income source. Consumption taxes in such an environment are redundant instruments.

Let us proceed by considering three possible restrictions: when the capitalist's labor is not taxed, when it is taxed at the same rate as capital and when it is taxed at the same rate as the worker's labor.

3.2 Optimal Policy when $\tau_t^2 = 0$

Suppose that no tax can be levied on the capitalist's wage income. This implies that $\tau_t^2 = 0$. Also in this case of agent heterogeneity, absent consumption taxes, the planner will choose to tax capital. However, once consumption taxes are introduced the result of Judd (1985) is recovered.

The planner maximizes a weighted average of utilities. The implementability set under this scenario is described by (30) and the implementability condition

$$\sum_{t=0}^{\infty} \beta^t [u_{ct}^2 c_t^2 + u_{nt}^2 n_t^2] = -\frac{u_{n0}^2}{f_{n2,0}} \mathcal{W}_0^2. \tag{43}$$

It can be understood that all other conditions can be satisfied by other variables. The equilibrium condition (31) can be satisfied by a transfer to the worker, (37) by a capital income tax τ_{t+1}^k, (38) by a labor tax τ_t^n, and (39) can be satisfied by a consumption tax τ_t^c. Finally, (40) defines a price r_{t+1}.

The optimal wedges for the Ramsey planner can be seen from the optimality conditions:

$$-\frac{u_{nt}^1}{u_{ct}^1} = f_{n1,t} \tag{44}$$

$$-\frac{u_{nt}^2}{u_{ct}^2} = \frac{\omega_2 + \mu(1 - \sigma_{2,t}^{cc} - \sigma_{2,t}^{nc})}{\omega_2 + \mu(1 - \sigma_{2,t}^{cn} - \sigma_{2,t}^{nn})} f_{n2,t}, \tag{45}$$

$$\frac{u_{ct}^2}{\beta u_{ct+1}^2} = \frac{\omega_2 + \mu(1 - \sigma_{2,t+1}^{cc} - \sigma_{2,t+1}^{nc})}{\omega_2 + \mu(1 - \sigma_{2,t}^{cc} - \sigma_{2,t}^{nc})} [1 - \delta + f_{kt+1}]. \tag{46}$$

We can understand from (45) that the asymptotic tax on consumption is constant.

Since a constant consumption tax yields no intertemporal distortion, and given that this margin should not be distorted when elasticities are constant, the asymptotic tax on capital income should be set to zero. The labor tax should be set to revert the distortion in household 1's intratemporal margin, caused by the consumption tax.

3.3 Optimal Policy when $\tau_t^2 = \tau_t^k$

Suppose a situation where the government cannot distinguish income from capital ownership from wages of managerial work. Thus, a single income tax must be levied on both capital returns and entrepreneur's labor income, i.e. $\tau_t^2 = \tau_t^k$. Should capital be taxed to also target that other source of income? Absent consumption taxes this is true. However, once consumption taxes are considered, these allow to recover the asymptotic zero tax.

The implementability set under this scenario is described by (30) and the implementability condition

$$\sum_{t=0}^{\infty} \beta^t [u_{ct}^2 c_t^2 + u_{nt}^2 n_t^2] = -\frac{u_{n0}^2}{(1-\tau_0^k) f_{n2,0}} \mathcal{W}_0^2. \tag{47}$$

It can be understood that all other conditions can be satisfied by other variables. The equilibrium condition (31) can be satisfied by a transfer to the worker, (37) can be met by a capital income tax τ_{t+1}^k, (38) can be satisfied by a labor tax τ_t^n, and (39) by a consumption tax τ_t^c. Finally, (40) defines a price r_{t+1}.

This yields essentially the same results as the previous scenario. The implementation will have the consumption tax constant when the relevant elasticities are constant and thus the capital tax should be set to zero.

3.4 Optimal Policy when $\tau_t^2 = \tau_t^n$

What if the entrepreneur's labor income is taxed at the same rate as the worker's? In this situation, just as in the analog representative agent economy, the planner is restricted by one more condition. The new restriction implies that the taxes on the intratemporal margins are the same for the two agents,

$$\frac{u_{ct}^1}{u_{nt}^1} f_{n1,t} = \frac{u_{ct}^2}{u_{nt}^2} f_{n2,t}. \tag{48}$$

The implementability set is characterized by (30), (48) and the implementability condition

$$\sum_{t=0}^{\infty} \beta^t [u^2_{ct} c^2_t + u^2_{nt} n^2_t] = \frac{u^2_{c0}}{1 + \tau^c_0} \mathcal{W}^2_0. \tag{49}$$

All other conditions can be satisfied by other variables. The equilibrium condition (31) can be satisfied by a transfer to the worker, (37) can be met by a consumption tax τ^c_{t+1}, given τ^c_t, and (38) can be satisfied by a labor tax τ^n_t. Finally, (40) defines a price r_{t+1}.

As in the representative agent economy, the implementation does not need to use the capital income tax, as time-varying consumption taxes can replace its use.

Furthermore, the inclusion of the extra restriction in the planning problem will imply an optimal distortion on the intertemporal margin, which can be seen by

$$1 - \delta + f_k = \frac{1}{\beta} - \eta \frac{u^1_c}{u^1_n} f_{n1} \Big(\frac{f_{n1k}}{f_{n1}} - \frac{f_{n2k}}{f_{n1}} \Big). \tag{50}$$

Also in this case the same considerations on the production function apply. If the production function is weakly separable in labor types, the intertemporal margin should not be distorted and, hence, capital should not be taxed.

4 Conclusions

The literature has provided several examples on how the tax system's incompleteness can overturn the Chamley-Judd result. Nevertheless, these papers have only focused on factor taxation, disregarding other tax instruments. This work discusses the consequences of introducing consumption taxes in representative agent and heterogeneous agent economies with incomplete factor taxation.

In the representative agent scenario, I modify the simple neoclassical growth model with taxes, to include two different types of labor. The tax restrictions considered are on how the second labor type can be taxed. For the heterogeneous agent economy, the framework of Judd (1985) is extended, allowing the worker to make an endogenous decision on labor and consumption. The capitalist can save in capital and government bonds and supplies a second labor input. Both capital and labor incomes are taxed. The same restrictions on the way this second labor is taxed are considered.

When the second labor is not taxed, the asymptotic tax on capital may be non-zero, whenever consumption taxes are not included in the problem. We show that once these taxes are introduced, they allow for a targeting of all income, even the one not taxed, thus recovering the steady state zero-tax on capital.

The second restriction analyzed is the case in which the government cannot distinguish capital income from wages of entrepreneurial labor. In this case, capital should be taxed to also target the second labor type. However, when the consumption tax is introduced the Chamley-Judd result is recovered, since the targeting of entrepreneurial labor can be done with that tax.

It can thus be concluded that if the relevant tax system restrictions are of this sort, the policymaker needs only use the consumption tax as a specific factor tax. By doing so it recovers the Chamley-Judd result of zero capital taxation and generates a welfare improvement.

The same ideas do not hold when the restriction is such that both labor types must be taxed at the same labor tax rate. In this case, the consumption tax does not provide an independent instrument. The planner cannot independently target each factor. A new restriction must be added to the planner's problem, which will then imply that capital should, in general, be taxed or subsidized.

The results suggest that factor taxation should not be analyzed independently of the remaining instruments. The consumption tax, for instance, as a tax on the use of income provides a very simple instrument to overcome potential tax restrictions.

References

[1] **Chamley, Cristophe**, "Optimal Taxation of Capital Income in General Equilibrium with Infinite Lives", *Econometrica*, 1986, 54 (3), pp. 607-622.

[2] **Chari, V. V., and Patrick Kehoe**, "Optimal Fiscal and Monetary Policy", In *Handbook of Macroeconomics*, vol. 1C, edited by John Taylor and Michael Woodford, 1999, North-Holland.

[3] **Chari, V. V., Juan Pablo Nicolini, and Pedro Teles**, "More on the Optimal Taxation of Capital", *mimeo*, 2016, Católica Lisbon SBE and Banco de Portugal.

[4] **Correia, Isabel**, "Should capital income be taxed in the steady state?", *Journal of Public Economics*, 1996, 60, 147-151.

[5] **Jones, Larry E., Rodolfo E. Manuelli and Peter E. Rossi**, "On the Optimal Taxation of Capital Income", *Journal of Economic Theory*, 1997, 93-117.

[6] **Judd, Kenneth L.**, "Redistributive Taxation in a simple perfect foresight model". *Journal of Public Economics*, 1985, 28, 59-83.

[7] **Lucas, Robert E., Jr. and Nancy L. Stokey**, "Optimal Fiscal and Monetary Policy In An Economy Without Capital". *Journal of Monetary Economics*, 1983, 12, 55-93.

[8] **Reis, Catarina**, "Entrepreneurial Labor and Capital Taxation". *Macroeconomic Dynamics*, 2011, 15, 326-335.

[9] **Straub, Ludwig and Ivan Werning**, "Positive Long Run Capital Taxation: Chamley-Judd Revisited", mimeo, MIT, 2015.